# MODAL ANALYSIS OF A BEAM-LIKE STRUCTURE USING PIEZOELECTRIC MATERIAL (PVDF)

**Prazzo, C. E., carloseduardoprazzo@gmail.com**
**Pereira, J. A., japereira@dem.feis.unesp.br**
**Turra, A. E., turra@dem.feis.unesp.br**
**Lopes Jr., V., vicente@dem.feis.unesp.br**
*Faculdade de Engenharia de Ilha Solteira, UNESP – Universidade Estadual de São Paulo, Departamento de Engenharia Mecânica, GMSINT*

***Abstract.*** *Nowadays, piezoelectric materials have been consolidating like a new technology that shows a large potential of application for different engineering areas. The structural tests are a prominent application area for piezoelectric materials, including, the experimental modal analysis, that is a consolidated technique to evaluate the dynamic behavior of the structures based on the input-output data relationship. The incorporation of the smart materials (piezoelectric materials as PFVD – Polyvinylidene Fluoride) in this technique is a natural tendency. However, the use of this technology in the experimental modal analysis area is still very incipient and there are only few papers using smarts materials as sensors and actuators. This paper aims to present some contribution in this area by using piezoelectric sensors, instead of the conventional ones like accelerometer for modal analysis of structures. It discusses the theoretical concepts and presents the results of modal analysis tests conducted in a beam like structure using piezoelectric material as sensor and as actuator.*

***Keywords****: modal analysis, pvdf, piezoelectric material*

## 1. INTRODUCTION

The experimental modal analysis is a part of the dynamic structures field interested in the structural behavior of the mechanical systems and structures. The technique is based in the input-output relationship, which requires the measuring of the excitation and the responses of the structure in order to calculate the frequencies response functions (FRF(s)) of the system or its equivalent time domain impulse response functions (IRF(s)). These functions are used to extract the modal parameters of the model. Many aspects of the technique have been investigated over the past thirty years, and there are an extensive literature covering the main aspects of the theory and experimental tests (Ewins, 1984, Maia and Silva, 1997; Mcconnell and Varoto, 2008).

One of the challenges of the technique is to quantify the excitation of the system, i.e., measure the value of the input forces. In some cases this is not used to estimate the FRF(s) or their equivalent time domain since most of the identification methods makes use of these functions to estimate the modal parameters of the models. For example, an airplane in operation is subjected to an enormous quantity of strength and different sources of excitations which are practically impossible to be measured. Alternatively, the modal analysis technique has been gained importance as technique that does not use directly the input forces, as can be seen in the literature. Various methods to estimate the parameters of the model without measuring the input force has been discussed (Peeters, and De Roeck, 2001; Herlufsen et. all, 2005, Freitas and Pereira, 2007) and this relatively new approach is becoming an important research line in the structure dynamic field.

In this context, the smart materials appear to be an interesting subject to be studied e used in the experimental modal analysis field. The Smart Material is defined as the materials that exhibit the coupling field between multi physics domains, such as mechanical, electrical, thermal or magnetic. They can convert, for example, an electric signal in mechanic strain and vice-versa. The coupling is exhibited because the piezoelectric material that provides an electric displacement when subject to a mechanical stress and also it provides a mechanical strain due to an applied electric field. This mechanic-electric coupling is called direct-effect and the electric-mechanical coupling is called indirect-effect (LEO, 2007).

The development of the measurement and instrumentation area in the last years has been enormous, and one of the biggest innovations is the possibility of the uses of piezoelectric materials, such as PZT (Lead Zirconate Titanate) or PVDF (Polyvinylidene Fluoride) as sensors or as actuators, taking the advantage of the direct and indirect effect of those piezoelectric materials. Therefore, smart materials in the experimental modal analysis tests appears to be a natural issue to be exploited since it can attend the demands of the conventional technique based in the input-output relationship and also it can be interesting for the output only-based modal analysis.

This paper has for purpose to contribute in this area, studying the modal analysis using piezoelectric sensor of the type PVDF that are flexible and could become part of the host structure instead of, the conventional sensors as accelerometers. The paper presents the experimental modal analysis of a beam like structure using different kinds of sensors/actuator and gives some insight of the difference of estimated modes shapes by using piezoelectric materials. Also are present a formulation that shows the relation between the displacement mode shapes and de slope difference modal shapes obtained with a piezoelectric material. Finally, it shows the results obtained for the different used approaches.

## 2. FUNDAMENTALS OF MODAL ANALYSIS

The study of the dynamic of the structures has gained prominence in the last decades for the scientific community and for the industry. The knowledge of the dynamic structural behavior of the structures is desired in almost projects. The structures can, depending of their operating conditions, enter into a phenomenon called resonance, where small forces can result in large strains and, so, introducing unexpected levels of vibration and noise, or even, some kind of damage in the systems. The resonances occur when the frequency of a force actuating in a structure coincide its natural frequencies, this phenomenon may lead to large oscillations of the system, as discussed in RAO (2008). The accident at Tacoma Narrows Bridge is a typical example of this kind of problem. The wings of in flight airplanes are also subjected to a similar phenomenon, called "flutter".

The resonance is a frequent, or at least a contributing factor, in most part of the problems associated to the vibration and noise that occur in structures and machines. For a better understanding of the structural vibrations problems, the natural frequencies, damping and modal residues of the structures must be identified and quantified. The development of a new product or the redesign, always involve many dynamics structural tests in prototypes to evaluate the real dynamic behavior of the structure (GUILLAUME, 2002). Modal analysis actually has been widely used to identify these parameters.

### 2.1 THEORETICAL MODEL FOR MODAL ANALYSIS USING PIEZOELECTRIC MATERIALS

The theoretical formulation of modal analysis with piezoelectric materials may use the bases of the conventional modal analysis using ordinary sensors. A proposal discussed by Wang (1998) shows that is possible to obtain an generic analytical model for different kinds of sensors and actuators. In these cases, the Eigen-functions of the model are defined taking into account the actuator and the sensor, in order to obtain the frequency response function (FRF) of the model. Different combinations of types of sensors/actuators are presented and it is possible to establish a physical interpretation of the meaning of the Eigen-functions for all type of combination of sensors/actuators. For only the interesting case, i.e., where the actuator is a force-point and the sensor is a PVDF response sensor, one may deduce a generic Eigen-function for the model (Wang, 1998). The expression of the harmonic response of a generic-sensor device, as shown in Wang (1998), is given by

$$S(P_i,t) = \sum_{r=1}^{\infty} \frac{A_j\{\int_D w_r(P)E(P_j)dD(P)\}\{Q[w_r(P_i)]\}}{(\omega_r^2-\omega^2)+i(2\zeta_r\omega_r\omega)} \qquad 2.1$$

Where: $A_j$ is the magnitude of the force; $E(P_j)$ is a spatial function; $P_i$ is the location a generic sensor device; $P_j$ is the location of the generic actuator device; Q is an operator related to the structure displacement of the model and $\omega$ is the excitation frequency.

The frequency transfer function of the model, defined between a generic sensor and the point-force can be calculated based in the formulation above. For this case, the FRF between the response of the *i-th* generic-sensor device located at $P_i$ and the magnitude of the *j-th* generic force-point applied at $P_j$ point, is given thought of the quotient of Eq. (2.1), by the force $A_j$, Eq. (2.2).

$$H_{ij}(\omega) = \frac{S(P_i)}{A_j} = \sum_{r=1}^{\infty} \frac{\{\int_D w_r(P)E(P_j)dD(P)\}\{Q[w_r(P_i)]\}}{(\omega_r^2-\omega^2)+i(2\zeta_r\omega_r\omega)} \qquad 2.2$$

This equation can be rewritten in a form similarly to the usual FRF equation, expression (2.3)

$$H_{ij}(\omega) = \frac{S(P_i)}{A_j} = \sum_{r=1}^{\infty} \frac{\Phi_{r,j}^A\,\Phi_{r,i}^s}{(\omega_r^2-\omega^2)+i(2\zeta_r\omega_r\omega)} \qquad 2.3$$

Where: $\Phi_{r,j}^A = \Phi_r^A(P_j) = \int_D w_r(P)E(P_j)dD(P)$

$\Phi_{r,i}^s = \Phi_r^s(P_i) = Q[w_r(P_i)]$

The terms $\Phi_r^A(P)$ $e$ $\Phi_r^s(P)$ are defined as the generic Eigen function of the actuator and the sensor, respectively and $\Phi_{r,j}^A$ $e$ $\Phi_{r,i}^s$ can be defined as the scalar value of the eigen-functions for the actuator and the sensor located at $P_j$ and $P_i$, respectively.

The FRF of the model is analogous to the conventional modal expression for discrete multi-degree of freedom system (mDOF). However, the shapes functions are different, the modal shape functions of the PVDF sensor are proportional to slope difference between the ends of the PVDF and the modal shape functions of the point-force are proportional to displacement (Wang, 1998).

### 2.1. Relation Between Displacement Mode Shapes and Slope Difference Mode Shapes

In this case, the use of piezoelectric material as sensor produces a slope difference mode shape instead of displacement mode shape but the force-point transducers produces a displacement mode shape. The relationship between displacement and slope difference mode shapes is discussed in Wang and Wang (1997). They relate the displacement mode shape and the slope difference mode shape of a cantilever beam, this paper, using the idea discussed in Wang and Wang (1997), it is developed a relationship for the displacement mode shape and the slope difference mode shape of a free-free beam structure. The expression of the modes shape of a beam (RAO, 2008), eq. (2.4), is used to relate the displacement and slope difference modes shape.

$$\phi_n(x) = A\sin(\alpha_n x) + B\cos(\alpha_n x) + C\sinh(\alpha_n x) + D\cosh(\alpha_n x) \qquad 2.4$$

By imposing the boundary conditions (free-free) for the beam and calculating the derivates of the expression above, its gives,

$$\phi_n(x) = \cosh(\alpha_n x) + \cos(\alpha_n x) - \sigma_n[\sinh(\alpha_n x) + \sin(\alpha_n x)] \qquad 2.5$$

where:

$$\sigma_n = \frac{\cosh(\alpha_n L) - \cos(\alpha_n L)}{\sinh(\alpha_n L) - \sin(\alpha_n L)} \qquad 2.6$$

In this case, using the above formulation to calculate the function of the PVDF, for a free-free beam, it gives

$$\phi_r^P(x_{pi}) = \alpha_n \left\{\sinh\left(\alpha_n\left(x_{pi}+\frac{l_p}{2}\right)\right) - \sin\left(\alpha_n\left(x_{pi}+\frac{l_p}{2}\right)\right) - \sigma_n\left[\cosh\left(\alpha_n\left(x_{pi}+\frac{l_p}{2}\right)\right) + \cos\left(\alpha_n\left(x_{pi}+\frac{l_p}{2}\right)\right)\right] - \sinh\left(\alpha_n\left(x_{pi}-\frac{l_p}{2}\right)\right) + \sin\left(\alpha_n\left(x_{pi}-\frac{l_p}{2}\right)\right) + \sigma_n\left[\cosh\left(\alpha_n\left(x_{pi}-\frac{l_p}{2}\right)\right) + \cos\left(\alpha_n\left(x_{pi}-\frac{l_p}{2}\right)\right)\right]\right\} \qquad 2.7$$

Solving each term and simplifying the equation in relation to the term $\sin\left(\alpha_n \frac{l_p}{2}\right)$, it gives,

$$\phi_r^P(x_{pi}) = 2\alpha_n \sin\left(\alpha_n \frac{l_p}{2}\right)\left\{\cosh(\alpha_n x_{pi})\frac{\sinh\left(\alpha_n \frac{l_p}{2}\right)}{\sin\left(\alpha_n \frac{l_p}{2}\right)} - \cos\left(\alpha_n \frac{l_p}{2}\right) - \sigma_n\left[\sinh(\alpha_n x_{pi})\frac{\sinh\left(\alpha_n \frac{l_p}{2}\right)}{\sin\left(\alpha_n \frac{l_p}{2}\right)} - \sin\left(\alpha_n \frac{l_p}{2}\right)\right]\right\} \qquad 2.8$$

Now, considering a small length sensor with $l_p$ approaching to zero, in the limit (applying L'Hosptal), the quotient in the expression (2.8), it gives

$$\lim_{l_p \to 0} \frac{\cosh\left(\alpha_n \frac{l_p}{2}\right)}{\cos\left(\alpha_n \frac{l_p}{2}\right)} = 1 \qquad 2.9$$

So, finally, the equation (2.8) can be simplified, eq. (2.10):

$$\phi_r^P(x_{pi}) = 2\alpha_n \sin\left(\alpha_n \frac{l_p}{2}\right)\left\{\cosh(\alpha_n x_{pi}) - \cos\left(\alpha_n \frac{l_p}{2}\right) - \sigma_n\left[\sinh(\alpha_n x_{pi}) - \sin\left(\alpha_n \frac{l_p}{2}\right)\right]\right\} \quad 2.10$$

The expression (2.10) is similar to original equation for the displacement of a free-free beam, unless the signal and the constant term $2\alpha_n \sin\left(\alpha_n \frac{l_p}{2}\right)$ that multiply the equation.

In this case, the form of the mode shape of the beam can be obtained plotting the mode shape expression for the point-force actuator or for the PVDF sensor point response. Figure (2.1) shows the vibration displacement modes of a continuous free-free beam and the figure (2.2) shows the respective slope difference modes shape.

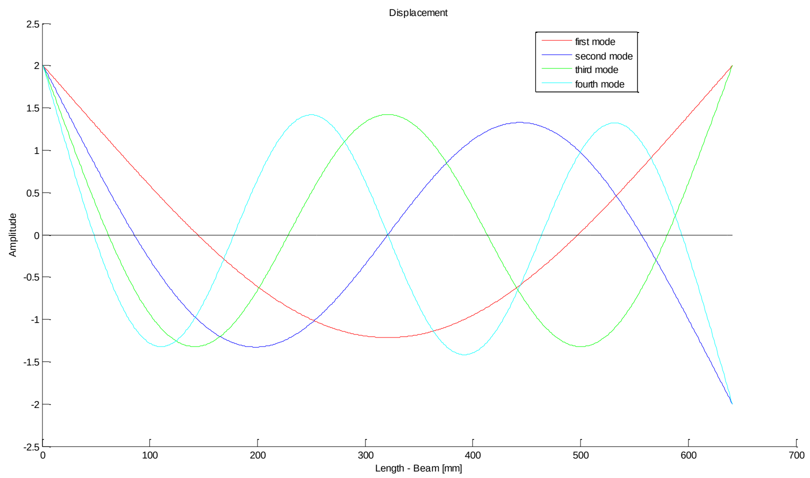


Figure 2.1 – Vibration modes for a continuous free-free beam (displacement).

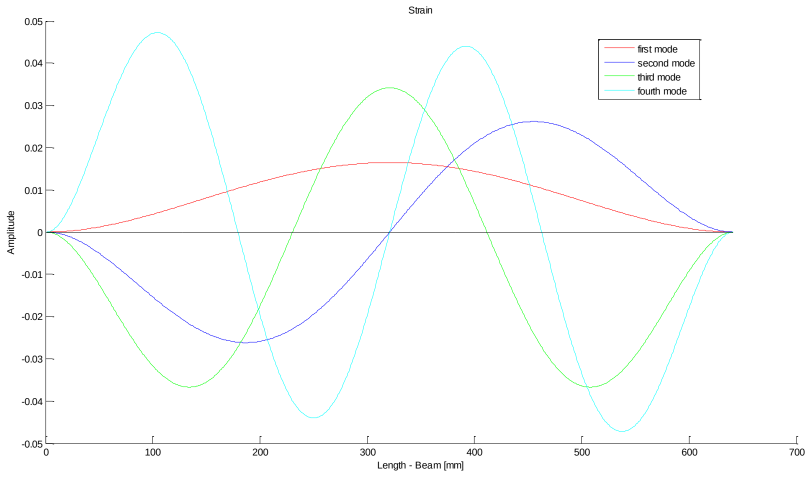


Figure 2.2 – Vibration modes for a continuous free-free beam (slope difference).

The figures show the behavior of a free-free beam in term of the actuator function (displacement mode shape) and in terms of the PVDF sensor function (slope difference mode shape) that is the main interest of this article.

## 3. MODAL ANALYSIS USING PVDF SENSORS

This section discusses the experimental modal analysis of a test structure of a steel beam (ANSI 1010), excited by a force-point actuator. The structure was discretized in 5 measuring points where the piezoelectric sensors of the model were installed (Fig. 3.1). The actuator point was defined in the point number 4 and it was used two different type of excitation, in the first case, an impact hammer and in the second one, a PZT-actuator. Differently of Wang and Wang (1997), that studied this problem for a cantilever beam it was used a free-free condition in the test. The table (3.1) shows the properties of the material of the beam.

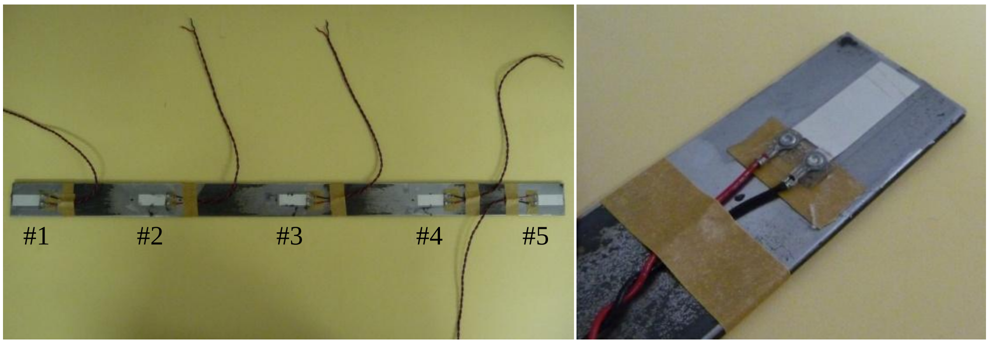


Figure 3.1 – Beam with the five PVDF(s) sensors bonded.

The modal parameters of the structure were calculated initially using finite element approach and after that, by experimental modal tests. Three different experimental tests were conducted. In the first one, the parameters were estimated by using the conventional modal analysis, i.e., using accelerometers as sensors and an impact hammer as actuator. In the second one, it was uses PVDF sensors as shown in the fig. (3.1) and the actuator, in the point 4, it was also the impact hammer. Finally, in the third one, it was used a PZT actuator to excite the structure and the response were measured using PVDF sensors like in previous test.

Table 3.1 – Material Properties for a Beam (ANSI 1010)

| **Beam Properties (ABNT 1010)** | **Value** |
|---|---|
| *Young's Modulus* | $205x10^9\ Pa$ |
| Density | $7860\ Kg/m^3$ |
| Poisson | 0.33 |
| Length | $640x10^{-3}\ m$ |
| Width | $37.7x10^{-3}\ m$ |
| Thickness | $3.1x10^{-3}\ m$ |

The acquisition system used was the SignalCalc ACE from DataPhisics and in all test the data were measured in the frequency range of 0 to 300 Hz. The data were processed in the OEMA software (2008) that uses the Matlab ambient to extract the modal parameters of the model.

## 4. RESULTS

The result shown below refers to the three tests and the finite element model. The experimental results are compared among them and with the finite element simulation. The finite element model was created initially aiming at to have some insight of the behavior of the structure and to provide some hints for the experimental tests. In a second moment, the model was redefined based in the experimental data and the density parameter was adjusted to 6470 kg/m^3.

Table (4.1) show the obtained values of the natural frequencies for the different tests, Case 01 refers to the results of finite element model, Cases 02, 03 and 04 refer to the experimental tests using, respectively, accelerometers/Impact Hammer, PVDF/Impact Hammer and PVDF/PZT.

Table 4.1 – Natural Frequencies of the beam [Hz].

| | *1ª Nat. Freq.* | *2ª Nat. Freq.* | *3ª Nat. Freq.* |
|---|---|---|---|
| Case 01 | 43.75 | 120.84 | 237.57 |
| Case 02 | 43.75 | 119.37 | 234.37 |
| Case 03 | 43.47 | 119.91 | 234.71 |
| Case 04 | 43.58 | 118.51 | 234.65 |

Table (4.2) shows the damping ratio estimated in the case of using piezoelectric material as sensor and the impact hummer as actuator and, finally, table (4.3) shows the percentage differences among the PVDF/Impact Hummer and the others three cases.

Table 4.2 – Damping Ratio of the beam to case PVDF sensor and Impact Hummer actuator (PVDF vs force-point).

| *Natural Frequency (Case 04)* | *Damping Ratio* |
|---|---|
| 1ª Nat. Freq. | 0.0081 |
| 2ª Nat. Freq. | 0.0018 |
| 3ª Nat. Freq. | 0.0011 |

Table 4.3 – Percentage difference of the natural frequencies for the different [%].

| *Error* | *1ª Nat. Freq.* | *2ª Nat. Freq.* | *3ª Nat. Freq.* |
|---|---|---|---|
| Case 03 vs. Case 01 | -0.6441 % | -0.7756 % | -1.2185% |
| Case 03 vs. Case 02 | -0.6441 % | 0.4503 % | 0.1449 % |
| Case 03 vs. Case 04 | -0.2530 % | 1.1675 % | 0.0256 % |

The vibrations modes shape were calculated for the case using PVDF as sensors and PZT as actuator by using the OEMA software. Figure (4.1-3) shows the first three mode shapes. It should be emphasize that they are slope difference mode shapes:

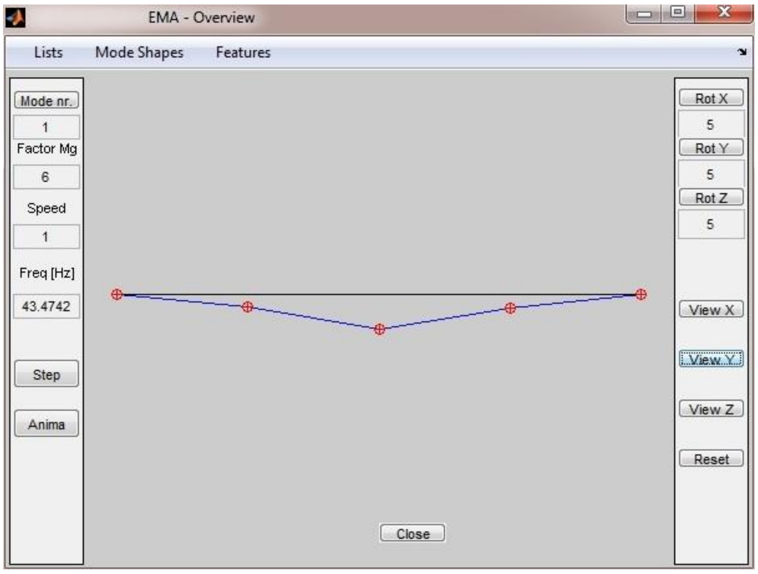


Figure 4.1 – First vibration mode using a PVDF sensor and Impact Hammer actuator.

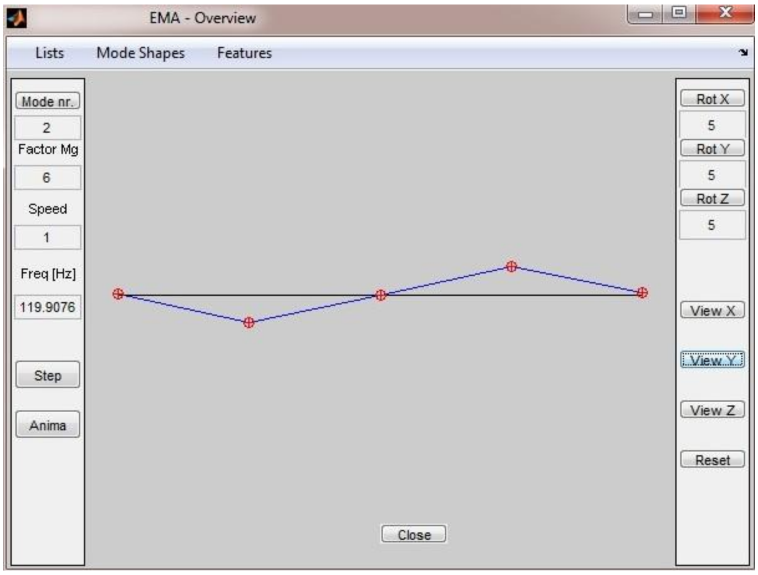


Figure 4.2 – Second vibration mode using a PVDF sensor and Impact Hammer actuator.

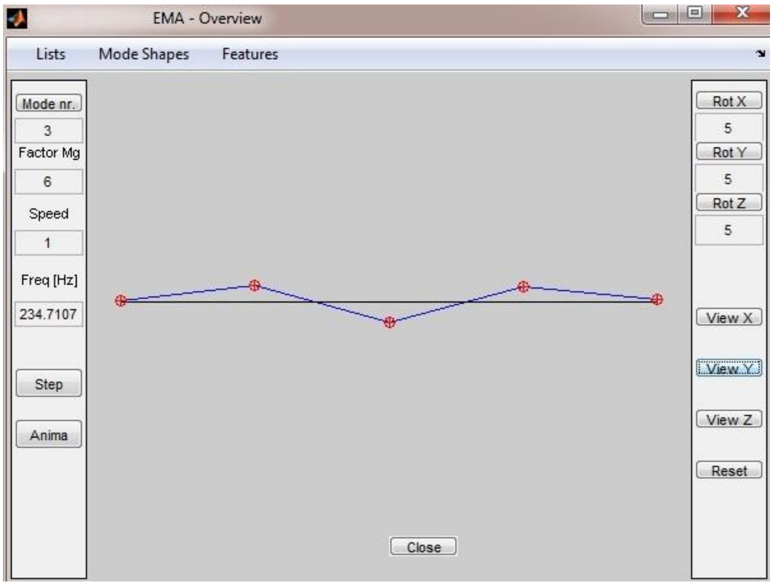


Figure 4.3 – Third vibration mode using a PVDF sensor and Impact Hammer actuator.

## 5. CONCLUSION

This paper discusses the modal analysis using piezoelectric material. The tests show that the results obtained by using PVDF sensors are similar to the ones obtained by using conventional sensors, when the interest is natural frequencies. The estimated modes shapes by using PVDF sensor are also reasonable and they agree with those theoretical modes plotted in the Fig.(2.1). This makes the modal analysis using piezoelectric material like PVDF as sensors an alternative possibility to the modal analysis using conventional sensor like accelerometers. It also opens a possibility to use this kind of sensor, with some advantage, in the output only-based modal analysis and operational modal analysis.

## 6. ACKNOWLEDGMENTS

The authors are thankful to CNPq and FAPEMIG for partially funding the present research work through the INCT-EIE.

## 7. REFERENCES

Ewins, D. J., 1984, "*Modal Testing: Theory and Practice,*" John Wiley & Sons Inc, New York

Freitas, T. C.; Pereira, J. A. Modal parameters identification using only response data – Stochastic Subspace Identification and Frequency Domain Decomposition. In: International Congress of Mechanical Engineering - COBEM, 19, 2007, Brasília – DF. Proceedings of COBEM 2007.

Guillaume, P. Modal Analysis. Encyclopedia of Life Support Systems, Oxford (UK), 2002. http://mech.vub.ac.be/avrg/publications/ModalAnalysis.pdf.

Herlufsen, H.; Andersen,P; Gade, S.; Møller, N.; Identification Techniques for Operational Modal Analysis - An Overview and Practical Experiences. Proceedings of the 1st International Operational Modal Analysis Conference (IOMAC), Copenhagen, Denmark, 2005.

Leo, D. J. Engineering Analysis of Smart Material Systems. 1. ed. Hoboken, New Jersey: John Wiley & Sons, 2007. 539 p. ISBN 978-0-471-68477-0.

Lieven, N. A. J.; Ewins, D. J. The Context of Experimental Modal Analysis. Philosophical Transactions of The Royal Society A, v. 359, p. 5-10, 15 January 2001. doi: 10.1098/rsta.2000.0710.

Maia, N. M. M. e Silva, J. M. M.; "Theoretical and Experimental Modal Analysis", Research Studies Press Ltda. John Wiley & Sons Inc, 1997.

Maia, N. M. M.; Silva, J. M. M. Modal Analysis Idenfitication Techniques. Philosophical Transactions of The Royal Society A, v. 359, p. 29-40, 15 January 2001. doi: 10.1098/rsta.2000.0712.

Mcconnell, K. G.; Varoto, P. S.; "Vibration Testing: Theory and Practice" 2. ed. Hoboken: John Wiley & Sons, 2008.

Peeters, B. and De Roeck, G., Stochastic System Identification for Operational Modal Analysis: A Review, J. of Dynamic System, Measurement and Control, V. 123, Dec. 2001

OEMA – Operational & Modal Analysis Software. Developed at Departamento de Engenharia Mecânica, UNESP, Faculdade de Engenharia Mecânica de Ilha Solteira by Prof. Dr. Pereira, J. A.

Rao, S. S. Vibrações Mecânicas. Tradução de Arlete Simille. 4. ed. São Paulo: Pearson Prentice Hall, 2008. ISBN 978-85-7605-200-5.

Wang, B.-T. Structural Modal Testing with Various Actuators and Sensors. Mechanical Systems and Signal Processing, 12, n. 5, September 1998. 627-639.

Wang, B.-T.; Wang, C.-C. Feasibility Analysis of Using Piezoceramic Transducers for Cantilever Beam Modal Testing. Smart Material and Structures, 6, n. 1, 1997. 106-116. doi: 10.1088/0964-1726/6/1/013.

## 8. RESPONSIBILITY NOTICE