DESIGN OF A NONLINEAR VIBRATION ABSORBER BY UTILIZING BIO-INSPIRED OPTIMIZATION METHODS

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Abstract. *Discrete dynamic vibration absorbers (DVAs) are mechanical devices developed in the beginning of the last century used normally to attenuate the vibration level of different structures and machines. They have been used in several engineering applications, such as ships, power lines, aeronautic structures, civil engineering constructions subjected to seismic induced excitations, etc. In the literature, different approaches based on optimization methods have been proposed to design dynamic vibration absorbers. The present work focuses the theoretical study and numerical simulations of a two degree-of-freedom nonlinear damped system, constituted of a primary mass attached to the ground by a linear spring and the secondary mass attached to the primary system by a nonlinear spring (nDVA). The sensitivity analysis of suppression bandwidth, namely, the frequency range over which the ratio of the main mass displacement amplitude to the amplitude of the forcing function is less than unity, with respect to design variables that characterize the nonlinear system based on the first order finite differences is proposed. The application is associated to the suppression bandwidth, where the interest is to maximize this bandwidth using two bio-inspired optimization methods recently proposed: Bees Colony Algorithm and Firefly Algorithm.*

Keywords*: Nonlinear dynamic vibration absorber, suppression band, nonlinear vibration, bio-inspired optimization methods.*

1. INTRODUCTION

The use of discrete dynamic vibration absorbers (DVAs) in the problem of vibration attenuation constitutes an important subject in modern engineering. The application of DVAs to reduce noise and vibration levels in various types of engineering systems such as compressors systems, robots, ships, power lines, airplanes, helicopters, etc., has been intensively investigated lately. Much of the knowledge available to date is compiled in the original patent by Frahm (1911), in the books by Den Hartog (1934) and Koronev and Reznikov (1993) and in some review papers such as those by Steffen Jr. and Rade (2001). In the last two decades, a great deal of effort has been devoted to the development of mathematical models for characterizing the mechanical behavior of nonlinear dynamic vibration absorbers (nDVAs) accounting for its typical dependence on parameters that control the nonlinearities. A particular type of nDVA is the so-called viscoelastic neutralizer as studied by Espíndola and Bavastri (1997). Different techniques have been proposed to design viscoelastic vibration absorbers, as shown by Espíndola et al. (2008, 2009). Besides the well-known complexity of the modeling strategy involved in nonlinear dynamics, which constitutes a simple and straightforward means of representing the dynamic behavior of nDVAs, some general methodologies have been suggested and have been shown to be particularly suitable to be used in combination with structural systems discretization. This aspect makes them very attractive for the modeling of nonlinear dynamic vibration absorbers. Among these strategies, it should be mentioned the theoretical study proposed by Nissen (1985) and Pai and Schulz (1998), in which some techniques to improve the stability and efficiency of nDVAs into a frequency band of interest have been proposed, leading to refined nDVAs. Also, Rice and McCraith (1987) and Shaw (1989), suggested optimization strategies to be applied to the design of nDVAs by applying an asymmetric nonlinear Duffing-type element incorporated in the suspension for narrow-band absorption applications.

In this context, different approaches based on optimization methods have been proposed to dynamic absorbers design. In this context, nature-inspired systems have contributed significantly to the development of new optimization techniques. Among the most recent bio-inspired strategies stands the Bees Colony Algorithm - BCA (Lucic and Teodorovici, 2001) and Firefly Algorithm – FA (Yang, 2008) for solving combinatorial optimization problems.

The BCA is based on the behavior of bees' colonies in their search of raw materials for honey production. According to Lucic and Teodorovici (2001), in each hive groups of bees (called scouts) are recruited to explore new areas in search for pollen and nectar. These bees, returning to the hive, share the acquired information so that new bees are indicated to explore the best regions visited in an amount proportional to the previously passed assessment. Thus, the most promising regions are best explored and eventually the worst end up being discarded. Every iteration this cycle repeats itself with new areas being visited by scouts.

The FA mimics the patterns of short and rhythmic flashes emitted by fireflies in order to attract other individuals to its vicinities. This optimization algorithm is formulated by assuming that: all fireflies are unisex, so that one firefly will be attracted to all other fireflies; attractiveness is proportional to their brightness, and for any two fireflies, the less brighter one will attract (and thus move) to the brighter one; however, the brightness can decrease as their distance increases and if there are no fireflies brighter than a given firefly, it will move randomly. The brightness is associated with the objective function.

In this sense, the present work is dedicated to presenting alternative optimization techniques for the design of nDVAs. For this aim, this contribution focuses on the theoretical study and numerical simulations of a two degree-of-freedom nonlinear damped system, constituted of a primary mass attached to the ground by a linear spring and the secondary mass attached to the primary system by a nonlinear spring (nDVA). The optimization techniques presented are applied to a nDVA for illustration purposes, however they are intended to be general in the sense that they can be applied to design different types of nonlinear mechanical systems. This work is organized as follows. The mathematical formulation of non-linear dynamic system and sensibility analysis are presented in Sections 2 and 3, respectively. A review of the BCA and the FA are presented in Section 4. The results and discussion are described in Section 5. Finally, the conclusions and suggestions for future work conclude the paper.

2. MATHEMATICAL MODELING OF NON-LINEAR DYNAMIC SYSTEM

Consider the two degree-of-freedom (d.o.f.) model shown in Fig. 1.

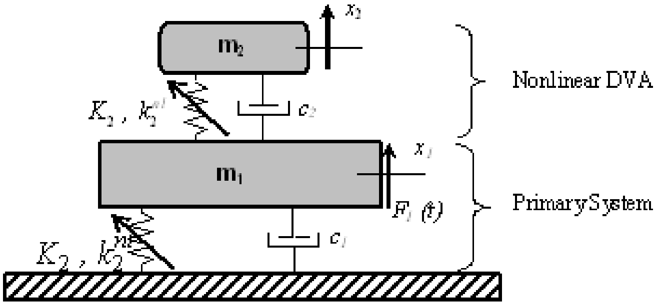

Figure 1: Two degree-of-freedom damped system.

This device consists of a damped primary system attached to the ground by a suspension that includes either a linear or a nonlinear spring, and a damped secondary mass coupled to the primary system by a spring with nonlinear characteristics. The external force applied to the primary system is given by the following expression:

$$F_1(t) = p\cos(\omega t) \tag{1}$$

The constitutive forces of the springs are given by:

$$r_i(x_i) = k_i x_i + k_i^{nl} x_i^3; \quad i = 1 \; to \; 2 \tag{2}$$

where x_1 represents de displacement of the primary system with respect to the ground, and x_2 is the displacement of the DVA with respect to the primary system. In the model above, the dampers are linear, however springs have nonlinear characteristics, where k_i and k_i^{nl} represent, respectively, the linear and nonlinear coefficients of the springs.

With the aim of obtaining the dimensionless normalized equations of motion for the nonlinear system, the displacements are normalized with respect to the length of a given vector x_c (Zhu, 1992), according to the following expression:

$$y_i = x_i / x_c \tag{3}$$

In addition, one introduces the following relations to the system:

$$\begin{gathered}\overline{\omega}_i^2 = \frac{k_i}{m_i}, \quad \omega_i = \frac{\omega_i}{\omega}, \quad \zeta_i = \frac{c_i}{2\sqrt{k_i m_i}}, \quad \delta_i = 2\zeta_i\omega_i, \quad \mu = \frac{m_2}{m_1}, \quad \eta_i = \omega_i^2, \\ \varepsilon_i = \frac{k_i^{nl} x_c^2}{m_i \omega^2}, \quad \rho = \frac{\omega_2}{\omega_1}, \quad P = \frac{p}{m_1 \overline{\omega}^2 x_c}, \quad \beta = \frac{P}{\eta_1}, \quad \Omega = \frac{\omega}{\overline{\omega}_1}.\end{gathered} \tag{4}$$

By applying the Newton's second law, and after some algebraic manipulations, the following normalized equation of motion of the nonlinear dynamic system can be expressed under the following matrix form:

$$\mathbf{M}\ddot{\mathbf{y}}(t)+\mathbf{C}\dot{\mathbf{y}}(t)+\mathbf{K}\mathbf{y}(t)=\mathbf{f}(t) \tag{5}$$

where the normalized mass, damping and stiffness matrices are expressed, respectively, by the following relations:

$$\mathbf{M}=\begin{bmatrix}1+\mu & \mu\\ \mu & \mu\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}\delta_1 & 0\\ 0 & \mu\delta_2\end{bmatrix},\ \mathbf{K}=\begin{bmatrix}\eta_1 & 0\\ 0 & \mu\eta_2\end{bmatrix}, \tag{6}$$

The normalized displacement and force vectors are given by:

$$\boldsymbol{y}=\begin{bmatrix}y_1\\ y_2\end{bmatrix},\ \boldsymbol{f}=\begin{bmatrix}P\cos\tau-\varepsilon_1 y_1^3\\ -\mu\varepsilon_2 y_2^3\end{bmatrix}, \tag{7}$$

2.1. Steady-State Response of the Nonlinear System

In this paper, the Krylov-Bogoliubov method (Nayfeh, 2000) will be used to integrate the matrix equation of motion (Eq. (1)). This method leads to an approximate solution of nonlinear differential equations. The process is based on the following transformation of variables:

$$\mathbf{y}(\tau)=\mathbf{u}(\tau)\cos\tau+\mathbf{v}(\tau)\sin\tau \tag{8}$$

where $\tau=\omega t$; the time dependence of $\mathbf{u}=(\mathbf{u}_1\,\mathbf{u}_2)^T$ and $\mathbf{v}=(\boldsymbol{v_1}\,\boldsymbol{v_2})^T$ is assumed to be small for high order terms, such as the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.

After mathematical manipulation, we obtain a nonlinear algebraic system with four equations and four variables u_1, u_2, v_1 and v_2 is obtained:

$$\begin{cases}\left(1+\mu-\omega_1^2\right)u_1+\mu u_2-2\zeta_1\omega_1 v_1-\dfrac{3\varepsilon_1\left(u_1^2+v_1^2\right)u_1}{4}+\beta\omega_1^2=0\\ \mu u_1+\left(\mu-\mu\rho^2\omega_1^2\right)u_2-\mu\left(2\zeta_2\rho\omega_1^2 v_2+\dfrac{3\varepsilon_2\left(u_2^2+v_2^2\right)u_2}{4}\right)=0\\ \left(\omega_1^2-1-\mu\right)v_1-\mu v_2-2\zeta_1\omega_1 u_1+\dfrac{3\varepsilon_1\left(u_1^2+v_1^2\right)v_1}{4}=0\\ -\mu v_1+\left(\mu\rho^2\omega_1^2-\mu\right)v_2-\mu\left(2\zeta_2\rho\omega_1^2 u_2-\dfrac{3\varepsilon_2\left(u_2^2+v_2^2\right)v_2}{4}\right)=0\end{cases} \tag{9}$$

The system represented by Eq. (9) should be numerically solved. Then, the values of n u_1, u_2, v_1 and v_2 can be calculated and the vibration amplitudes of the primary and secondary masses of the nonlinear DVA are obtained. The amplitude values are given by r_1 and r_2, according to the following equation:

$$r_i=\sqrt{u_i^2+v_i^2},\quad i=1\,\text{to}\,2. \tag{10}$$

3. SENSITIVITY ANALYSIS OF STRUCTURAL RESPONSES

In a mechanical system, the parameters of mass, stiffness and damping, establish the dependence with respect to a set of design parameters, which include physical and geometrical characteristics and the parameters that control the nonlinearities (Haug, 1986). Such functional dependence can be expressed in a general form as follows:

$$\boldsymbol{r}=\boldsymbol{r}(\boldsymbol{M}(\boldsymbol{p}),\boldsymbol{C}(\boldsymbol{p}),\boldsymbol{K}(\boldsymbol{p})) \tag{11}$$

where $\boldsymbol{r}$ and $\boldsymbol{p}$ designate vectors of structural responses and design parameters, respectively.

The sensitivity of the structural responses with respect to a given parameter $\boldsymbol{p_i}$, evaluated for a given set of values of the design parameter $\boldsymbol{p}^{\mathbf{0}}$ is defined as the following partial derivative:

$$\left.\frac{\partial \boldsymbol{r}}{\partial p_i}\right|_{p^0} = \lim_{\Delta p_i \to 0}\left[\frac{\boldsymbol{r}\left(\boldsymbol{M}\left(p_i^0+\Delta p_i\right),\boldsymbol{C}\left(p_i^0+\Delta p_i\right),\boldsymbol{K}\left(p_i^0+\Delta p_i\right)\right)}{\Delta p_i} - \frac{\boldsymbol{r}\left(\boldsymbol{M}\left(p_i^0\right),\boldsymbol{C}\left(p_i^0\right),\boldsymbol{K}\left(p_i^0\right)\right)}{\Delta p_i}\right] \quad (12)$$

where Δp_i is an arbitrary variation applied to the current value of parameter $\boldsymbol{p_i^0}$, while all other parameters remain unchanged. The sensitivity with respect to $\boldsymbol{p_i}$ can be estimated by finite differences by computing successively the responses corresponding to $\boldsymbol{p_i} = \boldsymbol{p_i^0}$ and $\boldsymbol{p_i} = \boldsymbol{p_i^0}+\Delta \boldsymbol{p_i}$.

Such procedure is an estimated approach enabling to calculate the sensitivity of the dynamic system responses with respect to small modifications introduced in the design parameters. Moreover, the results depend upon the choice of the value of the parameter increment $\Delta \boldsymbol{p_i}$. Another strategy consists in computing the analytical derivatives, as possible, of the structural responses with respect to the parameters of interest. This approach is not considered herein because of the numerical procedures used to solve the nonlinear equations.

4. BIO-INSPIRED ALGORITHMS

4.1. Bee Colony Algorithm

This optimization algorithm is based on behavior of colony of honey bees. It can extend itself over long distances and in multiple directions simultaneously to exploit a large number of food sources. In addition, the colony of honey bees presents as characteristic, the capacity of memorization, learning and transmission of information in colony, so forming the swarm intelligence (von Frisch, 1976).

In a colony the foraging process begins by scout bees being sent to search randomly for promising flower patches. When they return to the hive, those scout bees that found a patch which is rated above a certain quality threshold (measured as a combination of some constituents, such as sugar content) deposit their nectar or pollen and go to the "waggle dance".

This dance is responsible by the transmission (colony communication) of information regarding a flower patch: the direction in which it will be found, its distance from the hive and its quality rating (or fitness) (von Frisch, 1976). This dance enables the colony to evaluate the relative merit of different patches according to both the quality of the food they provide and the amount of energy needed to harvest it (Camazine *et al.*, 2003).

After waggle dancing on the dance floor, the dancer (i.e., the scout bee) goes back to the flower patch with follower bees that were waiting inside the hive. More follower bees are sent to more promising patches. This allows the colony to gather food quickly and efficiently. While harvesting from a patch, the bees monitor its food level. This is necessary to decide upon the next waggle dance when they return to the hive (Camazine *et al.*, 2003). If the patch is still good enough as a food source, then it will be advertised in the waggle dance and more bees will be recruited to that source.

In this context, Pham and co-workers (Pham *et al.*, 2006) proposed an optimization algorithm inspired by the natural foraging behavior of honey bees (Bees Colony Algorithm - BCA) and presented in Fig. 2.

Basic steps of the Bees Colony Algorithm
1. Initialize population with random solutions. 2. Evaluate fitness of the population. 3. While (stopping criterion not met) 4. Select sites for neighborhood search. 5. Recruit bees for selected sites (more bees for the best *e* sites) and evaluate fitnesses. 6. Select the fittest bee from each site. 7. Assign remaining bees to search randomly and evaluate their fitnesses. 8. End while.

Figure 2. Bees Colony Algorithm (Pham *et al.*, 2006).

The BCA requires a number of parameters to be set, namely, the number of scout bees (*n*), number of sites selected for neighborhood search (out of *n* visited sites) (*m*), number of top-rated (elite) sites among *m* selected sites (*e*), number of bees recruited for the best *e* sites (*nep*), number of bees recruited for the other (*m-e*) selected sites (*ngh*), and the stopping criterion.

The BCA starts with the *n* scout bees being placed randomly in the search space. The fitnesses of the sites visited by the scout bees are evaluated in step 2.

In step 4, bees that have the highest fitnesses are chosen as "selected bees" and sites visited by them are chosen for neighborhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to search near to the best *e* sites. The bees can be chosen directly according to the fitnesses associated with the sites they are visiting.

Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighborhood of the best *e* sites, which represent more promising solutions, are made more detailed by recruiting more bees to follow them than the other selected bees. Together with scouting, this differential recruitment is a key operation of the BCA.

However, in step 6, for each patch only the bee with the highest fitness will be selected to form the next bee population. In nature, there is no such a restriction. This restriction is introduced here to reduce the number of points to be explored. In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions.

In the literature, various applications using this bio-inspired approach can be found, such as: modeling combinatorial optimization transportation engineering problems (Lucic and Teodorovic, 2001), engineering system design (Yang, 2005; Lobato *et al.*, 2010), transport problems (Teodorovic and Dell'Orco. 2005), mathematical function optimization (Pham *et al.*, 2006), dynamic optimization (Chang, 2006), optimal control problems (Afshar *et al.*, 2001), parameter estimation in control problems (Azeem and Saad, 2004), among other applications (http://www.bees-algorithm.com/).

4.2. Firefly Algorithm

The Firefly Algorithm is based on the characteristic of the bioluminescence of fireflies, insects notorious for their light emission. According to Yang (2008), biology does not have a complete knowledge to determine all the utilities that firefly luminescence can bring to, but at least three functions have been identified: (*i*) as a communication tool and appeal to potential partners in reproduction, (*ii*) as a bait to lure potential prey for the firefly, (*iii*) as a warning mechanism for potential predators reminding them that fireflies have a bitter taste.

The bioluminescent signals are known to serve as elements of courtship rituals (in most cases, the females are attracted by the light emitted by the males), methods of prey attraction, social orientation or as a warning signal to predators (Lukasik and Zak, 2009).

It were idealized some of the flashing characteristics of fireflies so as to develop firefly-inspired algorithms. For simplicity the following three idealized rules are used (Yang, 2010):

1) all fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;
2) attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly;
3) the brightness of a firefly is affected or determined by the landscape of the objective function. For a maximization problem, the brightness can simply be proportional to the value of the objective function.

According to Yang (2008), in the firefly algorithm, there are two important issues: the variation of light intensity and formulation of the attractiveness. For simplicity, it is always assumed that the attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function.

This swarm intelligence optimization technique is based on the assumption that solution of an optimization problem can be perceived as agent (firefly) which "glows" proportionally to its quality in a considered problem setting. Consequently each brighter firefly attracts its partners (regardless of their sex), which makes the search space being explored more efficiently. The algorithm makes use of a synergic local search. Each member of the swarm explores the problem space taking into account results obtained by others, still applying its own randomized moves as well. The influence of other solutions is controlled by value of attractiveness (Lukasik and Zak, 2009).

According to Lukasik and Zak (2009), the FA is presented as follows. Consider a continuous constrained optimization problem where the task is to minimize cost function $f(x)$, find $x*$ such as:

$$f(x^*) = \min_{x \in S} f(x) \tag{13}$$

Assume that there exists a swarm of m agents (fireflies) solving above mentioned problem iteratively and x_i represents a solution for a firefly i in algorithm's iteration k, whereas $f(x_i)$ denotes its cost. Initially, all fireflies are dislocated in S (randomly or employing some deterministic strategy). Each firefly has its distinctive attractiveness τ which implies how strong it attracts other members of the swarm. As the firefly attractiveness one should select any monotonically decreasing function of the distance $r_j=d(x_i,x_j)$ to the chosen firefly j, e.g. the exponential function:

$$\tau = \tau_0 e^{-\gamma r_j} \tag{14}$$

where τ_o and γ are predetermined algorithm parameters: maximum attractiveness value and absorption coefficient, respectively. Furthermore every member of the swarm is characterized by its light intensity I_i which can be directly expressed as a inverse of a cost function $f(x_i)$.To effectively explore considered search space S it is assumed that each firefly i is changing its position iteratively taking into account two factors: attractiveness of other swarm members with higher light intensity, i.e., $I_j > I_i$, $\forall$ j=1, …, m, $j \neq i$ which is varying across distance and a fixed random step vector u_i. It should be noted as well that if no brighter firefly can be found only such randomized step is being used.

Thus, moving at a given time step t of a firefly i toward a better firefly j is defined as:

$$x_i^t = x_i^{t-1} + \tau\left(x_j^{t-1} - x_i^{t-1}\right) + \alpha\left(rand - \frac{1}{2}\right) \tag{15}$$

where the second term on the right side of the equation inserts the factor of attractiveness, τ while the third term, governed by α parameter, governs the insertion of certain randomness in the path followed by the firefly, *rand* is a random number between 0 and 1.

In the literature, few works using the FA can be found. In this context, is emphasized application in multimodal optimization (Yang, 2009), continuous constrained optimization task (Lukasik and Zak, 2009), solution of singular optimal control problems (Pfeifer and Lobato, 2010) and economic emissions load dispatch problem (Apostolopoulos and Vlachos, 2011).

5. RESULTS AND DISCUSSION

The following numerical example is presented to illustrate the application of the proposed methodology to obtain a robust design of a nDVA. Figure 1 depicts the test structure consisting of a primary mass attached to the ground by a nonlinear spring and coupled with a nDVA. The nominal values of the design parameters used to generate the dynamic responses of the nonlinear system are illustrated on Tab. 1. The computations performed consist in obtaining the driving point frequency responses associated to the displacement x_1.

Table 1 – Nominal Values of design variables.

Parameters	ε_1	ε_2	β	ζ_1	ζ_2	μ	ρ
Nominal Values	0.001	0.01	0.1	0.01	0.01	0.05	1.0

5.1. Sensitivities of the frequency response with respect to structural parameters.

To illustrate the computation procedure for the sensitivity of dynamic responses, numerical tests were performed by using the system configuration illustrated in Fig. 1. As previously mentioned, the computations are devoted to obtaining the sensitivities of the driving point frequency responses, which are given by the elements of $\boldsymbol{H}(\omega, \mathrm{p})$.

In this example, the normalized structural parameters ζ_1, ζ_2, ε_1, ε_2, β, μ and ρ, are considered as the design variables in the computation of the normalized sensitivities of the frequency responses with respect to a given parameter p, $\boldsymbol{S}_{\boldsymbol{H}}^{N}(\omega, p)$. The normalized real parts of the approximated complex sensitivity functions calculated by finite differences (according to Eq. (16)) are shown in Figs. 2 to 5, for which a variation of 20% of the nominal values of each design parameter was adopted. Also, in the same figures, the real parts of the frequency responses $\boldsymbol{H}(\omega, \mathrm{p})$, multiplied by convenient scale factors (SF), are shown. The sensitivity functions, denoted by $\boldsymbol{S}_{\boldsymbol{H}}^{N}(\omega, p)$, have been normalized according to the following scheme:

$$\boldsymbol{S}_{\boldsymbol{H}}^{N}(\omega, p) = \left.\frac{\partial \boldsymbol{S}_{\boldsymbol{H}}(\omega, p)}{\partial p}\right|_{(\omega, p^0)} \frac{p_0}{\boldsymbol{H}(\omega, p_0)} \tag{16}$$

Based on the amplitudes and signs of the sensitivity functions one can evaluate the degree of influence of the design variables upon the suppression bandwidth, in the frequency band of interest. In addition, the sensitivity analysis enables to decide among the design parameters those that will be retained in the optimization process. The parameters ζ_1 and ζ_2 and ε_1 do not have a significant influence on the evaluation of the suppression bandwidth (Borges, et al. 2010). Consequently, these parameters are not considered as design variables in the optimization run. However, as shown in Figures 3 and 4, the degrees of influence of the parameters ε_2, μ, and ρ on the suppression bandwidth are significant and will be considered as design variables in the optimization process.

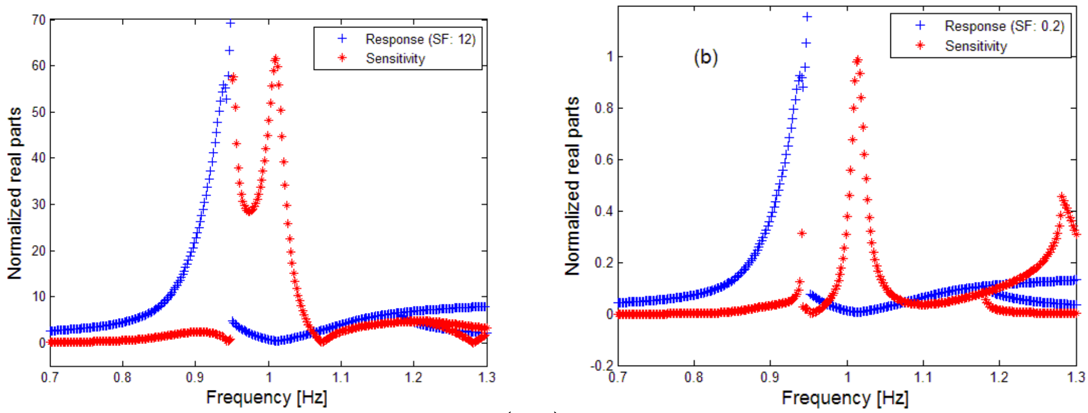

Figure 3 - Sensitivities of $\boldsymbol{H}(\omega, \mathrm{p})$ with respect to ρ (a) and ε_2 (b).

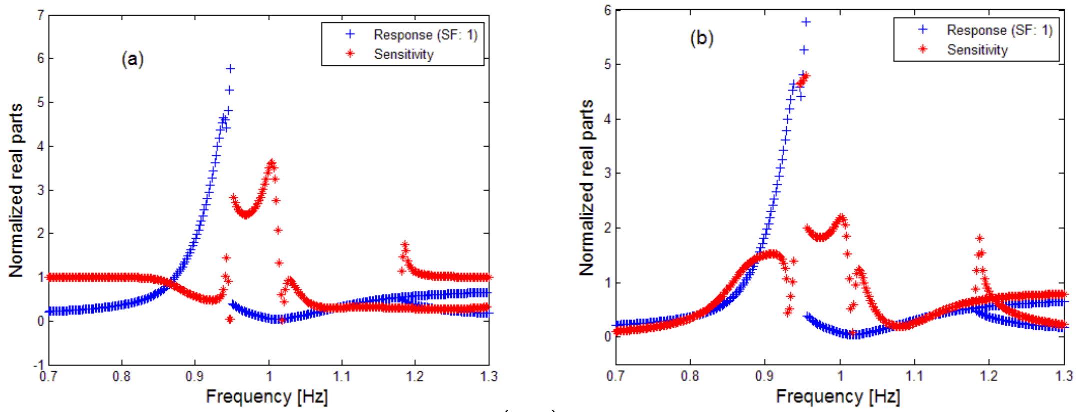

Figure 4 - Sensitivities of $\boldsymbol{H}(\omega, \mathrm{p})$ with respect to β (a) and μ (b).

After having verified the influence of each design variable on the dynamic response of the nonlinear system, the interest now is to maximize the suppression bandwidth, as illustrated in Figure 5, using the bio-inspired algorithms.

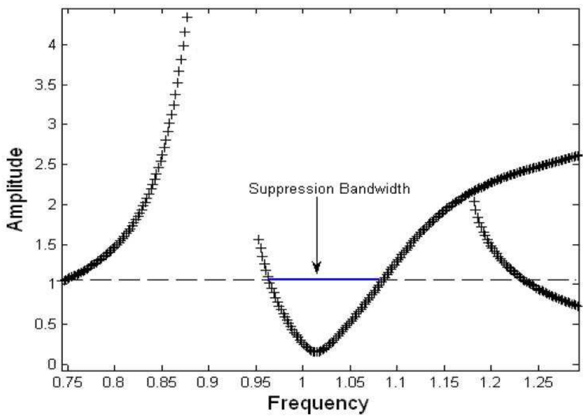

Figure 5 - Representation of the objective function (maximize the suppression bandwidth).

In these simulations, the following ranges to design parameters are considered: $0.9 \leq \rho \leq 1.2$, $0.04 \leq \mu \leq 0.06$, $0.09 \leq \beta \leq 1.2$ and $0.009 \leq \varepsilon_2 \leq 0.012$.

In order to evaluate the performance of the BCA to estimate both, the three test cases listed in Tab.(2) have been performed.

Table 2. Parameters used in bio-inspired algorithm to design of a nonlinear vibration absorber.

BCA parameters	
Number of scout bees	10
Number of bees recruited for the best *e* sites	5
Number of bees recruited for the other selected sites	5
number of sites selected for neighborhood search	5
number of top-rated (elite) sites among *m* selected sites	5
Neighborhood search (*ngh*)	[10^{-3} 10^{-4} 10^{-6}]
Generation Number	50
FA parameters	
Number of fireflies	15
Maximum attractiveness value	0.9
Absorption coefficient	[0. 7 0.9 1.0]
Generation Number	50

The stopping criterion used was the maximum number of iterations. Each case study was computed 20 times before calculating the average values. It should be emphasized that is necessary, with the parameters listed in this table, 1510 objective function evaluations in each algorithm.

In Table 3 the results (best, average and worst) obtained to design of nonlinear vibration absorber are presented.

Table 3. Results obtained by BCA and FA to design of nonlinear vibration absorber.

			ρ (??)	μ (??)	β (??)	ε_2 (??)	Objective Function
BCA	$ngh=10^{-3}$	Best	1.165047	0.053215	0.110475	0.017384	**0.236363**
		Average	1.189594	0.055915	0.102757	0.015343	0.230303
		Worst	1.027717	0.058962	0.091711	0.011162	0.218181
	$ngh=10^{-4}$	Best	1.099421	0.056000	0.103077	0.010199	**0.230304**
		Average	1.134470	0.058684	0.095935	0.011151	0.224242
		Worst	0.916268	0.045893	0.103029	0.011061	0.224255
	$ngh=10^{-6}$	Best	1.134469	0.058680	0.095952	0.011141	**0.224233**
		Average	0.981351	0.046040	0.095621	0.010047	0.218181
		Worst	0.961863	0.050917	0.105860	0.011697	0.218185
FA	$\gamma=0.7$	Best	1.158006	0.054298	0.110800	0.017410	**0.230767**
		Average	1.158007	0.054258	0.110883	0.017400	0.230769
		Worst	1.157047	0.053205	0.110533	0.017399	0.228762
	$\gamma=0.9$	Best	1.148089	0.055678	0.110122	0.018988	**0.230769**
		Average	1.148089	0.055678	0.110122	0.018988	0.230769
		Worst	1.148091	0.055680	0.110123	0.018989	0.230770
	$\gamma=1.0$	Best	1.156679	0.056758	0.111422	0.017777	**0.232775**
		Average	1.148089	0.055678	0.110122	0.018988	0.230769
		Worst	1.151179	0.057998	0.109898	0.015767	0.214715

In this table can be observed that both the algorithms presented good estimates for the unknown parameters. When the BCA is analyzed in terms of the neighborhood search parameter, the best result is obtained using 10^{-3}, i. e., a search region with smaller distances to exploit a large number of food sources. When the FA is analyzed in terms of the absorption coefficient, the best result is found using γ=1.0, i. e, this emphasized local search.

6. CONCLUSIONS

In this work, the Bees Colony Algorithm and the Firefly Algorithm were applied to design of a nonlinear vibration absorber. The nonlinearity was introduced in the springs that connect the primary mass to the ground and the absorber to the primary mass, respectively.

As observed in Tab. (3), both algorithms are able to obtain satisfactory results, in terms of the effectiveness of the nDVA configuration and the number of objective function evaluations. Although the results obtained, both through BCA and FA needs yet to be better studied, so that more definitive conclusions can be drawn, i.e., new mechanisms to diversity exploration should be proposed.

The choice of the design variables is based on previous knowledge regarding their sensitivities with respect to the suppression bandwidth. It is worth mentioning that these parameters are directly associated with the effectiveness of the nDVA.

In terms of the system resolution, the equations of motion of the nonlinear two d.o.f. system were numerically integrated by using the so-called *average method* that provides an approximate solution to nonlinear dynamic problems. The nonlinear algebraic equations obtained were solved numerically enabling to determine the roots of the nonlinear algebraic equations.

The nonlinearity factor is an important parameter to be investigated during the design procedure of nonlinear dynamic systems, due to its contribution to the reduction of the vibration level. However, care must be taken with high values of the nonlinearity because of the instabilities introduced in the nonlinear systems. This point motivates an important aspect regarding the proposed methodology: obtaining the optimal spring nonlinear coefficient that guarantees the best stable solution for a given system.

As further work, we intend to extend the algorithms to the multi-objective context and assess the sensitivity of the parameters with respect to the effectiveness of the solution.

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