A CONTRIBUTION TO THE MEASUREMENT OF CIRCULARITY AND CYLINDRICITY DEVIATIONS

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Abstract. *This paper presents and discusses the procedures used to estimate the measurement uncertainty when determining circularity and cylindricity deviations. Measuring systems and devices frequently used for geometric control of parts both in industrial and in research laboratories were analysed. The work was carried out according to the following steps: (i) analysis of the known Standards and documents used to uncertainty determination, in particular ISO GUM approach; (ii) analysis of the existing measurement systems and devices, with emphasis on their working principles, characteristics and error sources; (iii) identification of the variables that influence the determination of geometry deviations; and (iv) development of mathematical models with analysis and discussion. It was concluded that although the procedures carried out in circularity and cylindricity measurements using different measurement systems are similar, the mathematical models associated with them are different, because each measurement system presents different characteristics and working principles.*

Keywords*: form deviations, cylindricity, circularity, machining.*

1. INTRODUCTION

The main goal of any machining operation is to produce interchangeable parts with maximum functionality at reasonable costs. Such need requires each part or assembly of parts of a final product to be manufactured according to predefined specifications for dimensions, geometry and surface finish.

The evolution of machining tools has enabled the manufacturing of parts to be faster and more practical. New technologies implemented in machining processes have also contributed to improve final products significantly. Despite all of that, the occurrence of dimensional and geometrical deviations is inevitable. An appropriate understanding and use of the existing standards related to GD&T (*Geometric Dimensioning and Tolerancing*) and of the ISO-GPS (*Geometric Product Specification*) standards both in engineering departments and in metrology rooms are fundamental to guarantee that the deviations occurred during manufacturing do not jeopardize the proper assembly and functioning of the manufactured parts.

Sousa and Wandek (2009) have identified in Brazil deficiencies in the understanding and proper application of GD&T in design and manufacturing of parts. The main deficiencies found were related to a superficial knowledge of the existing standards for geometrical specification of parts and to a little experience of designers and manufacturing workers in the extrapolation of such knowledge to help defining good measurement practices, which have caused various problems for Brazilian industry. According to the authors, in addition to the difficulties related to GD&T specifications, difficulties to evaluate uncertainty measurements and to verify if they are adequate according to the specified tolerance are another problem. It is important to invest time and money in a series of actions to minimize such problems, in particular in the qualification of personnel involved in the whole design and manufacturing chain.

Also, dimensional and geometrical control goes beyond the manufacturing chain and gains vital importance in manufacturing research, which can enable many manufacturing processes to be improved.

This paper aims to present procedures to estimate circularity and cylindricity measurements according to ISO GUM (Guide to the Expression of Uncertainty in Measurement) requirements using three measurement systems: (i) a dial gauge attached to a tailstock device; (ii) a Coordinate Measuring Machine (CMM); and (iii) a roundness and cylindricity measurement equipment. We expect this work to help personnel involved in design, manufacturing and quality control to estimate measurement uncertainty and to adequate such measurements to NBR ISO/IEC 17025 (2005). We also expect to contribute to the improvement of scientific rigour in research and therefore in publications in the area.

2. THEORETICAL BACKGROUND

A significant amount of manufactured parts has cylindrical shape or at least some portions with a cylindrical section. Form tolerances (circularity and cylindricity) are frequently applied to such parts, which are particularly important in designs that require high accuracy. The development of instruments and procedures to verify if these tolerances are in accordance with the design requirements is of utmost importance to ensure the interchangeability and functionality of the manufactured parts.

A classic measurement system to verify the form tolerance of parts is composed of a dial gauge attached to a tailstock device. This system has a simple working principle and is relatively cheap, and therefore it has the largest application in industry. It allows measurements to be fast, reliable, and subject to minor influence by the operator.

However, the recent literature in the area shows that other measurement systems are more widely used in research related to form control of machined parts, in particular Coordinate Measuring Machines (Santos 2004, Tedesco *et al*. 2006, Barbosa 2007 and Cavalcante 2010) and Roundness Cylindricity Measurement Systems (Souza *et al*. 2004, Costa *et al*. 2007, Almeida 2008, Oliveira 2008 and Alves *et al*. 2009).

Independently of the measurement system and of the methodology used, the results will always be associated with a measurement uncertainty. Measurement results that are only presented as an arithmetic mean value are not meaningful, since they do not provide all the information about the measurement. Even when the standard deviation is also presented, various other variables that influence the measurements are not considered, such as the uncertainty related to the calibration of the measurement system and to distancing of the temperature in relation to 20 °C.

In order to estimate the measurement uncertainty, the concepts and recommendations presented in ISO GUM must be properly known. This document, published first in 1993, is the result of an international consensus about how to calculate measurement uncertainty.

The application of ISO GUM requires a mathematical model of the measurement process, e.g., the output variable (measurand) must be expressed as a function of the input variables, as shown Eq. (1).

$$Y = f(X_1, X_2, \ldots, X_3) \qquad (1)$$

Where Y represents the output variable and X_1, X_2, ..., X_N are the input variables.

For example, for length measurements, some of the input variables are: resolution of the measurement system; uncertainty associated with the calibration of the measurement system (reported in the calibration certificate); reading variability; distancing of the temperature in relation to 20 °C and temperature variation during measurement.

The calculation of the measurement uncertainty is more realistic if the operator is perfectly knowledgeable about the measurement system and the measurement procedure. Only a qualified operator is able to: define properly the measurand; choose the most adequate measurement system and procedure; define a correct measurement strategy; define environment factors that influence the measurement result; define a representative sample and interpret properly calibration certificates, manuals and catalogues in order to extract relevant information for the uncertainty calculation.

2.1. Circularity and cylindricity deviations

Circularity deviation is graphically equivalent to the minimum radial distance between two concentric circumferences within which the real profile of the part must be contained. As shown in Fig. 1a, the largest circle inscribed in the cross section of the part and the smallest circumscribed circle are considered in order to minimize the distance between the circles.

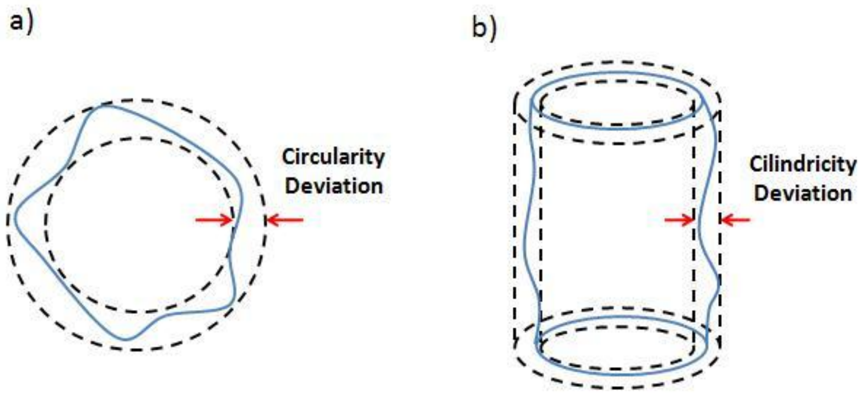

Figure 1. Graphic representation of circularity (a) and cylindricity (b) deviations.

The determination of circularity deviation is carried out in a circular section of a cylindrical part. It must be emphasized that some measurement systems take into account an infinite number of points of the analysed cross section, while others can only make a discrete evaluation by considering a determined number of points over a given cross section.

In many projects, only the application of circularity tolerances is not sufficient to guarantee a good performance of the manufactured parts. In such cases, cylindricity tolerances must also be used to limit the maximum deviations.

Cylindricity deviation is defined as the radial difference between two coaxial cylinders between which the real surface of the part must be contained (Figure 1b). This difference must be equal or less the specified cylindricity tolerance. It is important to point out that cylindricity deviation is a composed form deviation used to control both circularity and straightness of the generatrix.

The verification of circularity and cylindricity deviations is carried out in production lines during the form control of machined parts, as well as during research in machining. In order to follow the standards related to GPS, research has been conducted in the area. Sami *et al.* (2008) have described a method to analyse uncertainty in cylindricity measurements using a CMM. Zhao *et al.* (2010) have proposed a method to estimate uncertainty propagation during the verification of cylindricity deviations according to GPS. Sun *et al.* (2009) have estimated uncertainty in the measurement of form deviations in CMMs following the requirements of the new GPS system.

3. METHODOLOGY

Circularity and cylindricity deviations were measured for a cylindrical aluminium part with a length of 44.68 mm and a diameter of 18.82 mm using three different measurement systems (Figure 2): **System 1**: Dial gauge attached to a tailstock device; **System 2**: CMM and (iii) **System 3**: Roundness cylindricity measurement equipment.

The measurements were carried out by a single operator in only one day at the Dimensional Metrology Laboratory (Laboratório de Metrologia Dimensional), School of Mechanical Engineering (FEMEC), Federal University of Uberlandia (UFU), at a controlled room temperature of 20 ± 1 °C. A thermo-hygrometer with a digital increment of 0.1 °C and measurement range of -20 to 60 ºC was used to monitor the temperature. All the instruments and parts used in the measurement tests were exposed to this temperature for approximately 12 h before the measurements. In order to remove dust or other dirty particles that could interfere with the measurement results, all the instruments and parts were cleaned using isopropyl alcohol, gloves, cotton buds and dry cloths.

3.1. Measurement System 1

This system is composed of dial gauge manufactured by Mitutoyo with a measurement range of 1 mm, where the smallest division corresponds to 1 µm, attached to a tailstock device. The circularity and cylindricity measurements were carried out by positioning the parts as indicated in Fig. 2. This figure also shows a steel ruler with nominal range of 600 mm.

Figure 2. Dial gauge attached to a tailstock device.

In the case of circularity, the measurement used a single cross section of the part, which was turned around its own axis giving a complete turn. The maximum and minimum values indicated in the dial gauge were registered. The circularity deviation for a measurement cycle (D_{CIRi}) was calculated as indicated in Eq. (2).

$$D_{CIRi} = L_{Max} - L_{Min} \tag{2}$$

Where L_{Max} and L_{Min} represent the maximum and minimum values indicated by the dial gauge, respectively.

This procedure was repeated at least three times in order to detect possible errors and to perform a statistical analysis of the results. Therefore, the circularity deviation (D_{CIR}) was averaged as the arithmetic mean of the n measurement cycles, as shown in Eq. (3).

$$D_{CIR} = \frac{\sum_{i=1}^{n} D_{CIRi}}{n} \tag{3}$$

On the other hand, cylindricity evaluation must consider different cross sections along the cylinder length. The number of cross sections depends on the part dimensions, the measurement duration and the required measurement accuracy.

For one measurement cycle, cylindricity deviation is determined by the difference between the maximum and minimum values indicated in the dial gauge during the evaluation of all the cross sections considered in the measurement, Eq. (4). It is worth emphasizing that the dial gauge is only zeroed at the beginning of the measurement of the first cross section.

$$D_{CILi} = Max. - Min. \tag{4}$$

Where *Max.* and *Min.* represent the maximum and minimum values indicated by the dial gauge during the evaluation of all the cross sections considered in a measurement cycle, respectively.

In sequence, cylindricity deviation (D_{CIL}) is averaged as the arithmetic mean of the values found for the n measurement cycles, according to Eq. (5).

$$D_{CIL} = \frac{\sum_{i=1}^{n} D_{CILi}}{n} \tag{5}$$

A similar procedure was extrapolated for the two other measurement systems.

The variables that influence the measurements must be identified in order to estimate the measurement uncertainty associated with circularity measurements (D_{CIR}). The identified variables were: (i) the deviation variability considering the n measurement cycles ($Var(D_{CIRi})$); (ii) the dial gauge resolution (R_{DG}); (iii) the dial gauge hysteresis (H_{DG}); (iv) the uncertainty associated with the dial gauge calibration (UC_{DG}); (v) the distancing of the temperature in relation to 20 °C (ΔT_{20}) and (vi) the temperature variation during the measurements (δT). Therefore, a mathematical model was proposed for D_{CIR}, as shown in Eq. (6).

$$D_{CIRC} = Var(D_{CIRi}) + R_{DG} + H_{DG} + UC_{DG} + \Delta T_{20} + \delta T \tag{6}$$

If the measurements are carried out at a controlled room temperature of 20 ± 1 °C, ΔT_{20} can be neglected, since the geometrical deviation values are very small, only of the order of a few micrometers. If the measurements are carried out within a short period of time, δT can also be neglected. These observations are also valid for the two other measurement systems.

Neglecting the two last terms of Eq. (6) and applying the law of propagation of uncertainty, we obtain Eq. (7).

$$\begin{aligned} u_c^2(D_{CIRC}) = {} & \left(\frac{\partial D_{CIRC}}{\partial Var(D_{CIRi})}\right)^2 \cdot u^2(Var(D_{CIRi})) + \left(\frac{\partial D_{CIRC}}{\partial R_{DG}}\right)^2 \cdot u^2(R_{DG}) + \\ & + \left(\frac{\partial D_{CIRC}}{\partial H_{DG}}\right)^2 \cdot u^2(H_{DG}) + \left(\frac{\partial D_{CIRC}}{\partial UC_{DG}}\right)^2 \cdot u^2(UC_{DG}) \end{aligned} \tag{7}$$

In Eq. (7), the combined standard uncertainty is the sum of the product of the squared standard uncertainties for each influencing and their respective sensitivity coefficients (also squared). These coefficients (partial derivatives) describe how the output estimate (circularity or cylindricity) varies when the input estimates change. The calculation of the standard uncertainties is described in sequence.

The standard uncertainty related to the deviation variability $u(Var(D_{CIR}))$ can be calculated as shown in Eq. (8).

$$u(Var(D_{CIRi})) = \frac{s(D_{CIR})}{\sqrt{n}} \tag{8}$$

Where $s(D_{CIR})$ is the standard deviation of the deviation readings and n is the total number of measurement cycles.

This uncertainty is classified as Type A. It presents a normal probability with degree of freedom of (n-1), i.e., the total number of cycles minus 1.

The resolution of the dial gauge presents a Type B standard uncertainty $u(R_{DG})$ with an infinite number of degrees of freedom, considering a rectangular probability distribution. The uncertainty $u(R_{DG})$ can be estimated using Eq. (9).

$$u(R_{DG}) = \frac{resolution}{\sqrt{3}} \tag{9}$$

The uncertainty related to the dial gauge hysteresis $u(H_{DG})$ is of Type B, since information related to its expanded uncertainty can be found in the instrument calibration certificate. The degree of freedom is 4 and the value of k is 2. Eq. (10) can be used to calculate $u(H_{DG})$.

$$u(H_{DG}) = \frac{U_{P(hysteresis)}}{k} \tag{10}$$

Finally, the standard uncertainty related to the dial gauge calibration $u(UC_{DG})$ is classified as Type B, with a normal distribution, 4 degrees of freedom and k of 2.3. $u(UC_{DG})$ standard uncertainty is calculated according to Eq. (11).

$$u(UC_{DG}) = \frac{U_{P(calibration)}}{k} \tag{11}$$

The combined standard uncertainty calculated using Eq. (7) presents a coverage probability of only 68% and therefore the calculation of the expanded uncertainty (U_P) becomes necessary. For that, the combined standard uncertainty must be multiplied by a coverage factor k (Eq. (12). This factor is obtained from the T-Student Table according to the measurement effective degree of freedom v_{eff}, in order to increase the coverage probability to 95 %.

$$U_P = k \cdot u_c \tag{12}$$

The effective number of degrees of freedom is given by the Welch-Satterthwaite expression, as shown in Eq. (13). In this expression, the symbols $u_c(y)$ represents the combined standard uncertainty, $u_i(y_i)$ is the standard uncertainty for each variable, and v_i is the degrees of freedom associated with the distribution of each input variable.

$$v_{eff} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \frac{u_i^4(y_i)}{v_i}} \tag{13}$$

3.2. Measurement System 2

Measurement System 2 is a manual CMM, type moving bridge, manufactured by Mitutoyo. The resolution is 0.001 mm and the work volume is 300 mm (Axis X) x 400 mm (Axis Y) x 300 mm (Axis Z).

Despite the different possible configurations of CMMs, their working principle is similar. They work by storing digitally the coordinates of the measurement points (X, Y and Z). Computational programs use these coordinates to calculate the desired feature (circle diameter, sphere diameter, distance, angle, form deviations, etc.).

These programs are based on analytical geometry and vector analysis principles. They generally use least squares methods to adjust the part geometry. Therefore, the uncertainty associated with the adjustment method must also be considered to estimate the measurement uncertainty. For that, the law of propagation of uncertainty must be applied in the mathematical model until the calculation of the final characteristic.

All measurements must start with the definition of the number of measuring points on the part surface (sample size). In the case of circularity, these points must be determined in a cross section. During the measurement, a computer software stores the coordinates X, Y and Z of each point, projecting them in a so-called projection plane. In sequence, the radius (r) and the coordinates of the centre (x_c, y_c) of the circle that adjusts best to the n points projected onto the plane XY are determined (Eq. 14).

$$(x - x_c)^2 + (y - y_c)^2 = r^2 \tag{14}$$

To estimate the radius and coordinates of the centre of this circle, the expression given by Eq. (15) must be minimized.

$$\sum \varepsilon_i^2 = \sum \left(r_i^2 - r^2\right)^2 = \sum \left[-2 \cdot x_i \cdot x_c - 2 \cdot y_i \cdot y_c + \left(x_i^2 + y_i^2\right) + \left(x_c^2 + y_c^2 - r^2\right)\right]^2 \qquad (15)$$

Rearranging variables as shown in Eq. (16), the expression given by Eq. (15) can be rewritten in a linear form (Eq. (17)).

$$a = -2 \cdot x_c \qquad b = -2 \cdot y_c \qquad c = x_c^2 + y_c^2 - r^2 \qquad (16)$$

$$MQ = \sum \left[a \cdot x_i + b \cdot y_i + \left(x_i^2 + y_i^2\right) + c\right]^2 \qquad (17)$$

The least squares coefficients are determined by equating the partial derivatives of MQ in relation to a, b e c to zero, Eqs. (18)-(20).

$$\frac{\partial MQ}{\partial a} = 2 \cdot \sum \left[a \cdot x_i + b \cdot y_i + \left(x_i^2 + y_i^2\right) + c\right] \cdot \left(x_i\right) = 0 \qquad (18)$$

$$\frac{\partial MQ}{\partial b} = 2 \cdot \sum \left[a \cdot x_i + b \cdot y_i + \left(x_i^2 + y_i^2\right) + c\right] \cdot \left(y_i\right) = 0 \qquad (19)$$

$$\frac{\partial MQ}{\partial c} = 2 \cdot \sum \left[a \cdot x_i + b \cdot y_i + \left(x_i^2 + y_i^2\right) + c\right] \cdot 1 = 0 \qquad (20)$$

In the matrix form, we obtain the linear system (21).

$$\begin{bmatrix} \sum x_i^2 & \sum x_i \cdot y_i & \sum x_i \\ \sum x_i \cdot y_i & \sum y_i^2 & \sum y_i \\ \sum x_i & \sum y_i & n \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -\sum (x_i^3 + x_i \cdot y_i^2) \\ -\sum (x_i^2 . y_i + y_i^3) \\ -\sum (x_i^2 + y_i^2) \end{bmatrix} \qquad (21)$$

In sequence, the distance (Di) of each point P_i to the centre of the circle (P_c) is calculated, identifying the most distant point ($Pmax$) and least distant point ($Pmin$) in relation to P_c (22).

$$D_{Pi} = D\left(P_c, P_i\right) = \sqrt{\left(x_i - x_c\right)^2 + \left(y_i - y_c\right)^2 + \left(z_i - z_c\right)^2} \qquad (22)$$

Where (x_c, y_c, z_c) and (x_i, y_i, z_i) are the coordinates of the points P_c and P_i respectively.

The circularity deviation is given by the difference between the maximum and minimum distances, as shown by Eq. (23).

$$D_{Circ} = D_{P\max} - D_{P\min} \qquad (23)$$

This process must be repeated at least three times and the results must be averaged by calculating the arithmetic mean.

The deviation measurement uncertainty (D_{CIRC}) is influenced by the uncertainty of the coordinates of the points that generate the circle and by the uncertainty of the coordinates of the points P_{max} and P_{min}. Therefore, the law of propagation of uncertainty must be applied to express the combined standard uncertainty as:

$$u_c^2\left(D_{CIRC}\right)=\sum_{j=1}^{N}\left(\frac{\partial D_r}{\partial x_j}\right)^2\cdot u^2\left(x_j\right)+\sum_{j=1}^{N}\left(\frac{\partial D_r}{\partial y_j}\right)^2\cdot u^2\left(y_j\right)+\left(\frac{\partial D_{P\max}}{\partial x_{P\max}}\right)^2\cdot u^2\left(x_{P\max}\right)+$$
$$+\left(\frac{\partial D_{P\max}}{\partial y_{P\max}}\right)^2\cdot u^2\left(y_{P\max}\right)+\left(\frac{\partial D_{P\max}}{\partial z_{P\max}}\right)^2\cdot u^2\left(z_{P\max}\right)+\left(\frac{\partial D_{P\min}}{\partial x_{P\min}}\right)^2\cdot u^2\left(x_{P\min}\right)+ \tag{24}$$
$$+\left(\frac{\partial D_{P\min}}{\partial y_{P\min}}\right)^2\cdot u^2\left(y_{P\min}\right)+\left(\frac{\partial D_{P\min}}{\partial z_{P\min}}\right)^2\cdot u^2\left(z_{P\min}\right)$$

In sequence, all the partial derivatives in Eq. (24) must be calculated. The calculations are complex and require the use of applicative programming languages such as Mathematica. The whole mathematical development can be found in Vieira Sato (2003).

However, in this case a simplification can be used to calculate the measurement uncertainty, as proposed by Eq. (25).

$$D_{CIR}=Var\left(D_{CIR}\right)+R_{CMM}+E_{TPS}+UC_{CMM}+\Delta T_{20}+\delta T \tag{25}$$

Applying the law of propagation of uncertainty to equation (25), we have:

$$u_c^2\left(D_{CIRC}\right)=\sum_{j=1}^{N}\left(\frac{\partial D_{CIR}}{\partial Var\left(D_{CIR}\right)}\right)^2\cdot u^2\left(D_{CIR}\right)+\sum_{j=1}^{N}\left(\frac{\partial D_{CIR}}{\partial R_{CMM}}\right)^2\cdot u^2\left(R_{CMM}\right)+$$
$$+\left(\frac{\partial D_{CIR}}{\partial E_{TPS}}\right)^2\cdot u^2\left(E_{TPS}\right)+\left(\frac{\partial D_{CIR}}{\partial UC_{CMM}}\right)^2\cdot u^2\left(UC_{CMM}\right)+ \tag{26}$$
$$+\left(\frac{\partial D_{CIR}}{\partial \Delta T_{20}}\right)^2\cdot u^2\left(\Delta T_{20}\right)+\left(\frac{\partial D_{CIR}}{\partial \delta T}\right)^2\cdot u^2\left(\delta T\right)$$

It is important to consider that the uncertainty associated with circularity deviation depends on many variables: (i) Variability of the circularity values for the *n* measurement cycles; (ii) resolution of the CMM (R_{CMM}); (iii) errors of the CMM probe system (E_{TPS}); (iv) uncertainty associated with the CMM (UC_{CMM}); (v) distancing of the temperature in relation to 20 °C (ΔT_{20}); and (vi) temperature variation during measurement (δT).

3.3. Measurement System 3

This measurement system consists of a roundness cylindricity measurement equipment manufactured by Taylor Hobson, model Talyrond 131, with a resolution of 0.01 µm and a measurement range of 370 mm.

This equipment presents excellent metrological properties and high diagnosis power. During the circularity and cylindricity measurements, it also provides a graph related to the profile or effective surface of the part. Due to such characteristics, these equipments have experienced an increased use in research development in areas where the geometrical evaluation of parts is required.

During the measurements the parts are positioned in a rotating table. Adjustment screws and a computational program are used to level and centre the equipment. A stylus containing a ruby sphere touches the parts in different cross sections.

Circularity measurements are carried out in a cross section of the part considering all the infinite number of points of the profile. For cylindricity measurements, different cross sections along the length of the part are assessed. The measurement process is completely automated, which minimizes errors associated with the operator.

The variables that can influence the measurement results are: (i) variability of the circularity values found for the different measurement cycles (Var_{DCIR}); (ii) machine resolution (R_M); (iii) uncertainty associated with the machine calibration (UC_M); (iv) probing error (PE_M); (v) eccentricity deviation of the machine table (ED_M); (vi) distancing of the temperature in relation to 20 °C (ΔT_{20}); and (vii) temperature variation during measurement (δT), which were summed in Eq. (27).

$$D_{CIR}=Var_{DCIR}+R_M+UC_M+PE_M+ED_M+\Delta T_{20}+\delta T \tag{27}$$

Applying the law of propagation of uncertainty:

$$u_c^2(D_{CIRC}) = \sum_{j=1}^{N}\left(\frac{\partial D_{CIR}}{\partial Var(D_{CIR})}\right)^2 \cdot u^2(D_{CIR}) + \sum_{j=1}^{N}\left(\frac{\partial D_{CIR}}{\partial R_M}\right)^2 \cdot u^2(R_M) + \\ + \left(\frac{\partial D_{CIR}}{\partial UC_M}\right)^2 \cdot u^2(UC_M) + \left(\frac{\partial D_{CIR}}{\partial PE_M}\right)^2 \cdot u^2(PE_M) + \left(\frac{\partial D_{CIR}}{\partial ED_M}\right)^2 \cdot u^2(ED_M) \\ + \left(\frac{\partial D_{CIR}}{\partial \Delta T_{20}}\right)^2 \cdot u^2(\Delta T_{20}) + \left(\frac{\partial D_{CIR}}{\partial \delta T}\right)^2 \cdot u^2(\delta T) \quad (28)$$

In this case, the uncertainty associated with the adjustment method was not considered.

4. RESULTS AND DISCUSSION

In this section we only present the results obtained using the Measurement System 1. The results obtained with the two other measurement systems will not be published in this paper.

Table 1 presents the circularity values for the five measurement cycles carried out with Measurement System 1. Although the smallest division of the dial gauge was 1 µm, all the measurements were carried out with interpolation, which was possible due to both the excellent quality of the gauge visor and the experienced operator.

Table 1. Values obtained during the circularity deviation measurement tests.

Parameters (mm)	1st Cycle	2nd Cycle	3rd Cycle	4th Cycle	5th Cycle
Maximum reading	0.4075	0.4075	0.4080	0.4080	0.4080
Minimum reading	0.3620	0.3625	0.3630	0.3625	0.3625
Circularity deviation	0.0455	0.0450	0.0450	0.0455	0.0455
Mean	0.0453				
Standard deviation	0.0002				

Table 1 shows that the circularity deviation values are similar for the different measurement cycles, varying between 0.0450 and 0.0455 mm. After averaging the circularity deviation values obtained in the 5 measurement cycles, the circularity deviation of the part was calculated as 0.0453 mm, with a standard deviation of 0.0002 mm.

Similarly, the maximum and minimum values along the length of the part were obtained in order to calculate cylindricity deviation. The values for each measurement cycle are presented in Tab. 2.

Table 2. Values obtained during the cylindricity deviation measurement tests.

Parameters (mm)	1st Cycle	2nd Cycle	3rd Cycle	4th Cycle	5th Cycle
Max(Maximum reading)	0.4510	0.4425	0.4450	0.4440	0.4340
Min(Minimum reading)	0.3810	0.3705	0.3680	0.3580	0.3545
Cylindricity deviation	0.0700	0.0720	0.0770	0.0860	0.0795
Mean	0.0769				
Standard deviation	0.0063				

Table 2 shows that the cylindricity deviation values varied between 0.0700 and 0.0860 mm for the different measurement cycles. A mean of 0.0769 mm and a standard deviation of 0.0063 were obtained after averaging the values for the different cycles.

Comparing Tabs. 1 and 2, a clear difference between circularity and cylindricity deviation values is detected, where cylindricity deviation is shown to be more relevant than circularity deviation. The standard deviation associated with cylindricity measurements is also greater than that for circularity measurements, since it is not possible to analyse the same cross sections during the 5 cylindricity measurement cycles. During circularity measurements, the part is not translated, whereas the dial gauge is moved along the length of the part during cylindricity measurements in order to analyse all its relevant cross sections.

Table 3 summarizes all information related to the calculation of the circularity deviation measurement uncertainty.

Table 3. Uncertainty associated with the circularity deviation measurements.

Source of uncertainty (X_i)	Mensurand Estimation x_i	Probability Distribution	Sensitivity Coefficient	Uncertainty Type	Degrees of Freedom	Standard Uncertainty
Standard Deviation of L_{DG}	0.0002 mm	Normal	1	A	4	0.0894 µm
R_{DG}	0.5 µm	Rectangular	1	B	∞	0.2882 µm
H_{DG}	0.7 µm	Normal	1	B	4	0.3500 µm
UC_{DG}	0.6 µm	Normal	1	B	4	0.2609 µm
Combined standard uncertainty (u_c)						0.53 µm
Effective degrees of freedom (v_{eff})						16.02
Coverage factor (v_{eff}, 95 %)						k = 2.12
Expanded uncertainty (Up)						1.12 µm

The variable that exerts the strongest influence on combined standard uncertainty, and therefore on expanded uncertainty, is the dial gauge hysteresis (0.0004 mm). The weakest influence was exerted by the deviation variability, with a standard uncertainty of 0.1 µm.

The expanded uncertainty was calculated as 1.1 µm for a coverage probability of de 95 % and k of 2.12.

The uncertainty associated with cylindricity deviation measurements was also calculated and the results are presented in Tab. 4.

Table 4. Uncertainty associated with the cylindricity deviation measurements.

Source of uncertainty (X_i)	Mensurand Estimation x_i	Probability Distribution	Sensitivity Coefficient	Uncertainty Type	Degrees of Freedom	Standard Uncertainty
Standard Deviation of L_{DG}	0.0063 mm	Normal	1	A	4	2.8390 µm
R_{DG}	0.5 µm	Rectangular	1	B	∞	0.2882 µm
H_{DG}	0.7 µm	Normal	1	B	4	0.3500 µm
UC_{DG}	0.6 µm	Normal	1	B	4	0.2609 µm
Combined standard uncertainty (u_c)						2.89 µm
Effective degrees of freedom (v_{eff})						5.34
Coverage factor (v_{eff}, 95 %)						k = 2.53
Expanded uncertainty (Up)						7.30 µm

Deviation variability was the variable with the strongest influence on the final uncertainty during cylindricity measurements, assuming a value 2.8 µm. This occurred because the value of the standard deviation of the readings was high, in particular when compared with that obtained in the circularity measurements.

In order to reduce the standard uncertainty associated with deviation variability, it is possible to increase the number of cross sections analysed in each measurement cycle, which increases the sample size. However, this would result in higher costs and lengthier measurements.

The standard uncertainty associated with the dial gauge calibration had the weakest influence on the final uncertainty, only ± 0.0003 mm.

The expanded uncertainty associated with cylindricity measurements was calculated as ± 0.0073 mm, for a coverage probability of 95 % and k of 2.53.

5. CONCLUSIONS

Although the procedures carried out in circularity and cylindricity measurements using different measurement systems are similar, the mathematical models associated with them are different, because each measurement system presents different characteristics and working principles.

In the mathematical models presented for each measurement system, the following variables influenced the results obtained: Resolution of the measurement system; uncertainty associated with the calibration of the measurement system; distancing of the temperature in relation to 20 °C; and temperature variation during measurement.

For the measurements using the dial gauge, the mean value of the circularity deviation of the analysed part was 0.0453 mm, with a standard deviation of 0.0002 mm. On the other hand, cylindricity deviation measurements presented a mean value of 0.0769 and a standard deviation of 0.0063 mm, which were greater than those obtained for circularity measurements. This must have occurred because cylindricity deviation takes into account not only circularity but also the straightness of the generatrix.

The circularity expanded uncertainty was ± 0.0011 mm, for a coverage probability of 95 % and k of 2.12. The dial gauge hysteresis had the strongest influence on the final uncertainty, assuming a value of ± 0.0004 mm.

The cylindricity expanded uncertainty was ± 0.0073 mm, for a coverage probability of 95 % and k of 2.53. The variability between values obtained in different measurement cycles had the strongest influence on the final uncertainty, assuming a value of ± 0.0028 mm.

6. ACKNOWLEDGEMENTS

The authors are grateful to FAPEMIG/Brazil for financial support.

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