NUMERICAL ANALYSIS OF NANOFLUIDS FLOWING IN A STRAIGHT PIPE

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Abstract. *The main objective of this work is to initiate the analysis of thermal and hydrodynamic behavior of nanofluids, made by metallic oxides and metallic nanoparticles in water as a base fluid, flowing in a horizontal circular tube. A brief review about both experimental and theoretical approaches to the enhancement of thermo-physical properties is presented. For numerical simulation was considered a single-phase flow with enhanced thermal conductivity and viscosity, based on correlations found in the open literature. In these conditions, selected correlations were used in order to verify the influence of many parameters involved in the thermal process. A basic scheme was used to simulate nanofluids flowing through a horizontal straight pipe with smooth wall, using commercial software CFX. Finally, the results of simulation were compared with experimental data obtained for the nanofluids with alumina nanoparticles.*

Keywords*: nanofluids, heat transfer, thermal conductivity, numerical simulation, laminar flow, alumina.*

1. INTRODUCTION

Nowadays, with ever-increasing heat transfer in processes at all levels (from internal combustion engines to microprocessors heat dissipation), the efficiency improvement of such systems is a challenging task for scientist and engineers. The search of refrigerants with better thermal properties is a usual way to achieve that goal, without modifying drastically the operation scheme of conventional systems. On that matter, several studies have been developed to enhance the inherently poor thermal properties of some usual refrigerants, as water and oil.

Thermal conductivity of solids is much higher than those of liquids as shown in Tab. 1. Thus, the addition of metallic and non-metallic particles in a homogeneous fluid in order to improve its thermal properties is a long-time discussed topic. This enhancement on thermal properties has encouraged researchers to envision a further application of those suspensions. However, some problems emerged by the colloidal nature of the fluids. First of them was the low stability showed when mili and micro-sized particles were used, with the corresponding decay of previously enhanced properties. Another undesirable effect was the augmentation of viscosity (proper of this kind of colloids), which implies a major consumption of energy to pump the work fluid through all the system. These limitations have slowed down the research on this specific area. With the arise of nanotechnology (manipulation of matter on atomic and molecular scales) returned the interest of these colloids for heat transfer.

Named by Choi (1995), "*nanofluids*" are colloidal suspensions of nano-sized particles (with typical length scales between 1 and 100nm), dispersed in a base fluid in order to improve its thermal properties.

Table 1. Values of thermal conductivity for typical materials on nanofluids research.

	Material	k (W/m-K)
Solids	Carbon Nanotubes (CNT)	3000
	Graphite	110 - 190
	Alumina (Al_2O_3)	40
	Copper Oxide (CuO)	18
	Titania (TiO_2)	13,7
Liquids	Water	0,613
	Ethylene Glycol	0,253
	Propylene Glycol	0,182
	Engine oil	0,145

The most visible variation was on thermal conductivity, as reported by Lee *et al.* (1999), Eastman *et al.* (2001), Xie *et al.* (2002) and Chon *et al.* (2005). In their experimental tests were reached enhancements in thermal conductivity of almost 40% at less than 5% volume concentration of alumina and copper oxide in water. Also, it was noticed better stability for these new nanofluids, when compared to those suspensions of bigger particles.

All these features make of the study of nanofluids a large area of knowledge to develop, because expectations for its use have been long superseded by the experimental results. Product of interest in the scientific community worldwide, we have a steadily growing literature, which is shown in Fig. 1.

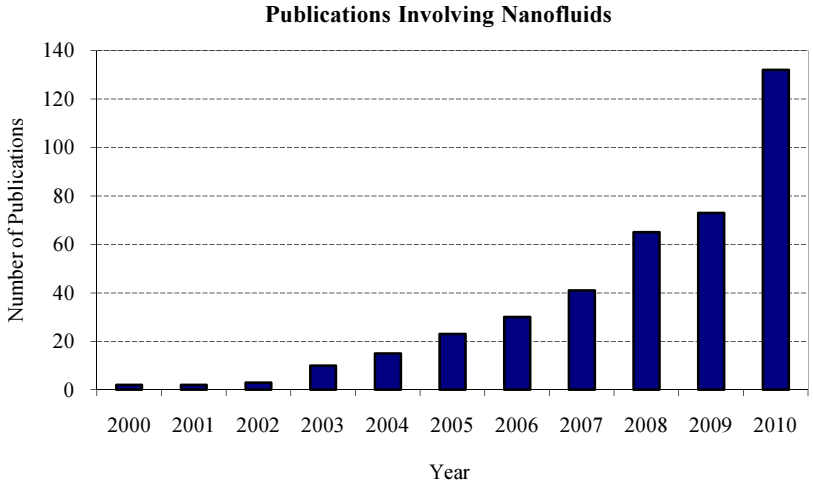

Figure 1. Number of international publications for the past decade.

Many of these studies presented the effects that would have the use of nanoparticles on the thermal conductivity and the convective heat transfer. Both topics were discussed from theoretical and experimental points of view in order to achieve a certain description of the phenomena.

2. EXPERIMENTAL RESEARCH

2.1. Thermal conductivity measurement

The measurement of enhanced properties was the first step to understand the nanofluids phenomena. As mentioned before, dispersing nanoparticles in a base fluid has a positive effect over the thermal conductivity, which could be measured by conventional and adapted methods. Usually, the measurement of thermal conductivity of liquids is based on Fourier's law of heat conduction, establishing a steady one-dimensional heat flow by the application of a known heat flux. Then, from the measures of temperature at also known locations, the thermal conductivity can be estimated. The main difficulty of using this principle is to provide a one-dimensional temperature field. Convection is also an obstacle to avoid for remaining the measurement errors at minimum. To overcome this, measurements on liquids must be done in a very small period of time for preventing the development of convection currents.

The transient hot-wire (THW) method is by far the most used for measuring thermal conductivity on nanofluids because of its balance between accuracy and simplicity. This method uses a platinum wire both as a heater and as a thermometer, immersed into the nanofluid. The temperature rise of the wire depends on the thermal conductivity of the surrounding fluid and is calculated from the variation in its resistance. Many experimental researches have been carried out using this method, including Murshed *et al.* (2005), Hong *et al.* (2006), Zhang *et al.* (2007), and Lee *et al.* (2008), who studied, mainly, metallic oxide and metallic nanoparticles dispersed in water.

Others methods (such as thermal constants analyzer technique, steady-state parallel-plate method, cylindrical cell method, temperature oscillation technique) show some comparative advantages, however are still less used than THW method. More information about these techniques is available in Paul *et al.* (2010).

2.2. Convective heat transfer coefficient

In terms of design, heat transfer coefficient is the proper indicator for heat exchange equipment. Experimental results have proved that heat transfer enhancement exceed the expectations due to the enhanced properties of nanofluids. Thus, Heris *et al.* (2009) reported experimental results of Al_2O_3 and CuO nanoparticles in water for laminar and turbulent regimes with gains of 40% and 15% for convective heat transfer and thermal conductivity, respectively.

Many schemes of facilities for convective heat transfer study are found in literature, but they can be divided by the boundary conditions set (constant heat flux or constant wall temperature), orientation (vertical or horizontal) and measuring characteristics (e.g. thermocouples distribution). Figs. 2 and 3 show some of this principal features:

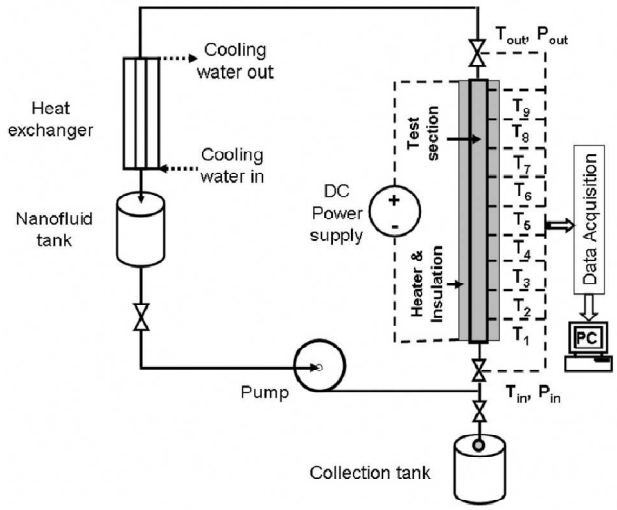

Figure 2. Facility with vertical test section, electrically heated . Chen *et al.* (2008).

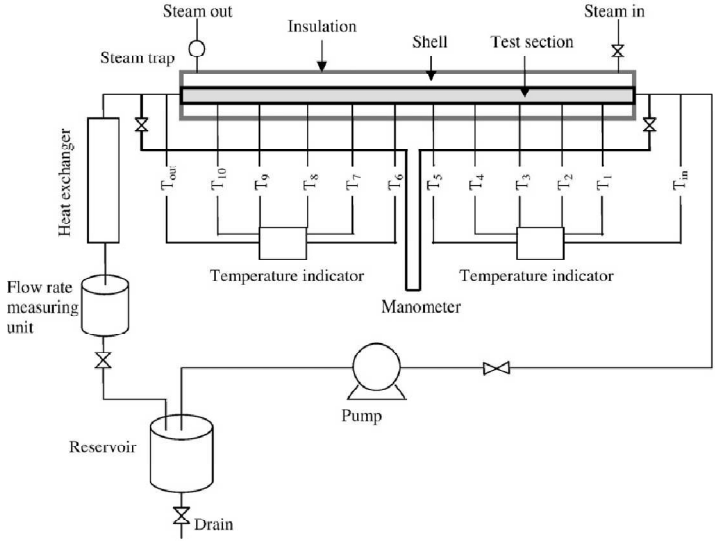

Figure 3. Facility with horizontal test section, heated by steam. Fotukian *et al.* (2010)

Generally, data processing is based on the modified properties of nanofluids and well-known correlations for heat transfer in internal flows. Equations (1) and (2) determine the properties for multi-component substances (models for the enhancement of thermal conductivity and viscosity are discussed later).

$$\rho_{nf} = (1 - \varphi_p)\rho_{bf} + \varphi_p \rho_p \tag{1}$$

$$(C_p)_{nf} = (1 - \varphi_p)(C_p)_{bf} + \varphi_p (C_p)_p \tag{2}$$

For determining the value of heat transfer coefficient, acquired temperatures along the test section are used in Eqs. (3) and (4), for constant heat flux and constant wall temperature, respectively. The Nusselt number is also calculated from the Eq. (5):

$$h = \frac{q''}{T_w - T_b} \tag{3}$$

$$h = \frac{C_p\ \rho\ u\ A\ (T_{b2} - T_{b1})}{\pi\ D\ L\ (T_w - T_b)_{LM}} \tag{4}$$

$$Nu = \frac{h\ D}{k} \tag{5}$$

3. THEORETICAL RESEARCH

For theoretical modeling, the nanofluid must be understood as a mixture formed by a continuous fluid (denominated base fluid) and a solid component that is dispersed nanoparticles. Thermal properties will depend on several factors, including the most important component properties, such as: volume concentration, size, distribution and shape of nanoparticles, among others. However, it is almost impossible to estimate the effective properties of nanofluids without knowing the detail of their intrinsic behavior. To avoid this complication factor, such properties are based on the reasonable assumptions of the structure of the mixture. In this paper, the theoretical approaches are showed of only two properties of nanofluids, highly relevant for the simulation to be performed.

3.1. Theoretical models for thermal conductivity and viscosity

The pioneering study by Maxwell (1873) considered the conduction phenomenon on a dilute suspension of spherical particles, neglecting the interactions between particles. The Maxwell equation, obtained as a solution to Laplace's equation for a temperature field surrounding the particles, is as follows:

$$k_{eff} = k_{bf} + 3\varphi_p \frac{k_p - k_{bf}}{2k_{bf} + k_p - \varphi_p(k_p - k_{bf})} k_{bf} \tag{6}$$

where k represents the thermal conductivity; φ, nanoparticle volumetric fraction; and subscripts *eff*, *p* and *bf*, mean *effective*, *nanoparticle* and *base fluid*, respectively.

Most of the further developed models was based on this model, with a few more considerations, such as the particle shape (Fricke, 1924; Hamilton & Crosser, 1962; Granqvist & Hunderi, 1978; Xue, 2000), distribution of particles (Wiener, 1912), particle-layer structure (Kerner, 1956, Lamb *et al.* 1978), contact resistance (Benvensite, 1987).

Among these, the Hamilton-Crosser model showed a major contribution, as it considers one parameter that evaluates the particle shape. The Hamilton-Crosser model is still used to compare some experimental results and the expression is as follows:

$$k_{eff} = k_{bf} + 3\,\psi\,\varphi_p \frac{k_p - k_{bf}}{(3\psi^{-1} - 1)k_{bf} + k_p - \varphi_p(k_p - k_{bf})} k_{bf} \tag{7}$$

ψ represents the particle shape parameter, as a function of sphericity factor. For spheric nanoparticles ψ=1

A different perspective for the intensification of thermal conductivity in nanofluids was discussed by assuming the presence of an interfacial *layer* formed by the arrangement of the base fluid molecules nearby the surface of the nanoparticles. Thus, this quasi-static structure has different properties from the base fluid and the nanoparticle. Yu and Choi (2003) employed this new parameter to modify the expression of Hamilton-Crosser and to obtain a model of thermal conductivity based on the shape of the particles and the properties that complex particle (formed by the nanoparticle and its interfacial layer):

$$k_{cp} = \frac{[2(1-\gamma)+(1+\beta)^3(1+2\gamma)]\gamma}{(1-\gamma)+(1+\beta)^3(1+2\gamma)} k_p \tag{8}$$

with $\beta = t/R$ and $\gamma = k_{lr}/k_p$, where t represents the thickness of the layer, R is the nanoparticle radius, and subscripts *lr* and *cp* mean *layer* and *complex particle*, respectively.

The Yu and Choi model is obtained by replacing k_{cp} on Hamilton-Crosser model for spheric nanoparticles:

$$k_{eff} = \frac{k_p + 2k_{bf} + 2\varphi_p(k_p - k_{bf})(1+\beta)^3}{k_p + 2k_{bf} - \varphi_p(k_p - k_{bf})(1+\beta)^3} k_{bf} \tag{9}$$

Another relevant property for establishing the practical application of nanofluids in conventional systems is the viscosity. Einstein (1906) began studying the increase of the viscosity of a heterogeneous medium, reaching the following expression:

$$\mu_{eff} = \mu_{bf}\left(1 + 2{,}5\varphi_p\right) \tag{10}$$

where μ represents the viscosity, and the subscripts remain

Equation (10) served as a basis for some later models as Bruijin (1942), Vand (1948), Brinkmann (1952). Outstanding of these is the model of Batchelor (1977):

$$\mu_{eff} = \mu_{bf}\left(1 + 2{,}5\varphi_p + 6{,}5{\varphi_p}^2\right) \tag{11}$$

Despite the many models developed for both thermal conductivity and viscosity, comparison with experimental data still does not allow to conclude which of them actually describes the thermal and hydrodynamic phenomena of nanofluids. Equations (6) and (11) were used to determine the modified properties of nanofluids used in this paper.

3.2. Numerical solutions for nanofluids flow

Although the results of theoretical approaches for the properties of nanofluids are inconclusive (and based on increments of convective heat transfer above the values predicted by the intensification of thermal conductivity) were carried out numerical simulations that took into account the dynamic effects that previous models neglected.

A nanofluid is, by the nature of its production, a substance with multiple components. Hence, most of the researches on literature about forced convection consider it as a homogeneous two-phase flow, with non-slip condition between the nanoparticles and base fluid. In forced convection, flows are dependent on both Reynolds and Prandtl numbers.

However, in the case of nanofluids flowing must be considered other factors that enhance the convective heat transfer above of the expected values. As observed before, it can be noted that such intensification will not only depend on the effective thermal conductivity of nanofluid and regime conditions studied, but also the geometric characteristics of nanoparticles and their concentration. According to this, Roetzel and Xuan (2000) proposed the following general form for the Nusselt number:

$$Nu_{nf} = f\left[Re, Pr, \frac{k_p}{k_f}\frac{(\rho C_p)_p}{(\rho C_p)_f}, \phi_p, particle\ shape\right] \tag{12}$$

where ρ is for density; C_p, specific heat, and the subscripts p and f represent nanoparticles and base fluid, respectively.

Considering the non-slip condition between both components of nanofluids, numerical study of nanofluids flowing in various regimes is divided into two approaches:

First approach: the hypothesis of nanofluid as a continuous medium is still valid, even with the suspended nanoparticles. This means that the flow could be considered single-phase, with only the thermal and physical properties modified. However, for these nanofluids, the variation between the use of constant temperature and dependent properties is considerable.

Second approach: it considers the nanofluid as a two-phase compound for a better description of the behavior of solid and liquid phases. This is a more closely way for studying the phenomena, since it assumes a nonzero slip velocity between the nanoparticles and base fluid, due to several factors such as gravity, friction between the base fluid molecules and nanoparticles, Brownian diffusion, sedimentation and dispersion. This approach applies a dynamic focus different for each phase, Eulerian for the case of base fluid and Lagrangian for nanoparticles (in order to capture the individual trajectories of these).

For this initial work, first approach was adopted by considering a single-phase flow of nanofluids with new thermal and rheological properties.

4. METHODOLOGY

One purpose of this work is to begin the study of nanofluids flow from the easier case to those more complicated. Based on the case study of He *et al.* (2007), it was supposed a tube heated by a constant heat flux of 4000 W/m^2. The tube length and internal diameter were set to 1 m and 1 cm, respectively. Initially, the study was for a flow of Alumina nanoparticles with 0,5% vol., 1.0% vol. and 3,0% vol. dispersed in water (named as NF1, NF2 and NF3, respectively). The next set of nanofluids was for silver nanoparticles with 0,5% vol., 1.0% vol. and 3,0% vol. dispersed in water (named as NF4, NF5 and NF6, respectively). All study cases were implemented with Re = 250. Volume concentration was set at that value to serve for further comparison with experimental results.

4.1. Mathematical modeling

Since considered the nanofluid as single-phase flow, governing equations for mass balance, momentum balance and energy balance are shown as follow:

$$\frac{\partial \rho_{nf}}{\partial t} + \nabla \cdot \left(\rho_{nf} \mathbf{v}_{nf}\right) = 0 \tag{13}$$

$$\frac{\partial \rho_{nf} \mathbf{v}_{nf}}{\partial t} + \nabla \cdot \left(\rho_{nf} \mathbf{v}_{nf} \mathbf{v}_{nf}\right) = -\nabla p + \nabla \cdot \mathbf{T}_{nf} - \mathbf{S}_{\mathrm{p}} + \rho_{nf} \boldsymbol{g} \tag{14}$$

$$\left[\frac{\partial T}{\partial t} + \mathbf{v}_{nf} \cdot \nabla T\right] = \nabla \cdot [\mathrm{k} \nabla T] \tag{15}$$

4.2. Geometry and nanofluid properties

Commercial software ANSYS Workbench was used for conditioning the problem and CFX for numerical solving. The problem geometry was modeled and meshed with the applications of Workbench. Figure 4 shows a capture with partial mesh for the tube. A refinement was made nearby the wall to get certain values of the boundary layer.

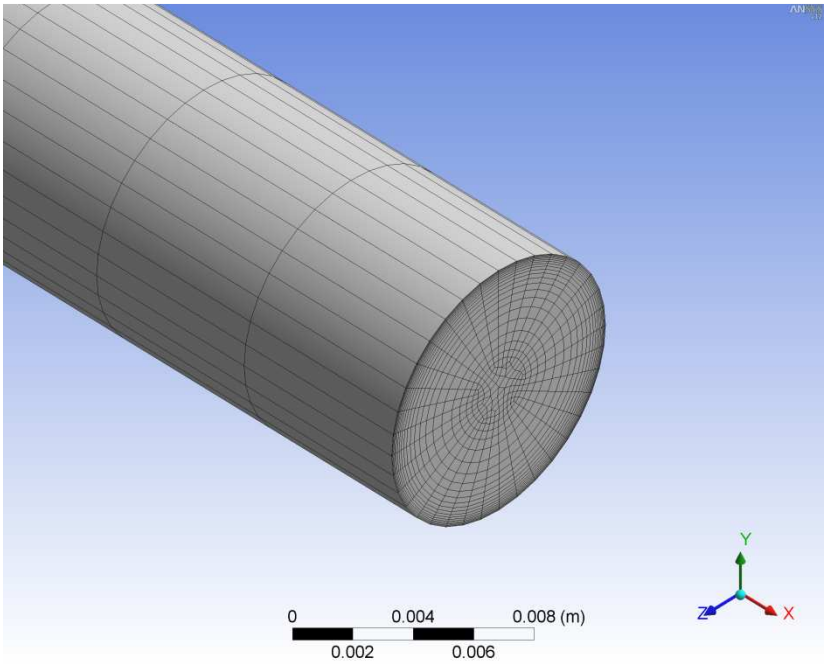

Figure 4. Mesh of the horizontal tube.

Initially, it was simulated a flow of water, with properties provided from the software materials library. Then, enhanced properties were calculated with the mentioned Eqs. (1), (2), (6) and (11). These new properties are shown in Tab. 2.

Table 2. Enhanced properties used in the current simulation.

Property	**Water**	**NF1** (Al_2O_3 0,5%)	**NF2** (Al_2O_3 1,0%)	**NF3** (Al_2O_3 3,0%)	**NF4** (Ag 0,5%)	**NF5** (Ag 1,0%)	**NF6** (Ag 3,0%)
k (W/m-K)	0,5948	0,6034	0,6120	0,6475	0,6037	0,6127	0,6498
ρ (kg/m^3)	997,10	1011,41	1025,73	1082,99	1044,56	1092,03	1281,89
Cp (J/kg-K)	4183,10	4166,43	4149,77	4083,11	4163,36	4143,62	4064,66
μ (mPa-s)	0,8905	0,9018	0,9134	0,9625	0,9018	0, 9134	0,9625

5. RESULTS

Numerical solution proportioned results for temperature over the whole domain. With these values and using the Eqs. (3) and (5), convective heat transfer coefficient was calculated. General trends are shown in the Figs. 5 and 6 for the alumina and water nanofluids and in for the Silver and water nanofluids, respectively. First values were removed to fit the scale.

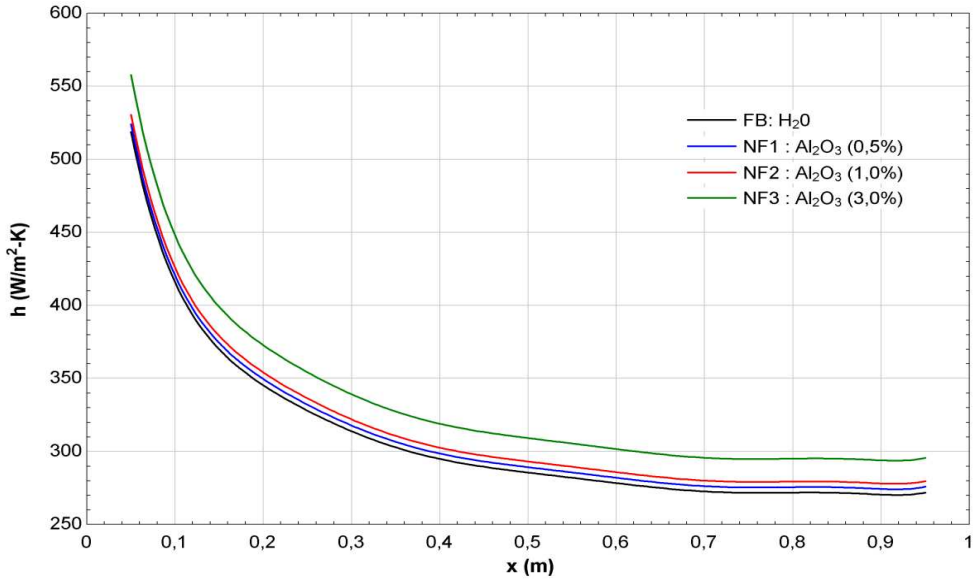

Figure 5. Calculated heat transfer coefficient for base fluid (black) and nanofluids (blue, red and green).

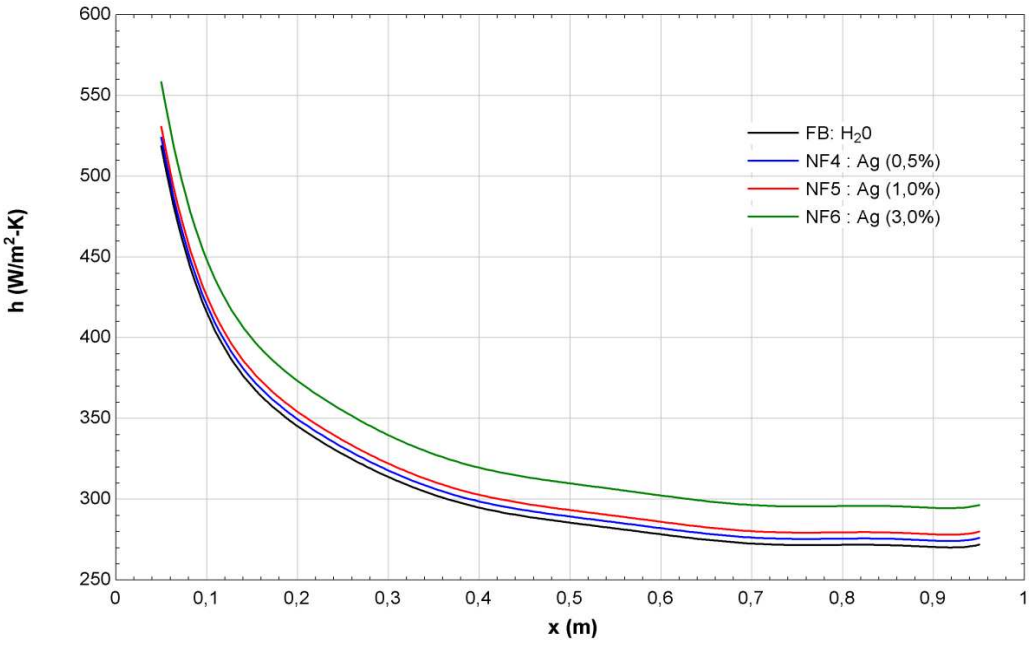

Figure 7. Calculated *h* for for base fluid (black) and nanofluids (blue and red).

6. CONCLUSIONS

The modified thermal conductivity of nanofluids NF1, NF2 and NF3 (those with nanoparticles of alumina) were 1,44%, 2,90% e 8,86%, respectively. For nanofluids NF4, NF5 and NF6 (those with nanoparticles of silver), enhancements were 1,47%, 2,90% and 8,86%. The enhancements of viscosity were 1,27%, 2,57% e 8,09%, for

nanofluids NF1, NF2 and NF3. These values were the same for nanofluids NF4, NF5 and NF6 because the Batchelor model is only dependent on volumetric concentration.

The calculated values of the heat transfer coefficient showed the expected trend and were validated with analytical solution of the problem. Nusselt numbers were close to the known value for laminar internal flow with constant heat flux, Nu = 4,36. The mean enhancement of the convective heat transfer coefficient for the cases NF1, NF2 and NF3 (alumina nanoparticles based) and for NF4, NF5 and NF6 (silver nanoparticles based) are showed in Tab. 3.

Tabela 3. Heat transfer coefficient enhancements for tested nanofluids

Nanofluid	Enhancement
NF1 (0,5% Al_2O_3)	1,47%
NF2 (1,0% Al_2O_3)	2,90%
NF3 (3,0% Al_2O_3)	8,76%
NF4 (0,5% Ag)	1,55%
NF5 (1,0% Ag)	2,98%
NF6 (3,0% Ag)	9,06%

As expected, enhancements on thermal behavior for nanofluids using silver nanoparticles were slightly higher than those which used alumina nanoparticles. It is important to notice that the low enhancements could be explained because the basic models used for determining the properties of each nanofluid. Actually, the measured values for those are much higher than calculated in present work.

Although that another goal of this paper was the comparison with experimental data, the difficulty to get certain values of the conditions set such as: heat flux, wall temperature and mass flow rate from literature, was present along all the literature review process. The next step for this study is to include the comparison with experimental data, since the nanofluid group of the Energy and Thermal System Laboratory at UFU is already building its own facility. However, it is for sure that more simulations using accurate models are needed for predicting the actual heat transfer enhancements on conventional systems.

7. ACKNOWLEDGEMENTS

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