COHERENT STRUCTURE IDENTIFICATION AND CLASSIFICATION IN TURBULENT FLOWS

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Abstract. *Coherent vortices morphology and evolution change with domain dimensions and the flow anisotropy in turbulent flows. The identification of such kind of structure is dependent not only on the correct mathematical parameter, but also on the threshold imposed into the flow for this parameter. The objective of present work is to evaluate those kinds of vortices, using strain-acceleration-based parameters and other criteria found in the literature on the flow across two different geometries: a backward-facing step and a non-confined jet. The strain-acceleration parameters are reasoned on the fact that, vortices are regions where the strain rate tensor is orthogonal (out-of-phase) with its acceleration, an objective tensor with respect to the changes in coordinates system. Isotropic and anisotropic parameters can be derived from this assumption, based on the principal strain directions. Turbulent jets can force the flow surrounding in the inflow region to produce coherent structures due to the expansion of the shear-dominated region. Another benchmark case for coherent vortex identification criteria is the turbulent production in the wake after the expansion in a backward-facing step. Those flows are solved using large-eddy simulation approach and its boundary conditions, as the domain are based on previous works in the area.*

Keywords: *Vortex Identification, Coherent Structures, Large-Eddy Simulation*

1. INTRODUCTION

Coherent vortices play an important role in turbulence studies and its behavior in some complex flows determines the consequences of related phenomena, like mixing, multiphasic interface motion, reaction and etc. Its morphology and evolution can change with domain topology and the kind of flow which is placed on this domain. Many analytical solutions for observed structures and criteria to identify those vortices have been proposed along the years and only few works connected these approaches. The main objective of the present work is to provide information about vortex morphology concerning two classical flows in which coherent vortex is continuously generated duo to its boundary conditions and geometry. The first one, the so-called backward-facing step flow, produces cylindrical vortices by the abrupt increase in shear in the expansion of domain section due to a vertical step. The second flow also produces coherent vortices by expansion, but the morphology of the most important structure is necessarily toroidal due to the shear layer around a circular inlet.

In order to identify coherent vortices in these flows, some existing vortex identification criteria were applied and a new set of vortex identification parameters were evaluated. The strain-acceleration parameters are reasoned on the fact that, vortices are regions where the strain rate tensor is orthogonal (out-of-phase) with its acceleration, an objective tensor with respect to the changes in coordinates system. Isotropic and anisotropic parameters can be derived from this assumption, based on the principal strain directions. The turbulent flows discussed in the present work were evaluated using the computational fluid dynamic solver ANSYS FLUENT version 13.

2. TURBULENT FLOWS

The boundary value problem proposed in §2.1 is solved by the commercial CFD package ANSYS FLUENT version 13.0 (ANSYS, 2010), which makes use of the finite volume method (Versteeg and Malalasekera (1995), Maliska (2004)). The inlet boundary condition in both cases, downstream to the step and inside the small hole in the center of jet domain, presents an average and fixed velocity condition in addition to a transient profile with null mass-flow, representing artificial fluctuations to the mean flow. These fluctuations are imposed at inlet boundary conditions and are based in the work of RăKraichnan (1970) and modified by RăSmirnov *et al.* (2001). Both problems presented in this work were solved using the large eddy simulation approach, using the sub-scale model proposed by Germano *et al.* (1991) and modified by Lilly (1991).

2.1 Backward-facing step flow

The backward-facing step flow is one of the most established flows in turbulence studies. The recirculation zone near the step due to detachment of boundary layer and consequent reattachment point are the targets of a great number of works. The works of Kuehn (1980), Durst and Tropea (1981) and (Armaly *et al.*, 1983) performed experimental analysis to evaluate de distance for reattachment of boundary layer. The present results were based on the analysis performed by (Le *et al.*, 1997), using direct numerical simulation to obtain de behavior and the reattachment point in a backward-facing step flow of Reynolds based on the step height equal to 5100. This work produced very good results, compared with the experimental data from the work of Jovic and Driver (1994).

Figure 1 shows the domain schematics. In the picture, the domain height, $H = 5h$, the domain length, $L = 31h$, the width, $t = 4h$ and the development length for the flow downstream to the step, $L_d = 3.12h$. The Reynolds number is the main non-dimensional number in the problem and its characteristic length is based on step height h.

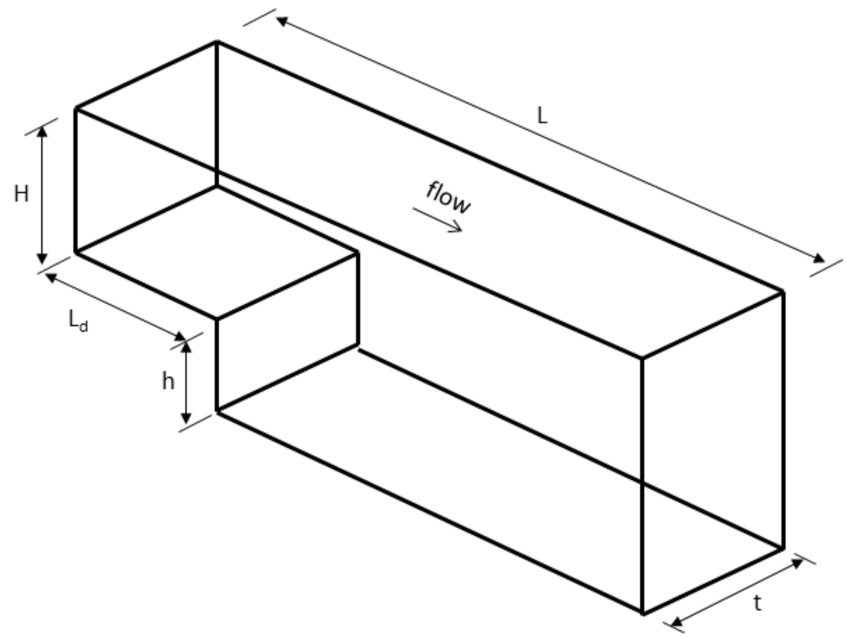

Figure 1. Domain schematics of backward-facing step geometry

Concerning the boundary conditions, the mean velocity condition in the inlet was based on the $1/7th$ profile, respecting the main velocity according to Reynolds number proposed. In both sides of domain, a periodic boundary condition was imposed. On the top surface a condition of free shear and no penetration was applied. In the outlet boundary, opposed by to the inlet b.c., an fixed average atmospheric pressure condition was inserted. All other surfaces were simulated as no-slip wall boundaries.

2.2 Round jet flow

Turbulent jet flows are another example of a simple configuration which can produce complex vortex structures. The main coherent structure produce by these flows are the so-called Rankine vortices, whose structure is typically toroidal. These structures are the result of a axisymmetric shear layer near the jet expansion. Far from the jet, some other types of coherent motions can be observed. The the turbulent round jet flow has been well investigated both analytically and experimentally (Wygnamski and Fiedler (1969) and Dimotakis *et al.* (1983)). Tso and Hussain (1989) measured the velocity field in the farfield region of a round jet by employing an array of several hot-wire probes. They tried to determine the main spatial modes of the vortex structure from the velocity correlation.

The present work is based on the experiment of Matsudaa and Sakakibara (2005). The authors applied stereo particle image velocimetry to observe turbulent vortical structures in a round free jet of water. They used a laser light sheet which illuminated a cross-sectional plane normal to the axis of the jet, and two charge-coupled device cameras captured particle images in the same region of interest but from different directions. Figure 2 ilustrastes the domain applied in CFD calculations with a axisymmetric section. Its important to highlight, however, that the domain simulated is really full tridimensional with no periodic or symmetry boundary conditions. In the picture, $D = 100d$, $L = 70d$ and $L_d = 10d$. Concerning the boundary conditions, the velocity in the inlet region was calculated to provide a Reynolds number based on duct diameter d equal to 3000. The farfield region was evaluated using a condition for pressure equal to the atmospheric value at sealevel, allowing entrainment. Both “wall bottom” and “duct wall” regions were evaluated using no-slip impermeable boundary conditions.

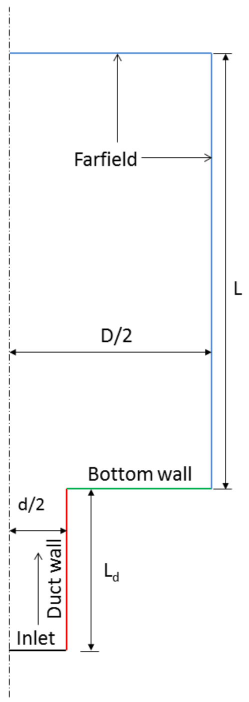

Figure 2. Domain schematics of round jet geometry

3. VORTEX IDENTIFICATION

Although the word vortex is frequently used when one wants to describe, understand, and explain flow patterns in fluid dynamic problems, the connection of this word to an entity which is unambiguously identified is still controversial. Concerning the proposed criteria in literature, some opposite ideas with respect to the formulation of vortex identification criteria can be observed from different authors. Those ideas can help to understand why a common vortex definition still not accepted by scientific community.

A first bi-polar strength present in the literature is the CAUSE X MANIFESTATION one. The approaches considered to identify a vortex can be, on one side, based on dynamics or force related quantities, entities related to the cause of the patterns of a flow. On the other side, the identification can be based on the manifestation, or the kinematics that is presented by the flow. A second opposition is the LAGRANGEAN X EULERIAN approaches. In fact, is not very clear in the literature if a vortex should be defined as a region in space which has certain instantaneous properties or a set of fluid particles that undergoes a particular trajectory in time.

3.1 Classical vortex identification criteria

The established criteria for vortex identification, known as the Q-criterion (Hunt et al., 1988), the Δ-criterion (Chong et al., 1990), and λ_2-criterion (Jeong and Hussain, 1995) are the most widely used in the literature. A great number of works that concern vortex identification or propose to evaluate coherent structures in a given turbulent flow commonly compares their results with those criteria.

3.1.1 Hunt *et al.* (1988) criterion

The criterion proposed by Hunt *et al.* (1988) is intrinsically related to a competition between vorticity and rate-of-strain where, in the case of a vortex, vorticity wins. The authors define a vortex as a connected region in space where

$$Q = \frac{1}{2}[\|\mathbf{W}\|^2 - \|\mathbf{D}\|^2] > 0 \qquad (1)$$

where $\mathbf{W}$ and $\mathbf{D}$ are, respectively the skew-symmetric and symmetric parts of the velocity gradient and the operator $\|\ \|$ indicates the Euclidean norm of a tensor. Therefore, the competition between rate of rotation and rate of deformation is translated by the difference between the the Euclidean norm of each part, symmetric and skew-symmetric, of the velocity gradient. A vortex is identified where vorticity dominates the rate of deformation.

It is worth noticing that the Q-criterion is strictly related to the Vorticity number introduced by Truesdell (1953), defined as

$$N_k = \frac{\|\mathbf{W}\|}{\|\mathbf{D}\|} \tag{2}$$

and interpreted as a Şmeasure of the quality of the vorticity". Its possible to observe that $Q > 0$ is equivalent to $N_K > 1$. In summary, the Q-criterion is not generally frame indifferent, since it is dependent on the vorticity. It is an Eulerian approach. It does not give a clear picture of how it can be extended to compressible flows, one can keep the same difference or work with the new second invariant. It has a non-subjective definition.

3.1.2 Chong *et al.* (1990) criterion

A second criterion was formulated by Chong *et al.* (1990). This criterion is based on the fact that, when vorticity vanishes, the eigenvalues and eigenvectors of the velocity gradient are (the same as the rate-of-strain) real, since the velocity gradient, in this case is symmetric. If we gradually increase the vorticity, there is a threshole which is eventually achieved, where there will be a real and two complex conjugates eigenvalues. Therefore, the importance of vorticity changes the nature of the eigenvalues of the velocity gradient and produce a rotation like behavior. The so-called Δ-criterion is given by a region where

$$\Delta = \frac{III_{\mathbf{L}}^2}{2} + \frac{Q^3}{27} > 0 \tag{3}$$

where $III_{\mathbf{L}}$ is the third invariant (determinant) of the velocity gradient. The Δ-vortex is a larger region than a Q-vortex, since $Q > 0$ is equivalent to $\Delta > 0$. This also shows that, to produce complex eigenvalues, the vorticity intensity measured by its norm, may not overcome the rate-of-strain intensity with the same measurer.

3.1.3 Jeong and Hussain (1995) criterion

Another very important criterion in the literature was proposed by Jeong and Hussain (1995). This criterion is based on a pressure minimum at the vorticity plane. The gradient of the Navier-Stokes equation can be separated into a symmetric and skew-symmetric parts. The skew-symmetric part is related to the evolution of vorticity, while the symmetric part is connected to the evolution of the rate-of-strain. The symmetric part of the equation is given by

$$\frac{D}{Dt}(\mathbf{D}) - \nu\nabla^2\mathbf{D} + \mathbf{WW} + \mathbf{DD} = -\frac{1}{\rho}\mathbf{P} \tag{4}$$

where $\nabla^2\mathbf{D}$ is the Laplacian of $\mathbf{D}$ and $\mathbf{P}$ is the pressure Hessian. According to Jeong and Hussain (1995), the principle of minimum pressure, can be corrected by discarding the terms related to unsteadiness of the flow and to viscous forces, this condition is satisfied, for an incompressible Newtonian fluid, when

$$\lambda_2^{\mathbf{D^2}+\mathbf{W^2}} < 0 \tag{5}$$

where $\lambda_2^{\mathbf{D^2}+\mathbf{W^2}}$ is the intermediate eigenvalue of tensor $\mathbf{D^2} + \mathbf{W^2}$. It is interesting to notice that, although expressed by kinematic quantities, this criterion is based on dynamical arguments.

4. NEW SET OF VORTEX IDENTIFICATION CRITERIA

It is very common, in many physical and mathematical situations, the identification of the necessity to compare the diagonal components of a matrix with its off-diagonal ones. One simple idea is to measure this competition by an overall ratio index. A parameter which has in the numerator and the denominator, the intensities of one and other sides of this balance: diagonal and off-diagonal components of the strain acceleration tensor, evaluated in the strain basis.

Here, we have developed two methods for an anisotropic comparison between the diagonal and off-diagonal components of a matrix. Following Haller (2005) and Thompson (2008) we use the matrix associated with the second Rivlin-Ericksen tensor. The first method which will be called here line-method is to compare, in the diagonal components of the tensor $\mathbf{A}_2^{\mathbf{A}_1}$, acceleration tensor on the basis of the strain tensor, $\mathbf{L}$, the part of each component that comes from the diagonal and off-diagonal component of tensor $\mathbf{A}_2^{\mathbf{A}_1}$

$$AR_i^A = \frac{(A|_{ii})^2}{(A^2)|_{ii}} \tag{6}$$

where $\mathbf{A} = \mathbf{A}_2^{\mathbf{A}_1}$ represents the second Rivlin-Ericksen tensor (strain acceleration) on the basis of the first one (strain rate tensor). An isotropic version was also formulated, based on the same idea provided in above relations

$$IR = \frac{A_{ii}A_{ii}}{[AA]_{jj}} \tag{7}$$

5. NUMERICAL SOLUTION

Large Eddy Simulation methodology is about filtering of the equations of movement and decomposition of the flow variables into a large scale (resolved) and a small scale (unresolved) parts. The filtering process is applied on the governing equations for separate the fields that contains the large and sub-grid scales. After performing the volume averaging, the filtered Navier-Stokes equations become

$$\frac{\partial \bar{\mathbf{U}}}{\partial t} + \nabla \cdot (\overline{\mathbf{U}\mathbf{U}}) = -\frac{1}{\rho_0}\nabla \bar{p} + \nu \nabla^2 \bar{\mathbf{U}} + f_B \tag{8}$$

Developing the non-linear transport term and introducing the sub-grid scale (SGS) stresses $\tau = \overline{\mathbf{u}\mathbf{u}} - \bar{\mathbf{U}}\bar{\mathbf{U}}$, the filtered Navier-Stokes equations can be rewritten as

$$\frac{\partial \bar{\mathbf{U}}}{\partial t} + \nabla \cdot (\bar{\mathbf{U}}\bar{\mathbf{U}}) = -\frac{1}{\rho_0}\nabla \bar{p} + \nu \nabla^2 \bar{\mathbf{U}} - \nabla \cdot (\tau) + f_B \tag{9}$$

The dynamic sub-grid scale model was used with the Large Eddy Simulation to obtain the sub-grid scales. In this sub-grid model the proportionality coefficient is computed as a function of time and space. As a consequence, some difficulties on finding a correct constant value in heterogeneous meshes, as in the Smagorinskyt's model are avoided. The expression that defines the turbulent viscosity, μ_T can be written as

$$\mu_T = C\Delta^2 \|\mathbf{D}\| \tag{10}$$

where C is the proportionality coefficient, calculated in FLUENT along time and space as a function of the velocity fluctuations and $\|\mathbf{D}\|$ the rate of strain tensor and Δ is the length scale of the grid filter. the round jet and the backward-facing step presented meshes with 2.6 and 1.8 million elements, respectively. The meshes presented refinements near the step and the expansion in jet.

6. RESULTS

6.1 Backward-facing step flow

The numerical results obtained in the present work were retrieved in $t = 100h/U_{\text{inf}}$. The timestep applied was equal to $0.005h/U_{\text{inf}}$ in order to obtain Courant number less than one in all elements in domain. Figure 3 presents the comparison in the average streamwise velocity field in four different regions with the direct numerical results from (Le *et al.*, 1997).

Figure 4 and 5 presents the iso-surfaces for the existing criteria and the parameters proposed in the present work. All entities were normalized in order to guarantee the same comparison. All iso-surfaces receives the value of 0.5 and, when the normalized criterion presents values below this threshold a vortex can be identified for this specific criterion. All iso-surfaces were obtained in a subdomain near the step in order present the recirculation effect on the vortex identification and to apply a contrast between identified coherent structures, the iso-surface were painted using streamwise coordinate variable.

Figure 6 presents the contours for the existing criteria and the parameters proposed in the present work. The isotropic ratio measures the global alignment between the acceleration of strain and the rate of strain. The new anisotropic numbers are able to classify the vortex into three categories. These are related to the cases where the vortex have one, two ore three directions where the acceleration of angular deformation, with respect to the plane defined by the eigenvectors of the rate of strain, overcomes the acceleration of linear deformation. Generally speaking, we can see that the structures identified are smaller than the traditional ones based on the velocity gradient.

6.2 Round jet flow

Figure 7 presents the comparison in the average streamwise, radial and azimuthal velocity components field in $20d$ line along the radial direction, compared with the experimental results from Matsudaa and Sakakibara (2005).

Figure 8 and 9 presents the iso-surfaces for the existing criteria and the parameters proposed in the present work. All entities were normalized in order to guarantee the same comparison. All iso-surfaces receives the value of 0.5 and, when the normalized criterion presents values below this threshold a vortex can be identified for this specific criterion. All iso-surfaces were obtained in a subdomain near the step in order present the recirculation effect on the vortex identification and to apply a contrast between identified coherent structures, the iso-surface were painted with streamwise coordinate.

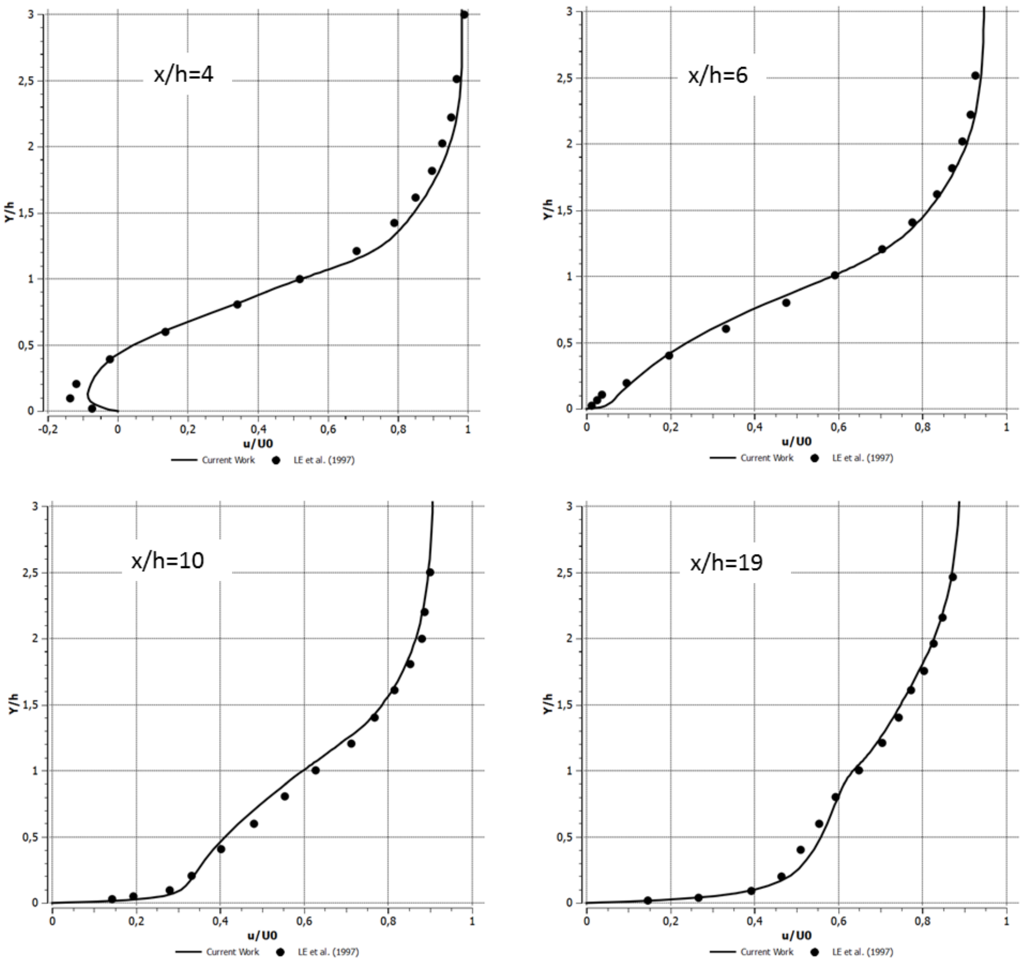

Figure 3. Comparison between present work and direct numerical simulation with respect to the average streamwise velocity.

7. FINAL REMARKS

We have presented a theoretical analysis to capture directional tendencies of stretching material elements. These directional quantities are able to delineate coherent structures that are present in turbulent flows. Besides that they are strongly related to flow-type classification criteria, giving an anisotropic version of previous criteria in the literature. The theoretical entities introduced are applied in a accompanied paper. We have presented also two application of the theory developed in an accompanied paper concerning flow classification. The general results are complex in nature and the full interpretation are in order. Besides that, the dimensionless numbers proposed are objective, a feature that needs to be addressed since the pioneering work of Haller (2005).

8. ACKNOWLEDGEMENTS

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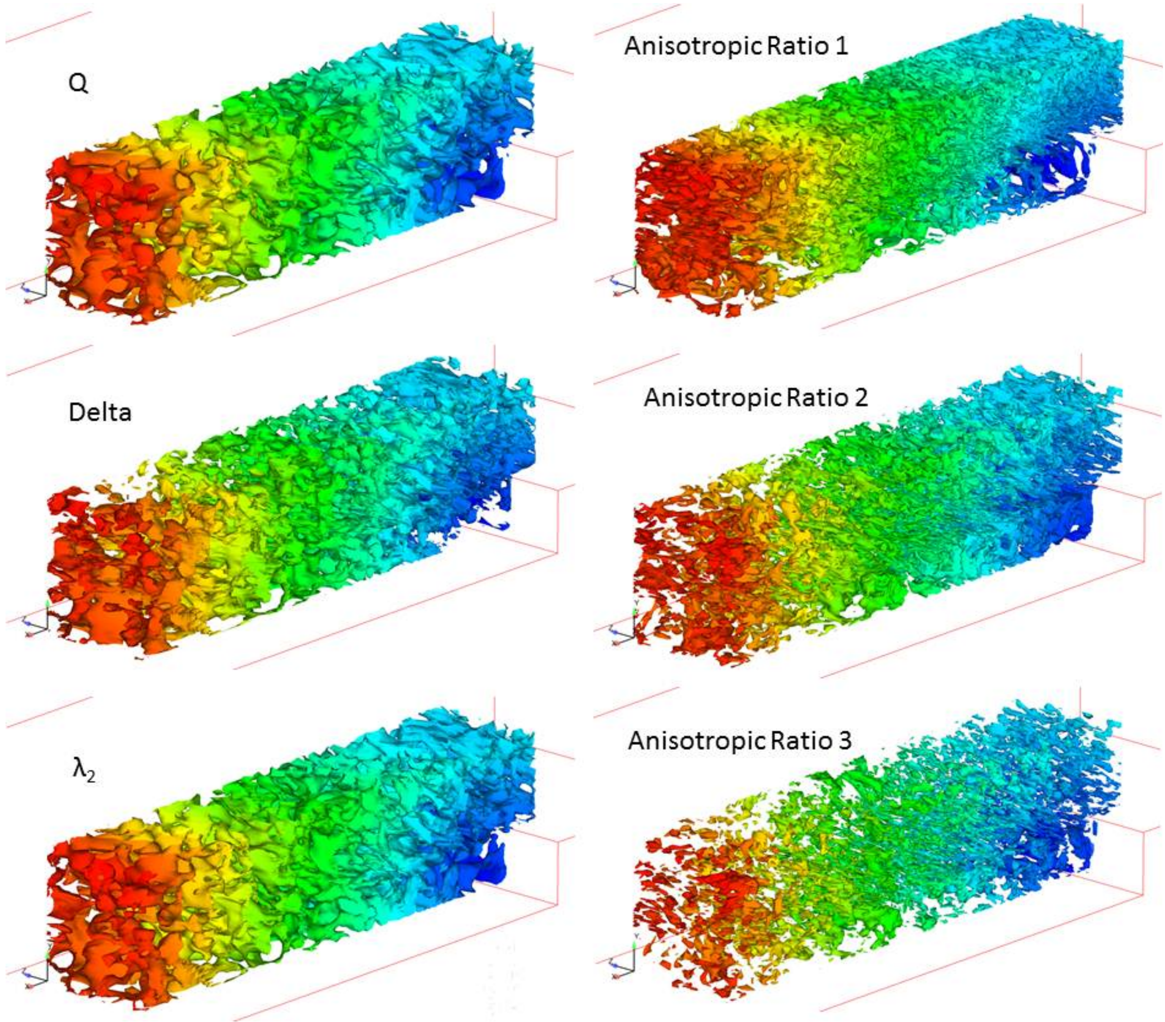

Figure 4. Iso-surface for existing and proposed coherent structure parameters.

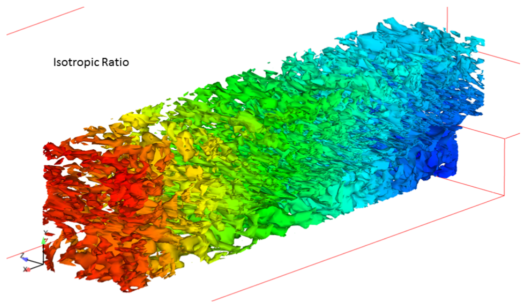

Figure 5. Iso-surface for the proposed isotropic parameter.

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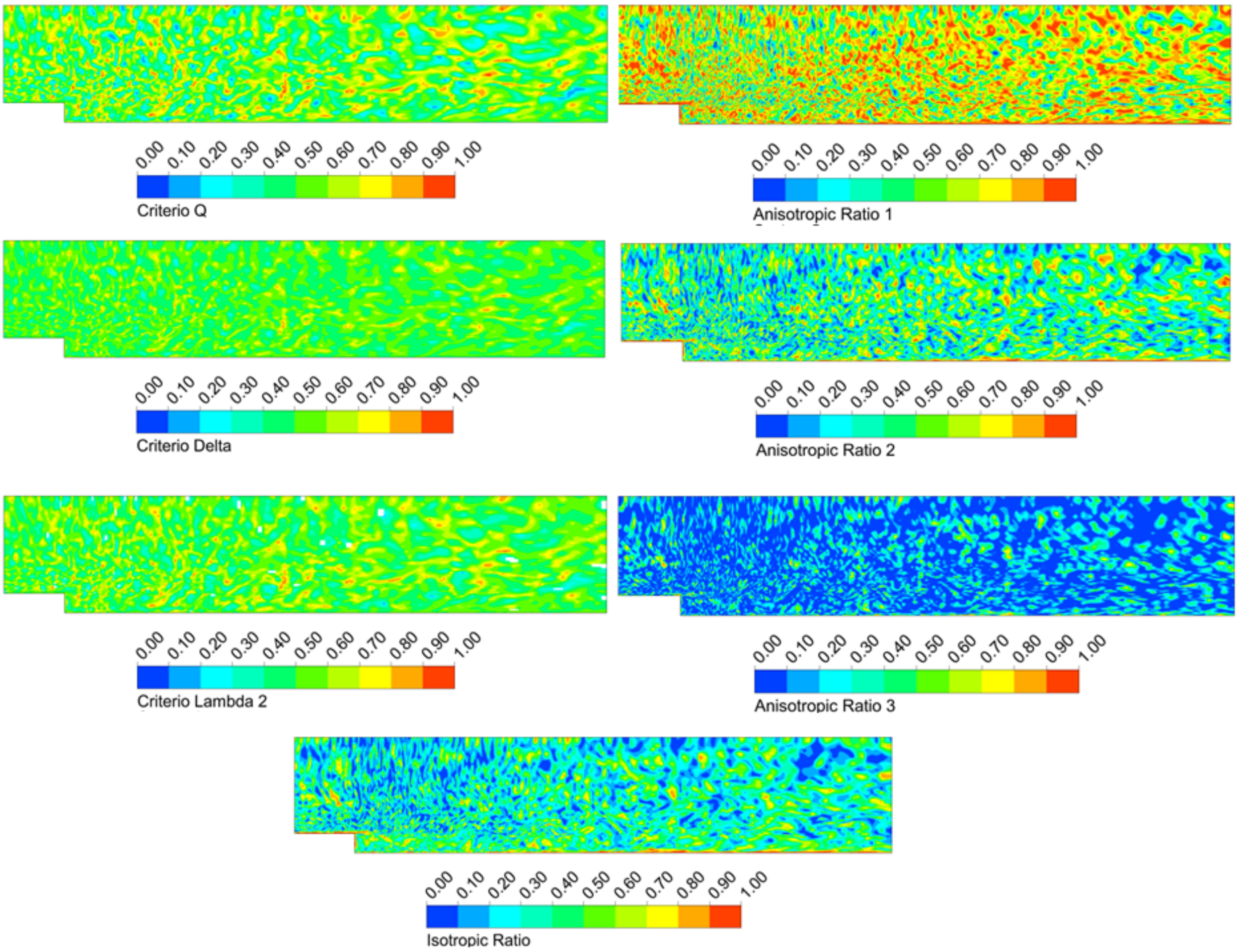

Figure 6. Contours of existing criteria and proposed parameters.

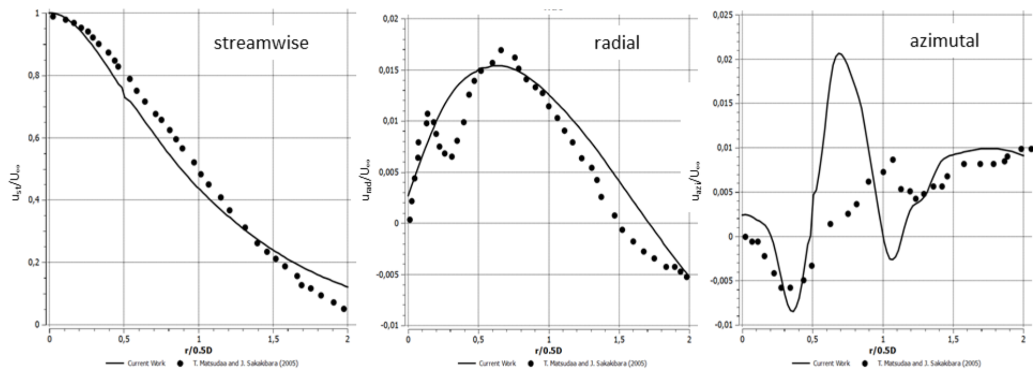

Figure 7. Comparison between present work and direct numerical simulation with respect to the average streamwise, radial and azimuthal velocity.

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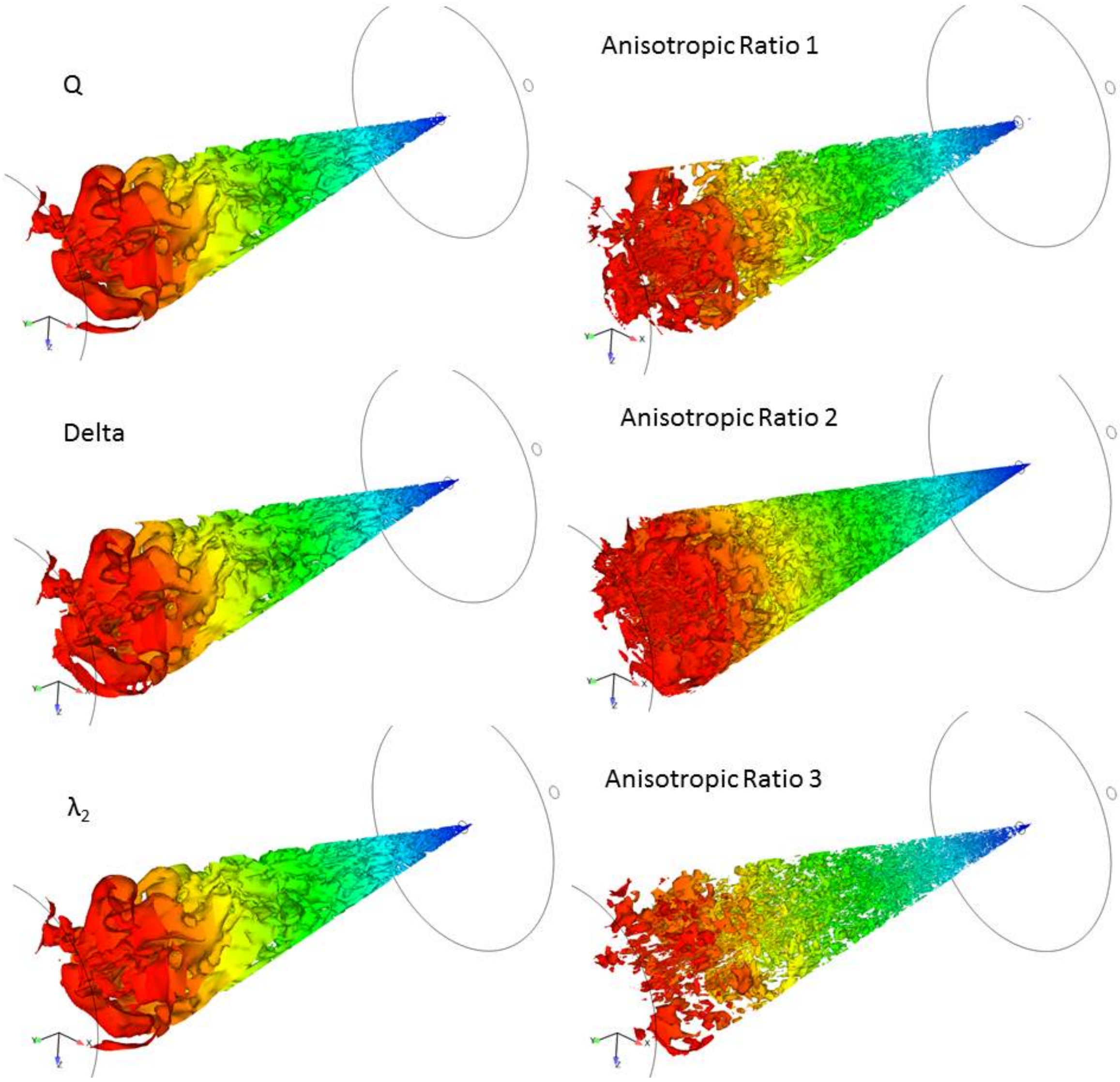

Figure 8. Iso-surface for existing and proposed coherent structure parameters.

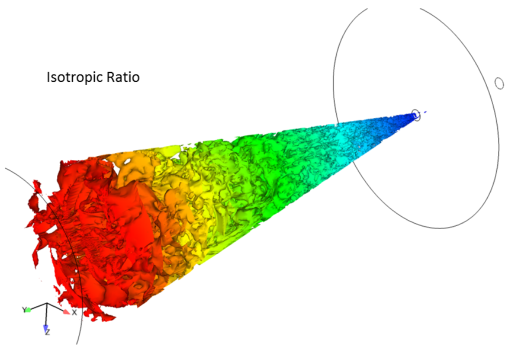

Figure 9. Iso-surface for the proposed isotropic parameter.

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10. Responsibility notice

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