NUMERICAL EVALUATION OF THE LONGITUDINAL ELASTICITY MODULUS IN TIMBER BEAMS OF STRUCTURAL SIZES OF THE *EUCALYPTUS CITRIODORA* SPECIE

André Luis Christoforo, alchristoforo@ufsj.edu.br
Árlei Huebra Póvoa, arleipovoa@hotmail.com
Túlio Hallak Panzera, panzera@ufsj.edu.br
Fabiano Bianchini Batista, fabianchini@ufsj.edu.br
Universidade Federal de São João del-Rei (UFSJ) – Dpto. de Eng. Mecânica (DEMEC) – 36307 – 352, Campus Santo Antônio (CSA), São João del-Rei – MG, Brasil.

Avelino Manuel da Silva Dias, avelino.dias@ct.ufrn.br
Universidade Federal do Rio Grande do Norte (UFRN) – Dpto. de Eng. Mecânica, Natal – RN, Brasil.

Abstract. *The modulus of elasticity for engineering materials is crucial in the development of structural or mechanical project. Among materials used in constructions, the wood is the most used for it's low density and good mechanical performance. However, the Brazilian code NBR 7190/1997 doesn't make reference to tests aimed to the determination of stiffness and strength of structural parts, restricting the analysis of test in small body-of-test that are faultless. In Brazil, the testing of structural parts are made according to the procedures of international normative documents. This paper aims to show an alternative methodology for the calculus of the modulus of elasticity in wooden beams with structural sizes. This methodology is numerically developed according to the employment of the finite element method combined with the least squares method and an optimization strategy. The results obtained for the Eucalyptus Citriodora wood type indicate that the calculation procedures presented in the Brazilian standard for calculating the modulus of elasticity in body-of-test can also be used for parts of structural dimensions*.

Keywords*: Beams, Finite Element Method, Least Squares Method.*

1. INTRODUCTION

The wood raw material is a naturally strong and relatively light. Its importance and versatility contributes to the great use of this material in many sectors of human activity in civil and rural construction, and in furniture industry. Its application as a structural element in Brazil has grown over recent years because of many research's trying to show it like a material more competitive among other used in building structures, such as steel and concrete.

Longing for the technological development, NBR 7190/1997 (Project of Timber Structures), of the Brazilian Association of Technical Standards (ABNT) is being reformulated in order to improve analyze and characterize the physical and mechanical properties of wood and structural systems, in which this material is used in order to enable more appropriate conditions for sizing method within the status limit.

Researchers' efforts for technological development from the most exact knowledge of wood properties are based on making it a more reliable material on its application as a structural element, thus diminishing the uncertainties or prejudgments about part of building professionals with regard to its application.

For the project of structures, regardless the materials used, the elasticity modulus is shown as an essential variable of the project, however, their determination according to NBR 7190/1997 is restricted to specimens of small size and free of defects , determined by static bending three-point, as seen in Fig. 1, using Eq. (1).

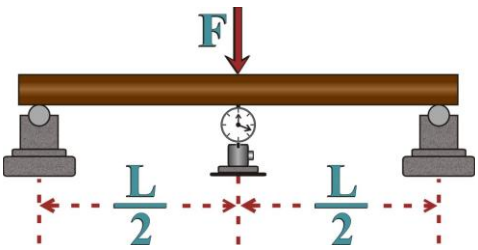

Figure 1. Bending test according to NBR 7190/1997.

$$\delta = \frac{\Delta F L^3}{48EI} \tag{1}$$

Where:
ΔF - ($F_2 - F_1$), An increase of strength;
E - modulus of elasticity or Young;
L - span of the element;
I - moment of inertia of cross section.

In terms of calculating the modulus of elasticity in bending, an extreme importance result is found in the work of Lahr (1983), that has evaluated the influence of *L/h* (rate between the length of the piece and its height) in determining the modulus of elasticity in lumber pieces, noting that for ratios $L/h \geq 21$, derived from the shear deformation can be neglected. This result validates the assumptions on Bernoulli Beam Theory, used in the determination of Eq. 1.

A survey comparing experimental values of elastic modulus obtained in bodies-of-proof and free from defects in structural dimensions of sawn pieces was carried out by Batista *et al.* (2000). In three species, two (*Eucalyptus* and *Cambará*) presented results for the tests of specimens used in evidence considered loyal to those obtained for the parts and structural dimensions. However, the same could not be said for the kind *Cupiúba*, which presented values for the models reduced about 30% lower than the structural models.

Researchers like Pigozzi *et al.* (2000) studied aspects such as better clearance for bending tests on structural components, the number of points to be tested in each piece, the accuracy of the values and the estimated cost of these tests for small businesses. Mentioning that the classification process is not to separate the pieces free of defects, but rather allow the greatest number of defects in each group, so that no declassify the parts for the use one wants.

Miná *et al.* (2004) evaluated the strength and stiffness of wood poles of *Eucalyptus* species by comparing the results with those obtained in proof-of-body free of defects. The results show that the values of modulus are superior to structural elements in bending tests and lower in compression parallel, yet the values for resistance were superior to structural parts in both trials.

The objective of this paper is present an alternative method of calculation, based on the Finite Element Method (FEM) combined with Inverse Analysis Method, the method of least squares and Newton's method (with quadratic approximation), for determining the longitudinal elasticity modulus (*E*) structural lumber *Eucalyptus Grandis*, in order to verify the differences in elasticity values found by the calculation method proposed here with those provided by the NBR 7190/1997.

2. MATERIALS AND METHODS

For the determination of the elasticity modulus were used 24 pieces of wood *Pinus caribaea* size 0,06 m *x* 0,16 m *x* 2 m, respecting the relationship $L \geq 21h$, where h = 0,06 m.

The modulus of elasticity in this work is evaluated by two different mathematical models, both using the three-point bending structural scheme. For the first, the value of elasticity is obtained through Eq. 1, and for the second, as an alternative form of calculation, it is proposed that the value of the modulus of elasticity "great" is determined according the structural scheme of test illustrated by Fig. 2.

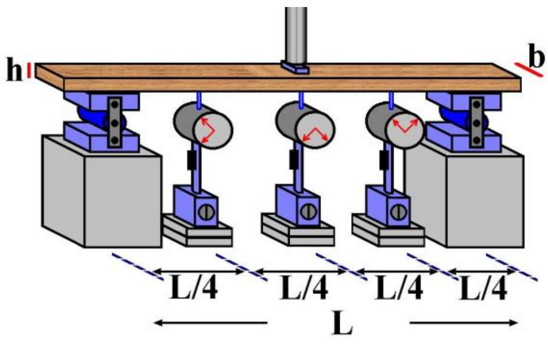

Figure 2. Alternate test for determining the *E* great.

For each structural test performed, three dial gauges were located to the right part, distant *L/4* from each other. The displacement reading for dial gauges equidistant shifts of support along the wooden parts are made when the displacement in the middle of the range in magnitude, approaching the rate *L/200*, where *L* is the length of the useful part (distance between supports), and this is the condition of small displacements accepted by NBR 7190/1997.

To calculate the modulus optimum, was developed a computer program (*Eotm*) Language Mathcad 2000®, according to the fundamentals of Finite Element Method (FEM) applied to the Virtual Work Principle (VWP), covering the kinematic model of deformation of Bernoulli beams, and ignoring in these calculations the forces per unit of volume and per surface area. Concerning the use of finite element analysis of wood structures, highlights the works of Christoforo (2006), Goes (2004), Cheung (2003), Parrini *et al.* (2002) and Mascia (1991).

The bar finite element used here has two degrees free per node, two translations and two rotations, see Fig. 3, developed using a third-degree polynomial function (form function) as an approximation of the displacement field.

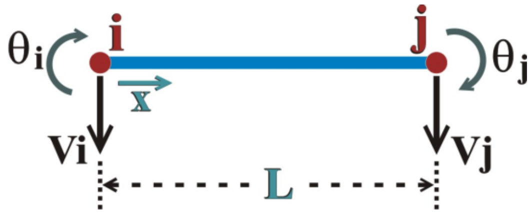

Figure 3. Degrees of freedom finite element.

Using the Inverse Analysis Method, the experimentally measured values of the displacements **(**$U^{(e)}$) are supplied to the program *Eotm*, in order to calculate the optimum value of the modulus of elasticity of the structural element.

The program *Eotm*, based on the fundamentals of the FEM, provides a numeric vector of displacements **(**$U^{(n)}$**)**' the dependent variable, the modulus of the structural element.

Holding the displacement vector determined by the program, and the vector displacement experiments, a function is constructed, based on the method of least squares whose objective is to determine the value of the elasticity modulus for the waste generated from both solutions, and numerical experimental, is minimal. Regarding the application of optimization techniques in structural engineering, some searches may be cited as Christoforo (2008), Rigo (1999), Soares & El Debs (1997) and Antunes & Alvarenga (1994).

The elastic modulus is obtained by minimizing the equation

$$f(E)=\frac{1}{2}\sum_{i=1}^{n}\left(\mathbf{U}_i^{(e)}-\mathbf{U}_i^{(n)}\right)^2 \tag{2}$$

with the support of Newton's method (quadratic approximation), as expressed in equation

$$\mathbf{x}^{k+1}=\mathbf{x}^{k}-\left[\nabla^2\mathbf{f}(\mathbf{x}^k)\right]^{-1}\bullet\nabla\mathbf{f}(\mathbf{x}^k) \tag{3}$$

where:

$\mathbf{x}^k$ - Vector estimate at iteration *k*;

$\mathbf{x}^{k+1}$ - Vector estimate at iteration *k +1*;

$\nabla^2\mathbf{f}(\mathbf{x}^k)$ - Hessian matrix;

$\nabla\mathbf{f}(\mathbf{x}^k)$ - Gradient vector.

Figure 4 illustrates the determination of the minimum value of *f (x)* according to Newton's method with quadratic approximation.

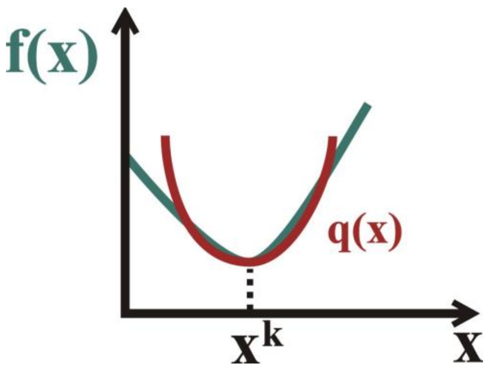

Figure 4. Newton method with quadratic approximation.

Newton's method with quadratic approximation was chosen for determining the minimum value of the objective function, as in the case of quadratic functions, independent of the initial estimate was adopted, the minimum of f is determined in only a single iteration.

Aiming to apply the methodology of NBR 7190/1997 in determining the modulus of elasticity also for parts of structural lumber, were determined the values of the module with the support of the Eq. (1) (E_{norm}) and also according to the methodology calculation presented here (E_o - modulus optimum). Checking the statistical equivalence between the values of E_{norm} and E_o for structural parts of lumber is accomplished using the confidence interval of differences of means expressed through the

$$\bar{x}_m - t_{\frac{\alpha}{2},n-1}\frac{S_m}{\sqrt{n}} \leq \mu \leq \bar{x}_m + t_{\frac{\alpha}{2},n-1}\frac{S_m}{\sqrt{n}} \tag{4}$$

Where:

μ - mean population differences;

$\bar{x}_m$ - Arithmetic mean of sample differences;

n - sample size;

S_m - sample standard deviation of differences;

α - significance level;

$t_{\frac{\alpha}{2},n-1}$ - Tabulated value for the distribution t of Student with $n-1$ degrees of freedom and significance level α.

The parameter t is equal to the tabulated $t_{2,5\%;23}$ = 2,069 with 95% reliability and degree of freedom ($\nu = n-1$) = 23.
The Fig. 5 show an abstracted scheme of the Eotm method.

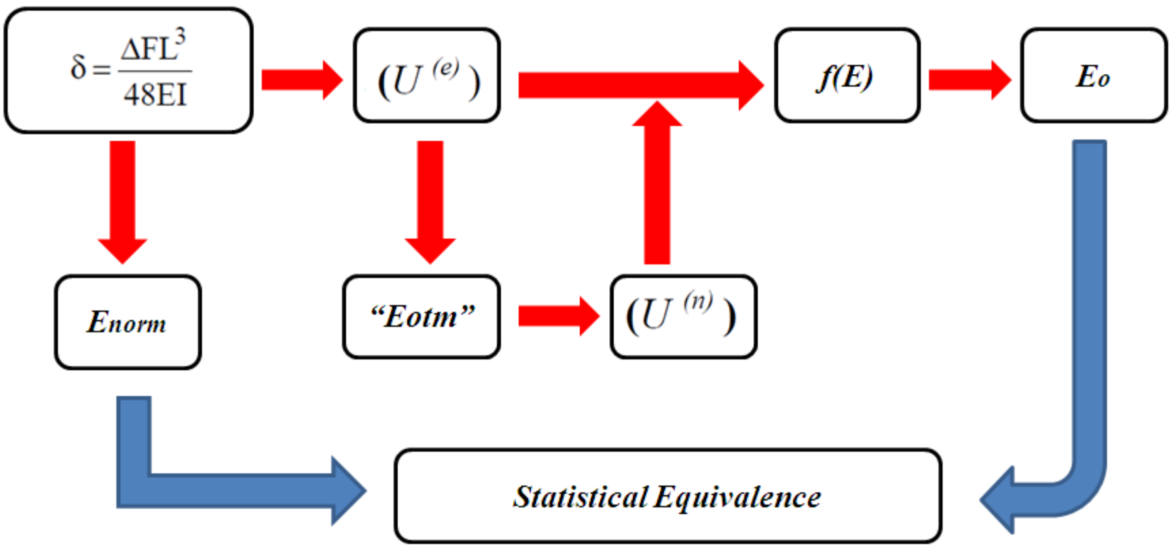

Figure 5. Scheme of the Eotm method.

3. RESULTS

The values of the elastic modulus E_{norm} and E_o obtained for the woods of *Eucalyptus Grandis* in this paper are shown in Fig. 6.

The mean, standard deviation and coefficient of variation for the values E_{norm} are respectively equal to 153,6; 22 and 0,14.

The mean, standard deviation and coefficient of variation for the values of E_o are respectively equal to 154,5; 22,70 and 0,15.

The confidence interval among the values of E_{norm} and E_o is $-7,17 \leq \mu \leq 24,76$ and, as the zero belongs to the interval, it is argued that these are statistically equivalent.

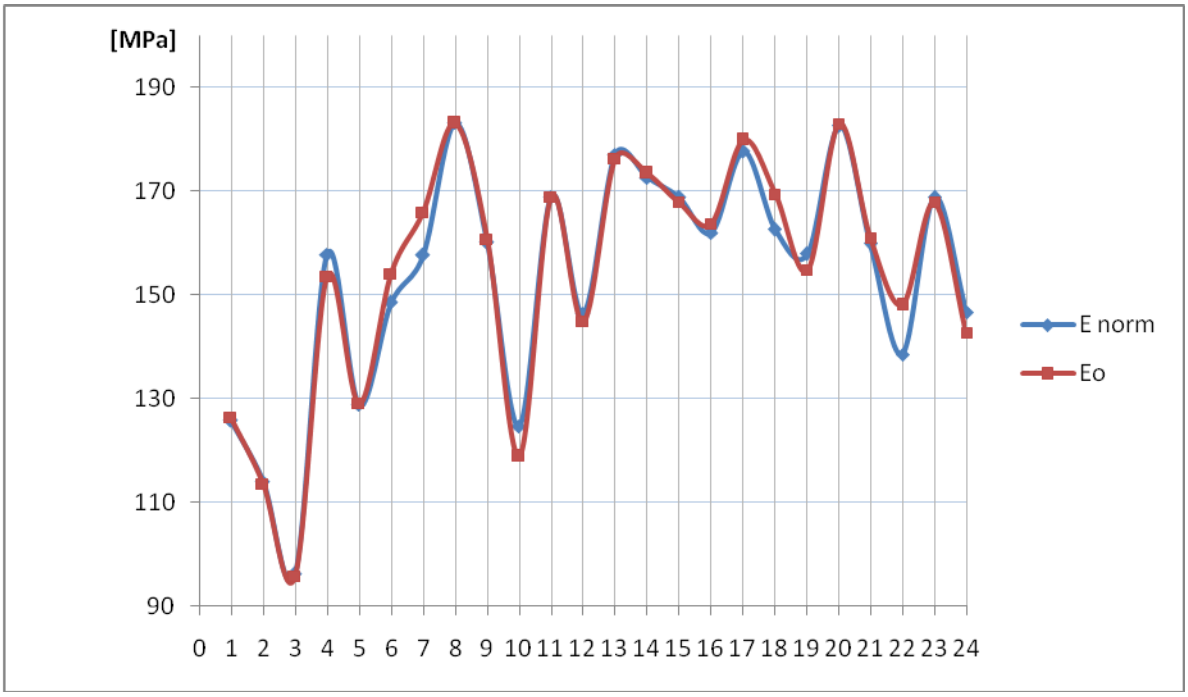

Figure 6. Values E_{norm} and E_o for *Eucalyptus Grandis.*

4. CONCLUSIONS

The alternative method of calculation proposed here allows to determine the modulus of elasticity of structural parts with greater reliability when compared to other methodologies, since this is based on concepts of optimization, and for this, the use of numerical tools.

Found by the statistical equivalence between the modulus of elasticity of wood from *Eucalyptus grandis* by both methods of calculation, it appears that the pieces can even be provided with some defects or imperfections not visible on the condition of small displacements (premise project), little affect the calculation of the elastic modulus, implying in this case, the validity of the use of NBR 7190:1997 for parts of structural dimensions too.

5. ACKNOWLEDGEMENTS

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