

## STUDY AND EVALUATION OF ENERGY HARVESTING FOR A PIEZOELECTRIC BEAM-LIKE STRUCTURE

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**Abstract.** *Mechanical vibrations have been shown one effective forms of generation energy by deforming piezoelectric materials or moving electromagnetic coil for energy harvesting in the smart material area. In the case of ambient vibration the piezoelectric material is deformed through a condition of operation generating a voltage/current which could be used like a natural energy source, mainly, for electronics devices of low power. However, the energy generated through piezoelectric effect usually is not enough to operate directly the most electronics circuits. Therefore, the development and implementation of methods to accumulate and store the energy captured in these systems (smart materials) until an usable level is a key for the success of this technology. This article discusses the study and evaluation of a theoretical-experimental modeling of a beam structure with bounded PZTs subjected to mechanical vibration, aiming at evaluating the behavior of the electromechanical coupling of the system and the positioning of sensor for energy generation, as well as, to quantify the amount of energy and to storage this energy in a type capacitive device.*

**Keywords:** *Piezoelectric material, Mechanical vibration, Energy Harvesting*

### 1. INTRODUCTION

Actually, a large number of studies are in progress aiming at the process of generating and storing electricity from renewable energy sources, mainly, energy generated by natural activities or movements induced by the operation conditions of systems and equipments. These include photonics (solar or artificial light), thermal (thermal gradient), kinetic (movement), fluid (hydraulic and wind) and vibrating (piezoelectric materials or electromagnetic) sources of energy. The techniques of harvest and storage these kinds of energies are actually known as *Energy Harvesting* (Anton and Sodano, 2007).

In this context, piezoelectric materials are getting a great importance in the development of energy harvesting devices, mainly, due to its ability to act as a sensor or actuator, i.e., when subjected to an external electric potential they present a deformation and at the same time, when subjected to a deformation they have the ability to generate electric energy. The use of these piezoelectric materials as sensors of deformation caused by the operational conditions of systems and equipments enables them to function as a capture device of some environment energy, thereby, making possible to take the advantage of unused energy sources. The development of systems for capturing and storing energy due to they own operation conditions opens up a huge range of options for their use as an alternative sources of energy, mainly to control small electrical devices. These harvesting devices, aims to take the advantage of using energy from vibration ambient.

Additionally, the integration of energy harvesting devices within portable electronic equipments and sensors can eliminate the need of periodic recharge, ensuring an uninterrupted operation of these equipments or even the replacement of electrochemical batteries (Anton and Sodano, 2007).

The viability of the energy harvesting from vibration ambient has been investigated by using different approaches and mathematical models (Adachi et al., 2009; Ajitsaria et al., 2007; Liang et al., 1996 and Lument et al., 2009), as well as, experimental results (Erturk et al., 2009 and Liao et al., 2008) to support those proposed approaches and mathematical models.

The analyses of the adaptability, life span, energy density of Piezoelectric Energy Harvesting Systems as discussed in Guan et al. (2007) shows that the super capacitors has been more suitable than rechargeable batteries as energy storage devices. Actually, different types of piezoelectric materials have been used to study the relative efficiency of the time and the maximum capacity of the device to be charged (Sodano et al., 2005a and Sodano et al., 2006). Sodano et al. (2005b) tested three types of piezoelectric material commonly used, the Lead-Zirconate-Titanate (PZT), the bimoto

Quick Pack (QP) and the Macro-Fiber Composite (MFC) and the presented results showed the PZT presents higher efficiency compared to the other materials tested.

This article presents a theoretical-experimental modeling of a beam-like structure with a bounded piezoelectric material, more specifically, PZT patches. The beam structure is analyzed in a fixed-free end condition. The structure is subjected to a deformation provoked by a controlled mechanical vibration aiming at to evaluate its ability to convert mechanical energy into electrical one, as well as, to quantify the amount of electrical energy generated due to the vibration of the structure.

## 2. ELECTROMECHANICAL COUPLING FOR PIEZOELECTRIC MATERIAL

The electromechanical behavior of a system formed by a mechanical structure and piezoelectric materials such as PZT can be obtained from the constitutive relations of the structure and the PZT, together with their electromechanical coupling. For the PZT material the equations involving the constitutive relations are given by expressions (1) and (2) (Leo, 2007).

$$\boldsymbol{\sigma}_p = \mathbf{c}^E \mathbf{S} - \mathbf{e} \mathbf{E} \quad (1)$$

$$\mathbf{D} = \mathbf{e}^T \mathbf{S} + \boldsymbol{\epsilon}^S \mathbf{E} \quad (2)$$

where  $\boldsymbol{\sigma}_p$  and  $\mathbf{S}$  are the mechanical stress and strain of piezoelectric material,  $\mathbf{E}$  is the electric field,  $\mathbf{D}$  is the electric displacement,  $\mathbf{c}^E$  is the elastic stiffness matrix of piezoelectric material on a constant electric field,  $\mathbf{e}$  is the piezoelectric coupling coefficient matrix and  $\boldsymbol{\epsilon}$  is electric permittivity matrix of the material under a constant mechanical strain.

The motion equations of the electromechanical coupled systems are obtained through of the Lagrange motion equations. In matrix form, the equilibrium equations of an element (Marqui et al., 2007), in a local generalized coordinates, is given by:

$$\begin{cases} (\mathbf{M}_s^e + \mathbf{M}_p^e) \ddot{\mathbf{u}}_i + (\mathbf{K}_s^e + \mathbf{K}_p^e) \mathbf{u}_i - (\mathbf{K}_{u\phi}^e) \boldsymbol{\phi}_i = \mathbf{F}^e \\ -((\mathbf{K}_{\phi u}^e) \mathbf{u}_i + (\mathbf{K}_{\phi\phi}^e) \boldsymbol{\phi}_i) = \mathbf{Q}^e \end{cases} \quad (3)$$

where  $\mathbf{M}_s^e$  is the elemental mass matrix of structure,  $\mathbf{M}_p^e$  is the elemental mass matrix of PZT,  $\mathbf{K}_s^e$  is the elemental stiffness matrix of structure,  $\mathbf{K}_p^e$  is the elemental stiffness matrix of PZT,  $\mathbf{K}_{u\phi}^e$  is the elemental electromechanical coupling matrix,  $\mathbf{K}_{\phi u}^e$  is the elemental electromechanical coupling matrix,  $\mathbf{K}_{\phi\phi}^e$  is the elemental piezoelectric capacitance matrix,  $\ddot{\mathbf{u}}_i$  is the vector of acceleration nodal,  $\mathbf{u}_i$  is the vector of displacement nodal,  $\boldsymbol{\phi}_i$  is the vector of electric potential nodal,  $\mathbf{F}^e$  is the vector of external forces applied on the element and  $\mathbf{Q}^e$  is vector of electric charge induced on the piezoelectric element.

The global system of motion equations of a piezostucture with the incorporated electrical-mechanical coupling effect (Marqui et al., 2007) is given by:

$$\begin{bmatrix} \mathbf{M} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\phi}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{\phi u} & \mathbf{K}_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\phi} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{Q} \end{Bmatrix} \quad (4)$$

where the global matrices are defined by

$$\mathbf{M} = \sum_{i=1}^{ne} (\mathbf{M}_s^e)_i + \sum_{j=1}^{np} (\mathbf{M}_p^e)_j \quad (5)$$

$$\mathbf{K}_{uu} = \sum_{i=1}^{ne} (\mathbf{K}_s^e)_i + \sum_{j=1}^{np} (\mathbf{K}_p^e)_j \quad (6)$$

$$\mathbf{K}_{u\phi} = \mathbf{K}_{\phi u}^T = - \sum_{j=1}^{np} (\mathbf{K}_{u\phi}^e)_j \quad (7)$$

$$\mathbf{K}_{\phi\phi} = - \sum_{j=1}^{np} (\mathbf{K}_{\phi\phi}^e)_j \quad (8)$$

where  $ne$  is the number of elements of the host structure and  $np$  the number of PZTs inserted in the structure. The symbol of addition in the above equations means the conventional finite element assembly.

Equation (4) defines the relationships of the electromechanical coupling system. If this structure is subjected to a condition of operation that causes a deformation of the electromechanical system, there is a generation of energy. Since there is no external electric potential actuating in the system, the energy is only due to deformation and it can be

calculated (Marqui et al., 2007) by making the electrical charge  $Q$  equal zero in the Eq. (4). In this case, the electric potential  $\phi$  can be decomposed in two dependent parcels, one referent to the piezoelectric material used as actuator  $\phi_a$ , which is not used in this work, and the other referent to the piezoelectric material used as sensor  $\phi_s$ . Knowing that  $\phi_s$  is the electric potential generated by sensor due to a piezoelectric strain and is given as a function of the electroelastic rigidity  $K_{\phi u}$ , the piezoelectric capacitance of the structure  $K_{\phi\phi}$ , and the displacement due to the efforts actuating in the structure  $u$ , from the second Eq. (4).

$$\phi_s = -K_{\phi\phi}^{-1} K_{\phi u} u \quad (9)$$

The system equations (4) do not consider damping. However, structures generally have a degree of damping. This value is difficult to be defined with accuracy, but can be expected. From this work, the damping is considered as proportional damping defined as proportional to the mass and stiffness matrices, Eq. (10).

$$D_a = \alpha M + \beta K \quad (10)$$

with:

$$K = K_{uu} - K_{u\phi} K_{\phi\phi}^{-1} K_{\phi u} \quad (11)$$

The global system of motion equations including the damping is given by:

$$M\ddot{u} + D_a\dot{u} + Ku = F - K_{u\phi} K_{\phi\phi}^{-1} Q \quad (12)$$

where  $M$ ,  $D_a$  and  $K$  are the global matrices of mass, damping and stiffness, respectively.

### 3. THEORETICAL AND EXPERIMENTAL EVALUATION OF THE ELECTRIC ENERGY OF THE PIEZOELECTRIC STRUCTURE

The electromechanical system studied is a beam-like structure of aluminum with PZT patches bonded on the beam. Table 1 presents the main properties of the beam structure and PZT (Piezo systems, Inc. PSI-5H4E). The piezoelectric structure was analyzed in a clamped condition by using the finite element approach and experimental test. In both cases, the potential electric generated by the structure was obtained and the results were compared aiming at to define a reliable finite element model that could be used to study different configurations and positions of the sensor and the actuator in the structure.

Table 1 – Dimensions and properties of the beam and the PZT.

Beam Parameters	Value	PZT Parameters	Value
Young's modulus (GPa)	70	Young's modulus YE1(GPa)	62
Poisson's ratio	0.33	Young's modulus YE3(GPa)	50
Density (kg/m <sup>3</sup> )	2690	Piezoelectric constant d33 (m/V)	650e-12
Length (mm)	400	Piezoelectric constant d31 (m/V)	-320e-12
Width (mm)	30.5	Relative dielectric constant KT3	3800
Thickness (mm)	2.1	Coupling coefficient k33	0.75
		Coupling coefficient k31	0.44
		Density (kg/m <sup>3</sup> )	7800
		Length (mm)	20
		Width (mm)	20
		Thickness (mm)	0.267

#### 3.1 Finite element model

The finite element model of the electromechanical structure was defined by using the program ANSYS®. The beam structure was discretized in 245 solid elements of 8 nodes, with displacement in the direction x, y and z (SOLID 45). The PZT patch was modeled also using 100 solid elements of 8 nodes but now with displacement in the direction x, y and z and also electric potential (SOLID5). The PZT patch was bonded in the other side of the aluminum beam, 60 mm from the fixed end of beam structure. The frequencies of the model, Tab. 2, were obtained through a theoretical modal analysis, solving the problem of eigenvalues/eigenvectors. Figure 1 shows vibration modes of the structure with PZT elements.

Table 2 – Numeric natural frequencies.

Modes	Numeric $f_n$ (Hz)
1	11.0
2	68.5
3	192.4
4	381.5

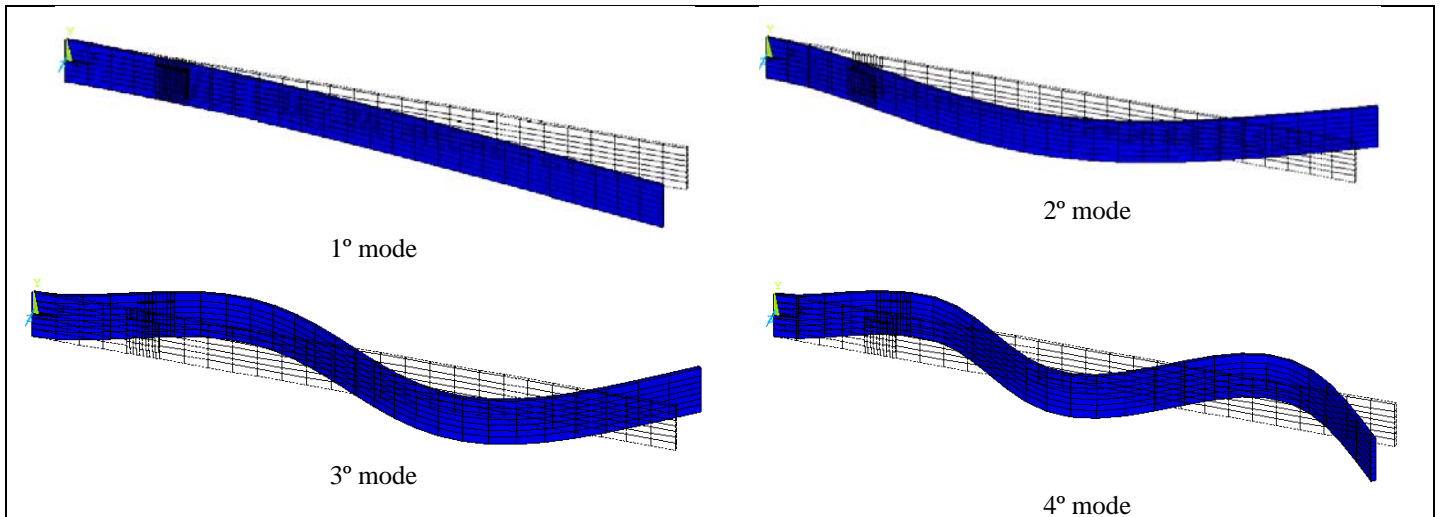


Figure 1. Vibration modes of the structure with PZT

The responses of the model were also obtained from the finite elements matrices, in a frequency range from 0 to 100 Hz, with increments of 0.1 Hz. The response was derived using as excitation force (sine) of unity amplitude, applied at a point 30 mm from the fixed extremity of the beam, in the z-direction. The response of the structure was obtained at various points, in the same direction of the excitation. Figure 2 shows the frequency responses of the structure containing the first two modes, calculated for three different points, located in the beginning, middle and end of the beam.

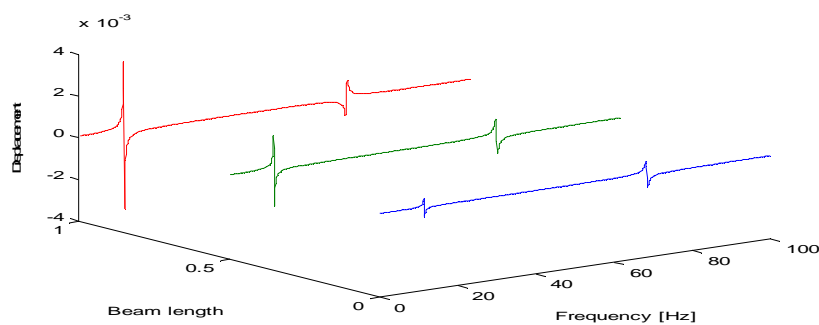


Figure 2. Structural response of the system

The electric potential generated by the system it is related to the applied efforts in the beam structure and the position of the PZT. In the case of frequency excitation close to the resonance frequencies, there will be major deformations in some specific regions of the structure concerning to the shape of the excited mode. Depending on the position of the PZT, the electric potential generated can be larger or smaller for that mode, as also discussed in Leo (2007).

The electric potential generated by structure due to application of a sine force, close to the first resonance frequency, it was investigated. In the various analyses it was used the proportional damping, Alpha coefficient equals 0.9 and Beta coefficient equals  $1 \times 10^{-4}$ . The positioning and the sensor size have shown a variation of the electric potential

generated by the structure, which makes these choices an important issue to be studied. For the excitation force at the frequency of 11.06 Hz and unity amplitude the calculated electric potential it was of 9.223V.

The concept of spatial filter (Leo, 2007) could be used in this case to study the positioning of sensor/actuator. Since in this approach, the electromechanical coupling is correlated with the integral of the deformation of the material and, in the case of an element of a single dimension, the term of coupling is proportional to the difference of the slope of the ends of the element. If the wavelengths of the vibrating modes of the structure are long compared with the size of the piezoelectric element (sensor), the change of the deformation is small and the slope difference will also be small. So, depending of the positioning of the sensors, some vibrating modes can be filtered, such that the value of the slope difference in relation to those modes is negligible if compared with the values of the remaining vibrating modes. The use the numeric finite element model in this case emerges as an obvious approach and, it appears to be more reasonable than the use of experimental tests. However, it demands a representative finite element model, which can be obtained by comparing and updating the response of the numeric modeling with the experimental data (Pereira and Borges, 2002).

### 3.2 Experimental test

The experimental test of the structure was carried out aiming at to measure the electric potential generated by the vibrating structure, as well as, to validate the results of the finite element model used to simulating some force conditions, concerning to its capacity of generate energy. Figure 3 shows the experimental set-up used for the tests. The system consists of a beam structure with a PZT patch bonded at a position 30 mm way from the clamped end. The model parameters and boundary conditions are equivalent to the used in the finite element model, show in Tab. 1. The structure excitation is controlled by an electromechanical shaker that will simulate the operating condition of the system.

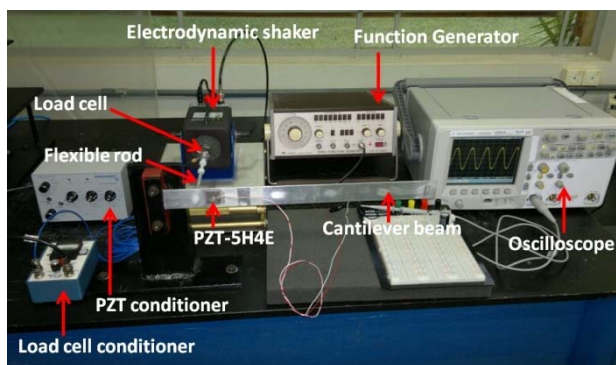


Figure 3. Experimental set-up

The structure was excited using a flexible rod fixed in the shaker and the force it was measured with a load cell mounted between the shaker and the flexible rod aiming at to minimize the effect of the mass of the cell in the structure itself. The acquisition system used was SignalCalc Ace Dynamic Analyzer, from Data Phisics and a PZT conditioner Piezo Film Lab Amplifier from Measurement Specialties<sup>TM</sup>. The parameters and the frequency range of analysis, in the first moment, they were been defined based on the finite element analysis.

Preliminary tests were conducted at first seeking to observe and identify the modal parameter of the structure, previously, identified in the numerical modeling. In this case, the structure was excited with an instrumented hammer in the z-direction and the responses were measured by using the own PZT material as sensor, Tab. 3 shows the identified experimental natural frequencies.

Table 3 – Experimental natural frequencies.

Modes	Experimental $f_n$ (Hz)
1	10.2
2	62.0
3	173.5
4	343.0

Once identified and extracted the modal parameters of interest of the structure, the experimental tests were conducted to measure the electric potential generated by the piezoelectric structure.

The structure was excited with a sine force at the frequency of 10.2 Hz (resonance) and the electric potential generated by the piezoelectric structure was measured. The frequency of the excitation force and amplitude were

controlled by a signal generator used to feed the shaker and the force applied in the structure was measured using the force cell between the shaker and structure. To measure the output voltage of the PZT it was used a multimeter. The value of the measured force was 0.4795 N and the measured output voltage of the PZT was 4.446 V.

### 3.3 Comparison of the models

The comparison of the models, in general, aims to define the representativity of the numeric finite element model in order to study and evaluate other features of the system. In this case, the comparison was to validate the results obtained by finite element approach in order to evaluate the capacity of the system to generate energy and also to quantify the amount of the electric potential generated by the model, which could be stored in a storage device such as capacitor. The results showed that the first four numeric modes calculated were the same of those experimentally identified and they differ in frequency a maximum of 10.09%, Tab. 4. The models show a reasonable correlation and the finite element model updating technique was not used, since the goals, in this case, was not to obtain a very reliable model. The frequency response functions of the two models also were qualitatively compared. Figure 4 shows the calculated and measured FRF curves of the piezoelectric structure.

Table 4 – Experimental and numeric natural frequencies.

Modes	Experimental $f_n$ (Hz)	Numeric $f_n$ (Hz)	Difference %
1	10.2	11.0	7.27
2	62.0	68.5	9.49
3	173.5	192.4	9.82
4	343.0	381.5	10.09

$$Difference = (Experimental f_n - Numeric f_n / Numeric f_n) * 100$$

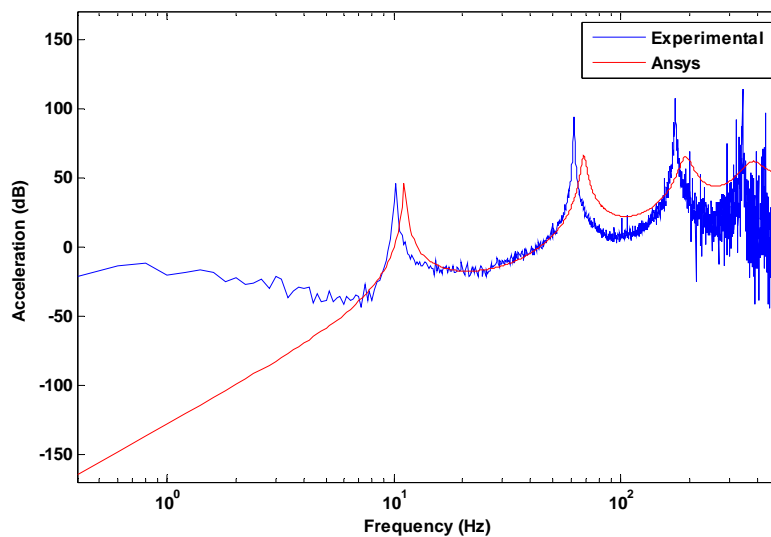


Figure 4. Frequency response function for the beam with a bonded PZT for numerical and experimental test

Identified the parameters of the model it was conducted a test to obtain the electric potential generated by the system for some specific conditions of excitation. For a sine force of frequency close to the first resonance of the model, the value of the electric potential measured in the PZT it was 4.446 V. The excitation force was measured by the force cell between the shaker and structure and the value was 0.4795 N and frequency 10.2 Hz. The value of this measured force it was used in the finite element modeling in order to reevaluate the electric potential generated of the simulated model to compare it with the experiment one. The Figure 5.a shows the numerical and the experimental electric potential generated in PZT for the two cases and Figure 5.b shows the respective spectrums of responses. The electric potential calculated by using the value of the measured force in the numerical model at resonance (11.06 Hz) it was 4.415 V

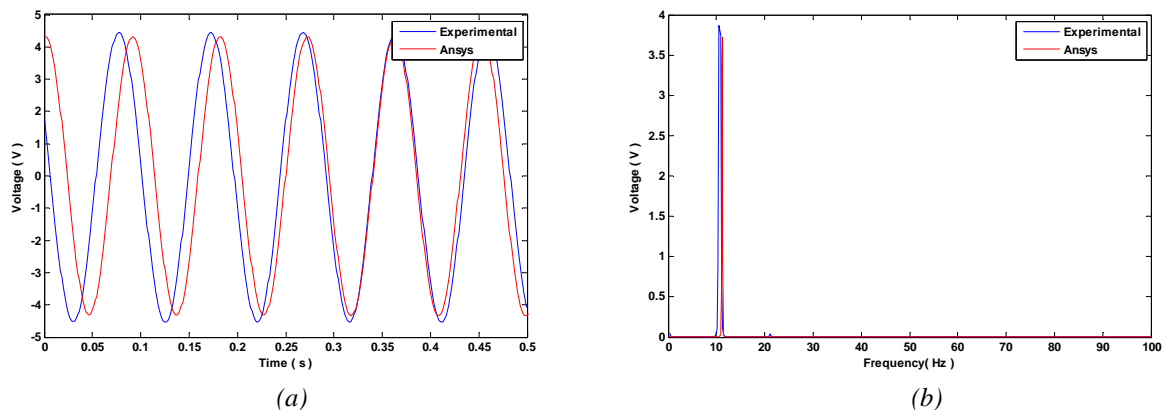


Figure 5. (a) Time signal of the experimental and numerical response in the PZT and (b) spectrum of the experimental and numerical responses

Comparison of the two models shows a reasonable correlation of the responses curves. The electric potential calculated by the finite element model was 4.415 V and the measured in the experiment it was 4.446 V. Note that in both cases, the excitation frequency is close to the first natural frequency of the models and the electric potential generated is almost the same. This means that the numerical model could be used to study and evaluate others conditions and features of the piezoelectric structure.

Preliminary experiments to store the electric potential generated of the system were also conducted by using a supercapacitor. The amount of the electric potential generated stored was approximately 3.5 V. However to understand better the whole energy harvesting system it is necessary to include in the modeling a storage component. The next step is to use the numerical model to study other conditions of operating of the system and modeling directly the coupling of the storage device in the system and also to use the stored energy to drive a low power device component.

#### 4. CONCLUSIONS

In this study, the modeling of a beam-like structure with bounded PZTs subjected to mechanical vibration was evaluated experimentally and numerically. It was verified the possibility of the utilization the piezoelectric structure for energy generation. The comparison of the numerical results with experimental data indicated that the adopted finite element modeling was reasonable to represent the system aiming at studying its potentiality. The next step would be to integrate the storage circuit directly in the modeling of the system and to use the stored electric potential generated in order to drive a low power device component.

#### 5. ACKNOWLEDGMENTS

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