# NUMERICAL EVALUATION OF THE SHEAR EFFECT IN BEAMS MADE OF POLYMERIC COMPOSITES REINFORCED WITH WOOD SAWDUST

**Árlei Huebra Póvoa, arleipovoa@hotmail.com**
**André Luis Christoforo, alchristoforo@ufsj.edu.br**
**Túlio Hallak Panzera, panzera@ufsj.edu.br**
**Fabiano Bianchini Batista, fabianchini@ufsj.edu.br**
Universidade Federal de São João del-Rei (UFSJ) – Dpto. de Eng. Mecânica (DEMEC) – 36307 – 352, Campus Santo Antônio (CSA), São João del-Rei – MG, Brasil.

**Avelino Manuel da Silva Dias, avelino.dias@ct.ufrn.br**
Universidade Federal do Rio Grande do Norte (UFRN) – Dpto. de Eng. Mecânica, Natal – RN, Brasil.

***Abstract.*** *Beams are elements commonly used in structural engineering and mechanical designs. The use of polymeric composite materials reinforced with wood sawdust has been the focus of many researches in engineering construction. The determination of variables such as displacements, stresses and strains is essential for the accurate design of beams. However, the mathematical formulations which are responsible to predict the mechanical behavior of the beams are developed under certain calculation hypothesis. This paper aims to determine the relation between height and length of the beam profile made of this sustainable composite, in order to neglect the effect of shear forces in the calculation of displacements, ensuring the validity of the Bernoulli beam theory, usually employed in structural analyses. This study was conducted using a Finite Element Method software, which was developed in accordance to Bernoulli and Timoshenko beam theories.*

***Keywords****: Polymeric composites, beams theory, Finite Element Method.*

## 1. INTRODUCTION

The structural engineering is a sub-area of civil engineering analysis and design responsibility for them. Excluding bar elements, we can highlight the beams, which are commonly found in mechanical design and civil construction.

The mechanical design of beams requires knowledge of the stresses and displacements that develop in the piece. The stresses and displacements are inherent to mathematical models for calculating defined.

The theory of Bernoulli beams, commonly employed in many of the mechanical designs, takes into account only the portion of the bending moment in the calculation of displacements, so that, in this theory, flat sections remain plane and orthogonal to the axis of the deformed bar after being properly the case for long beams, where the normal stresses are significantly larger than the shear.

Timoshenko theory also account for the effect of stress bending moment, also consider shear stress. In this theory, flat sections, no longer orthogonal to the axis deformed. The effect of shear force in the computation of the displacement becomes significant as the length of the beam, or rather, that ratio of height *(H)* by the section length (*L*) becomes smaller and smaller.

This work aims to mirror the range of values for the ratio *H/L* for which the effect of shear becomes significant in calculation of displacements in a beam model of polymer composite material reinforced with particulate sawdust. This analysis is designed according to the fundamentals of Finite Element Method.

## 2. PARTICULATE REINFORCED BY POLYMERIC COMPOSITE WOOD SAWDUST

The increase of domestic waste, industrial and minerals, plus the difficulty of recycling, has been subject of debates and conferences all around the world, in order to reuse these materials and reduce the social and environmental impacts. Grounding and incineration practices are condemned and unused in most developed countries, which are major sources of pollution through the contamination of soils and the release of toxic gases, respectively. For this reason the possibility of reuse these materials is crucial to the quality of living and sustainability. The waste from timber industry, known as sawdust, will be investigated in this work as the dispersed phase polymeric particulate composites.

Due to its low cost and high availability, wood fibers are being increasingly used as reinforcement in composite materials instead of synthetic fibers (Stark and Rowlands, 2003). The incorporation of wood waste in particulate composites not only reduces the cost of the final product, but also can substantially improve the performance of the material on mechanical properties such as bending strength, traction and stability dimensionless (Bengtsson *et al*., 2005).

Composites made from wood waste presents a great potential for application in the aeronautical, automotive, sports and construction have been used in various types of polymer matrices (Bledzki *et al*., 2005, Robin and Breton, 2001 and Bledzki and Fauk, 2004), and cementitious matrices (Kavvouras, 1987, Teixeira and Guimarães, 1989 and Batista 2003).

Vianna *et al.* (2004) found that fine particles of wood can be used efficiently in substitution of mineral fillers and glass fibers, exhibiting adequate mechanical properties for many engineering applications. However, one of main problems in the manufacture of composite polymer/wood fiber is the adherence to the polymer matrix. The chemical affinity between the cellulose and the polymer matrix can be improved by modifying the surface of the fiber or the polymer with the use of various additive elements, such as maleic anhydride (Maldas and Kokta, 1990 and Lyons and Mallik, 2005).

On grounds variety of wood species and differents processes they are subjected, the wood fibers may present with different physical and chemical characteristics. Such characteristics may influence the properties of composite polymer/wood products (Raj *et al.*, 1989).

Panzera *et al.* (2009) studied the addition of different particle sizes (10/20 and 100/200 Tyler US-US-Tyler) and different mass fractions (40%, 60% and 80%) of the composite particles of sawdust. The composites made from sawdust 100/200 US-Tyler exhibited satisfactory mechanical properties and superior to those of US-gradings 10/20 Tyler. The increase in the fraction of sawdust factor promoted the decrease in density and strength of the composites, however the properties presented by the fraction of 60% were satisfactory for many engineering applications, enabling the reuse of industrial waste as filler in a composite material development. The present study developed in this paper uses a beam that has mechanical properties for the composite made from sawdust-Tyler U.S. 100/200 and 60% weight fraction of sawdust, shown in Tab. 1.

Table 1. Properties of the composite made with 60% mass fraction of sawdust 100/200 US-Tyler

| | |
|---|---|
| Volumetric Density | 1050 kg/m ³ |
| Longitudinal Modulus of Elasticity | 1.45 GPa |
| Poisson ratio | 0,39 |
| Transverse Modulus of Elasticity | 0.43 GPa |
| Bending Strength | 30 MPa |

## 3. THEORY OF BEAMS TIMOSHENKO

The fundamental hypothesis of the bending theory beams of Timoshenko, as shown above, is that flat sections remain flat after deformation, but no more orthogonal to the axis of the bar. It occurs because of the distortion or due to deformation by shear. The other assumptions are the same as Euler-Bernoulli theory. This hypothesis presents a closer approximation of the actual deformation of the beam, particularly for non-slender beams. The kinematic of the beam illustrated in Fig. 1.

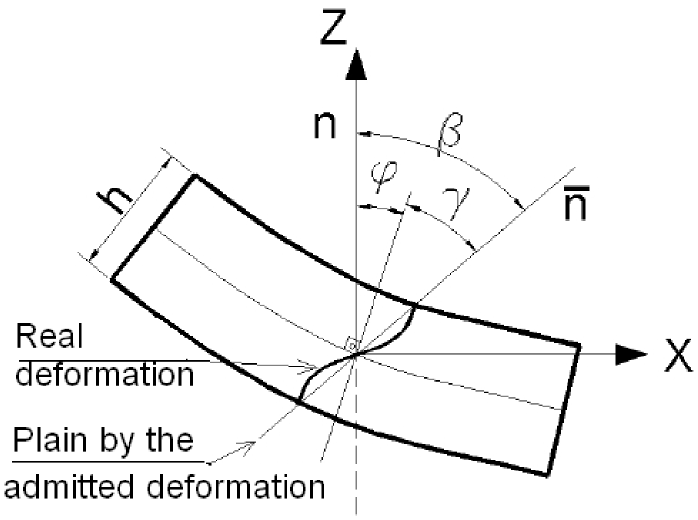


Figure 1. Kinematics of Timoshenko beam.

Kinematics of the beam can be stated:

$$\beta = \varphi + \gamma \; ; \; \varphi = -w_x ; \; \beta = -w_x + \gamma \qquad (1a, 1b, 1c)$$

Upon verification of the conditions of equilibrium of an element with infinitesimal length *dx* subjected to a uniformly distributed load of intensity *q* perpendicular to its axis, one can write the following equilibrium equations:

$$\frac{dV}{dx} = -q \ ; \ \frac{dM}{dx} = V \ ; \ V = -E\,I\,\frac{d^3w}{dx^3} \qquad (2a, 2b, 2c)$$

The following, constitutive relations, assuming the hypothesis of a material with linear elastic conduct:

$$M = E\,I\,\frac{d\varphi}{dx} \qquad (3)$$

$$V = f\,G\,A\gamma = f\,G\,A(\frac{dw}{dx} + \beta) \qquad (4)$$

Where *E* is the modulus of elasticity, *G* the transverse modulus, *I* the moment of inertia of the bending section, *A* is the cross sectional area and *f* is a factor that takes into account that $\gamma$ is an average distortion of section. For rectangular sections *f* takes the value of 5/6. Further details on obtaining the form factor *f* can be found at Oñate (1993).

**3.1. Determination of the Stiffness Matrix for Finite Element under the theory of Timoshenko**

The simplest finite element that includes the bending theory beams of Timoshenko is that one where we employ approximately linear functions for both vertical displacements and for the rotations. Oñate (1993) presents a finite element obtained with linear approximation for the fields of displacements and rotations. However, this element has the lock-up problem for a few beams for shear deformable.

Gere and Weaver (1981) present a deduction of the stiffness matrix for Timoshenko beam which does not present the phenomenon of locking through the use of so-called direct method (application of unitary displacements and rotations at the ends of the element). In this work the element stiffness matrix is deduced with the same assumptions made by Gere, necessitating that the balance equation between moments and biting should be respected in the element field and limits, but the method will be employed to obtain energy functions curvature and distortion along the length of the element.

Figure 2 illustrates the degrees of freedom (vertical displacement and a rotation per node) combined with the directions taken as positive in the finite element formulation.

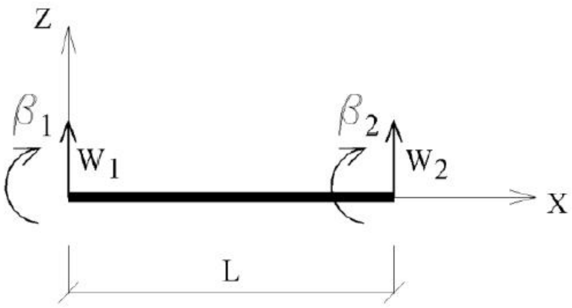


Figure 2. Finite Element Beam.

The vertical displacement *w* along the element are described by a polynomial of third degree, as shown in Eq. (5).

$$w = a_1x^3 + a_2x^2 + a_3x + a_4 \qquad (5)$$

The equilibrium equation, Eq. (2), which expresses the relationship between the shear and vertical displacements *w* may be rewritten as Eq. (6) shows.

$$-EI\,\frac{d^3w}{dx^3} = f\,GA\gamma \qquad (6)$$

Thus it becomes possible to write the relationship between the $\gamma$ distortion and vertical displacements *w*, thus:

$$-EI\frac{d^3w}{dx^3} = f\,GA\gamma \tag{7}$$

being $g = 6EI/fGAL^2$ Weaver's constant. It is perceived that $\lim_{A\to 0} g = \infty$.

The rotations $\beta$ are written from the Eq. (1c), thus:

$$\beta(x) = -3a_1x^2 - 2a_2x - a_3 - gL^2a_1 \tag{8}$$

The constants $a_i$ polynomials can be determined from the imposition of boundary conditions on the element nodes. These conditions can be written in matrix form expressed by Eq. (9).

$$\begin{bmatrix} w_1 \\ \beta_1 \\ w_2 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -gL^2 & 0 & -1 & 0 \\ L^3 & L^2 & L & 1 \\ L^2(-3-g) & -2L & -1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{9}$$

By solving the equations above, results:

$$a_1 = \frac{1}{L^3(1+2g)}(2w_1 - 2w_2 - L\beta_1 - L\beta_2) \tag{10}$$

$$a_2 = \frac{1}{L^2(1+2g)}(-3w_1 + 3w_2 + L\beta_1 - L\beta_2) \tag{11}$$

$$a_3 = \frac{1}{L(1+2g)}(-2w_1 + 2w_2 - L(1+g)\beta_1 + Lg\beta_2) \tag{12}$$

$$a_4 = w_1 \tag{13}$$

Then the curvature of the section can be written directly in terms of nodal parameters according to Eq. (14), as expressed in Eq. (15).

$$\frac{1}{r} = -\frac{d^2w}{dx^2} \tag{14}$$

$$\frac{1}{r} = -6\frac{(L-2x)}{(1+2g)L^3}w_1 + 2\frac{\left[2L(1+\frac{g}{2}) - 3x\right]}{(1+2g)L^2}\beta_1 + 6\frac{(L-2x)}{(1+2g)L^3}w_2 + 2\frac{[L(1-g)-3x]}{(1+2g)L^2}\beta_2 \tag{15}$$

Starting from the Eq. (7) and using the Weaver’s constant, one can write the distortion this way:

$$\gamma = g\frac{[2(w_2 - w_1) - L(\beta_1 + \beta_2)]}{[L(1+2g)]} \tag{16}$$

From the formulation of the finite element method via potential energy and integrating the tensions at the height of the element we can write the strain energy by:

$$U = \frac{E\,I}{2}\int_0^L \left(\frac{1}{r}\right)^2 dx + \frac{f\,G\,A}{2}\int_0^L (\gamma)^2\, dx \tag{17}$$

Applying the principle of minimum energy (Eq. (18)) in the expression of strain energy (Eq. (17)) and performing the integration along the element length, one obtains finally the stiffness matrix for Timoshenko beam, expressed by Eq. (19).

$$\frac{d\Pi}{dV_i} = 0 \tag{18}$$

$$K = \frac{E\,I}{1+2g}\begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L}(2+g) & \frac{6}{L^2} & \frac{2}{L}(1-g) \\ -\frac{12}{L^3} & \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L}(1-g) & \frac{6}{L^2} & \frac{2}{L}(2+g) \end{bmatrix} \tag{19}$$

From Eq. (18), $\Pi = U + W$. $W$ is the external potential energy and $V_i$ represents the nodal degrees of freedom. For the vector of nodal loads is employing the same formulation obtained by Euler-Bernoulli, thus:

$$F^T = \begin{bmatrix} \frac{q\,L}{2} & -\frac{q\,L^2}{12} & \frac{q\,L}{2} & -\frac{q\,L^2}{12} \end{bmatrix} \tag{20}$$

### 3.2. Crash or lock Solution

The finite element bending of beams according to Timoshenko beam theory obtained from the form as presented, ie, by imposing the equilibrium conditions between moments and shear forces is completely free from the phenomenon of blocking solution, or blocking.

The lock is basically the solution for excessive hardening of relationship between the small beam height and length. Imagining a rectangular base $b$ and height $h$, the Weaver's constant is shown by Eq. (21).

$$g = \frac{E}{2G}\left(\frac{H}{L}\right)^2 \tag{21}$$

It is clear that in the limit situation where $(H/L) \rightarrow 0$, $g \rightarrow 0$, and the stiffness matrix expressed in Eq. (19) tends to the stiffness matrix considering Euler-Bernoulli hypothesis, thus the locking is completely removed.

Concerning the use of FEM to evaluate the performance of mechanical structures and in determining the elastic properties of materials, some works can be cited such as de Alvarenga and Antunes (1994), Cheung and Lindquist (2004), Christoforo (2007), Garcia (2004), Mascia (1991), Rigo (1999) among others.

## 4. PROBLEM MODEL

In this example, we intend studying the influence of shear deformation of a beam clamped at one end and cantilevered at the opposite end subjected to a concentrated load $F$ = *180 N*, as shown in Fig. 3. Therefore, it was determined the cross section of the beam and vary the length of the balance sheet so that the ratio of height *(H)* and the beam length *(L)* takes values between 0.8 and 0.05. The characteristics of polymeric composite materials reinforced with wood sawdust, a constituent of the beam are presented in Tab 1. The values for the width ($b$) and height of the cross section are respectively equal to 0,1 m and 0,4 m.


Figure 3. Problem model.

## 5. SOLUTION OF MODEL PROBLEM

We employ the finite element method for analysis of displacements and the beam was discretized into a single finite element. He likens himself to study the effect of shear deformation, results obtained with the theories of Euler-Bernoulli and Timoshenko, whose exact values of vertical displacement of the free end are given by Eq. (22) and Eq. (23), respectively.

$$w = \frac{F\,L^3}{3E\,I} \tag{22}$$

$$w = \frac{F\,L^3}{3E\,I} + dw(V) \tag{23}$$

The results are presented in Tab. 2 and in Fig. 4, which are related to vertical displacements of the free end of the beam. The use of finite element method leads to identical values to exact for this case and in the case of linear analysis is not necessary to use more than one element.

Table 2. Shifts in the balance

| ***H / L*** | **Bernoulli** | **Timoshenko** | **Rate (%)** |
|---|---|---|---|
| **0,8** | 0,0097 | 0,0160 | 65,088 |
| **0,7** | 0,0145 | 0,0217 | 49,833 |
| **0,6** | 0,0230 | 0,0314 | 36,612 |
| **0,5** | 0,0397 | 0,0498 | 25,425 |
| **0,4** | 0,0776 | 0,0902 | 16,272 |
| **0,3** | 0,1839 | 0,2007 | 9,153 |
| **0,25** | 0,3178 | 0,3380 | 6,356 |
| **0,2** | 0,6207 | 0,6459 | 4,068 |
| **0,15** | 1,4713 | 1,5049 | 2,288 |
| **0,1** | 4,9655 | 5,0160 | 1,017 |
| **0,075** | 11,7701 | 11,8374 | 0,572 |
| **0,05** | 39,7241 | 39,8251 | 0,254 |

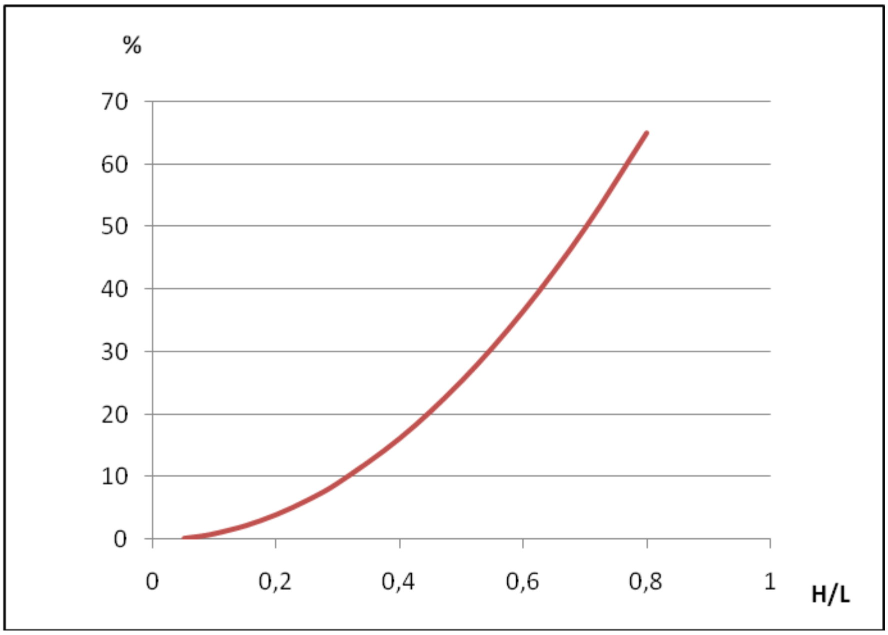


Figure 4. Increase the percentage shift in the balance sheet due to sharp.

## 6. CONCLUSIONS

The use of wood waste in particulate composites, beyond reduce the cost of the final product and improve the performance of the material in some mechanical properties, helps prevent grounding and incineration of waste, avoiding contamination of soils and the release of toxic gases, contributing for the sustainability of the planet.

From the result analysis of model problem, we can conclude that the deformation by cutting should be considered for relationship *H/L* greater than 0,2 for beams fabricated with polymeric composites reinforced with wood sawdust, i.e., for a higher reliability of the project, the model of the Timoshenko beam theories should be used.

Of course, this number is only an indicative, because the width of the beam also influences their conduct. It’s important note that rates for *H/L* higher than 0,8, we must use two-dimensional finite elements, because it’s no longer possible to admit that the sections remain flat.

## 7. ACKNOWLEDGEMENTS

The authors thank UFSJ (Federal University of São João del-Rei) and FAPEMIG (Research Support Foundation of Minas Gerais) for their support and financial support.

## 8. REFERENCES

Batista, A.M., 2003, “The influence of the treatment on wood-cement composites”, Journal Psico-USF, vol. 8, n. 2, pp. 103-114.

Alvarenga, R.C.S.S. and Antunes, H.M.C., 1994, “Optimization of trusses”, 15th Ibero Latin-American Congress on computational methods for engineering, Belo Horizonte, Brazil, pp. 1699-1708.

Bengtsson, M., Gatenholm, P. and Oksman, K., 2005, “The effect of crosslink on the properties of polyethylene/wood flour composites”, Composites Science and Technology, Vol. 65, pp. 1468-1479.

Bledzki, A.K. and Fauk, O., 2004, “Creep and impact properties of wood fibre polypropylene composites: influence of temperature and moisture content”, Composites Science and Technology, Vol. 64, pp. 693-700.

Bledzki, A.K., Letman, M., Viksne, A. and Rence, L., 2005, “A comparison of compounding processes and wood type for fibre-PP composites”, Composites Part A: applied science and manufacturing, Vol. 36, pp. 789-797.

Cheung, A.B., Lindquist, D. and Calil, C.J., 2004, “Calibration of elastic properties of an ortótropa plate using genetic algorithms”, Journal of South American Structural Engineer, Vol. 1, No. 2, University of Passo Fundo. Passo Fundo, Brazil. pp. 74-92.

Christoforo, A.L., 2007, "Influence of irregularities of form in pieces of wood in the determination of longitudinal elasticity module", PhD Thesis, EESC-USP. São Carlos, Brazil.

Gere, J.M. and Weaver Jr., W., 1981, “Analysis of Cross-linked Structures”, Guanabara. Rio de Janeiro, Brazil.

Garcia, J.L.N., 2004, "Theoretical models for sizing of bridges with wooden protendida multicellular Board", XXXI Jornadas Sud-American Structural Engineering. Facultad de Ingeniería. Universidad Nacional de Cuyo. Mendonça. Argentina.

Kavvouras, P.K., 1987, "Suitability of Quercus conferta wood for the manufacture of cement-bonded flakeboards", Holzforschung, Vol. 41, pp. 159-163.

Lyons J.S. and Ahmed M.R., 2005, "Factors Affecting the Bond Between Polymer Composites and Wood", Journal of Reinforced Plastics and Composites, Vol. 24, pp. 404-405.

Maldas, D. and Kokta, B.V., 1990, "Influence of polar monomers on the performance of wood fiber reinforced polystyrene composites. I. Evaluation of critical conditions", International Journal of Polymeric Materials, Vol. 14, pp. 165-189.

Mascia, N.T., 1991, "Considerations of anisotropy of wood", PhD Thesis – EESC-USP. São Carlos, Brazil.

Panzera, T.H., Rubio, J.C.C., Bowen, C., Vasconcelos, W.L. and Strecker K., 2008, "Correlation between structure and pulse velocity of cementitious composites", Advances in Cement Research, Vol. 20, pp. 101-108.

Raj, R.G., Kokta, B.V., Maldas, D. and Daneault, C., 1989, "Use of wood fibers in thermoplastics: VII. The effect of coupling agents in polyethylene/ wood fiber composites", Journal of Applied Polymer Science, Vol. 37, pp. 1089-1103.

Rigo, E., 1999, "Optimization methods applied to the analysis of structures", Master Dissertation – EESC-USP. São Carlos, Brazil.

Robin, J.J. and Breton, Y., 2001, "Reinforcement of recycled polyethylene with wood fibers heat treated", Journal of Reinforced Plastics and Composites, Vol. 20, pp. 1253-1262.

Oñate, E., 1993, "Calculus of Structures by finite element Method: analysis of Elastic-lineal", International Center for Numerical Methods in Engineering. Barcelona, Spain.

Stark, N.M., Rowlands R.E., 2003, "Effects of wood fiber characteristics on mechanical properties of wood/polypropylene composites", Wood Fiber Science, Vol. 35, pp. 167-174.

Teixeira, D. E. and Guimarães, T. L., 1989, "Treatment of particles Acacia mearnsii wild for the production of wood-cement sheets", Ministry of Interior, Brazilian Institute of Environment and Renewable Natural Resources, Laboratory of Forest Products, LPF-series Technical Publication, Brasilia, Brazil, No. 11, pp. 9.

Vianna, W.L., Correa, C.A. and Razzino, C.A., 2004, "Effects such as high impact polystyrene in the properties of thermoplastic compositeswith flour, wood waste", Polymers: Science and Technology, Vol. 14, No. 5, pp. 339-348.

## 9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.