EFFECTS OF AXIAL CONDUCTION IN THE WALL ON CONJUGATE HEAT TRANSFER IN CIRCULAR TUBES WITH SLIP FLOW

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Abstract. *A fully analytical solution, based on the ideas of the Generalized Integral Transform Technique, is obtained for an internal fully developed laminar slip-flow regime subjected to a convection process to the ambient. Here, the duct wall effect is taken into account by a radially lumped energy balance which results in a more involved boundary condition. An extensive collection of results are scrutinized in order to study the influence of the Knudsen and Biot numbers and also that of the conjugation parameter which encompasses the Peclet number, fluid to wall thermal conductivity ratio and inner and outer tube radius ratio. Numerical values for this conjugation parameter reflect the physical situation encountered in typical silicon microtubes conveying nitrogen under the incompressible slip-flow regime. These results are also critically compared with the traditional fully developed parabolic profile and an assessment of the three dimensionless parameters is made on the axial distributions of the bulk and wall temperatures and local Nusselt number.*

Keywords*: slip flow, Knudsen number, conjugate problem, microtubes*

1. INTRODUCTION

The analysis of heat and fluid in microchannels has received a great attention during recent years due to its relevance in cutting edge applications related to micro-electrical mechanical systems (MEMS), the cooling of the latest generation of microchips used in personal computers, spacecraft thermal control and micro heat exchangers. There are also studies in this field related to biological systems and a better appreciation of such diverse applications can be found in review papers and monographs such as those of Avelino and Kakac (2004) and of Kakac *et al.* (2005). As already pointed out by early surveys (Sparrow and Haji-Sheik, 1964 and Tunc and Bayzitoglu, 2001), the mathematical modeling of such heat transfer situations must take into account the so-called "slip velocity" boundary condition at the wall channel.

A brief literature review suggests that considerable effort has been made over the years to provide for closed-form analytical solutions to the single-phase thermal entry problem in microchannels. Barron and his collaborators (1996, 1997) have devised a scheme to calculate the eigenvalues and related eigenquantites basic to the Graetz problem with slip-flow. Mikhailov and Cotta (1997) have employed the Mathematica® package in order to address the same eigenvalue problem but with a more accurate numerical precision by employing the "sign-count" method to the Sturm-Liouville problem related to the thermal entry in microchannels of circular cross section. Castellões *et al.* (2004) utilized the Generalized Integral Transform Technique (GITT) to simulate the transient thermal entry region in a parallel plate cross section microchannel where viscous dissipation and heat diffusion along the axial direction are considered. An inspection of convergence behavior indicates that a truncation order of 15 terms is enough to warrant validation with previously published results and the influence of the Brinkman number in the average fluid temperature and Nusselt number distribution is assessed.

An interesting aspect related to the contributions discussed above is that they do not take into account wall conjugation effects. Guedes *et al.* (1989) and Guedes and Cotta (1991) discussed this important effect in internal convection heat transfer problems especially in regions close to the channel inlet. Castilho (2003) extended this original research work to the slip flow regime including fluid viscous dissipation and temperature jump at the walls. In this contribution, we present the analysis and solution of the thermal entry problem in a circular cross section microchannel, which exchanges heat to an external environment through a convection process and at the same time including the heat diffusion at the wall channel. As it shall be discussed in the following sections, once the problem formulation is obtained, the ideas of the GITT are employed to obtain a fully analytical solution, which is then utilized for studying relevant parameters such as the fluid average, wall temperature and Nusselt distributions for various combinations of the conjugation parameter, dimensionless external heat transfer coefficient and slip flow regimes.

2. ANALYSIS

This analysis starts out by briefly recalling some basic aspects related to the determination of the velocity profile with slip flow considerations. By constraining the Navier-Stokes equations to a steady state, parallel flow in a circular duct of radius r_1 and taking into account an independent viscosity coefficient, one finds:

$$d\left[r\frac{du}{dr}\right]=\frac{1}{\mu}\frac{dp}{dz}rdr \tag{1}$$

which is subjected to symmetry and slip flow boundary conditions such as (Kakac *et al.*, 2005)

$$\left.\frac{du}{dr}\right|_{r=0}=0 \tag{2}$$

$$u(r_1)=-\frac{2-F_v}{F_v}\lambda\left(\frac{du}{dr}\right)_{r=r_1} \tag{3}$$

By integrating Eq (1) twice and employing the restrictions described in Eqs. (2) and (3), it is a relatively simple matter to determine the dimensionless fully developed velocity profile as:

$$U(R)=\frac{u(r)}{u_m}=\frac{2(1-R^2+4Kn\beta_v)}{1+8Kn\beta_v} \tag{4}$$

where $R=\dfrac{r}{r_1}$ $\quad \beta_v=\dfrac{2-F_v}{F_v}$ $\quad Kn=\dfrac{\lambda}{2r_1}$

An inspection on Eq. (4) reveals that the velocity profile is influenced by the Knudsen number (Kn) which is a ratio of the molecular mean free path (λ) to the tube radius (r_1) and by the dimensionless wall velocity slip coefficient (β_v) whose numerical value is taken as 1 (Barron *et al.*, 1997) since the momentum accommodation coefficient (F_v) is typically close to the unity. Also worth mentioning is the fact that the standard laminar profile is recovered for the limiting case of Kn = 0. Figure 1 depicts the influence of the Knudsen number in the velocity profile distribution and a marked departure from the standard laminar flow behavior is observed as the effect of the slip flow becomes more pronounced.

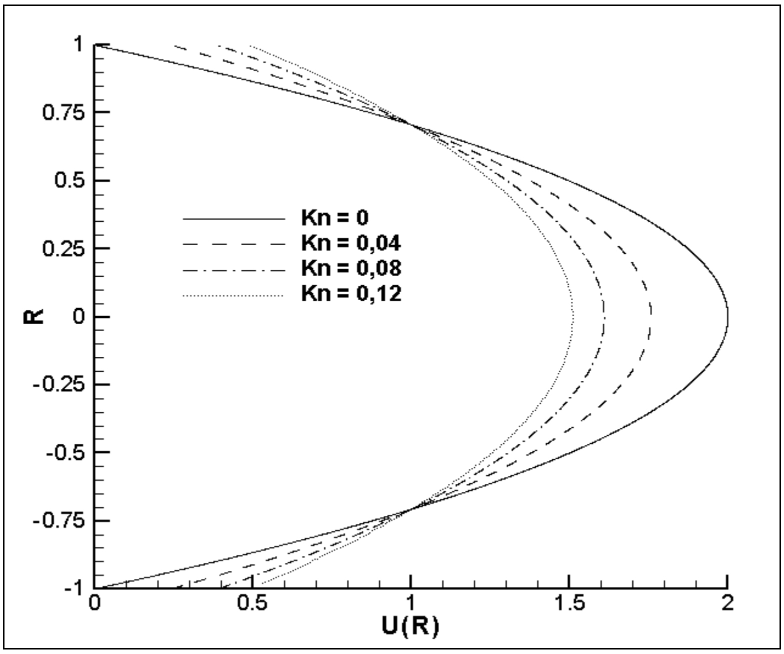

Figure 1. Slip flow velocity profile

Our next step is to determine the set of equations that govern the temperature distribution in both the inner fluid and tube wall as illustrated in Fig. 2. Here, we closely follow the assumptions discussed in Guedes *et al.* (1989) and consequently steady state thermally developing flow with negligible viscous dissipation and axial diffusion effects in the fluid flow are assumed. Moreover, the outer wall is supposed to exchange heat by convection with an external environment with known temperature T_∞ and heat transfer coefficient h. A uniform fluid inlet temperature T_e is also considered. Accordingly, the temperature distributions for the fluid (T_f) and wall (T_s) regions are given by:

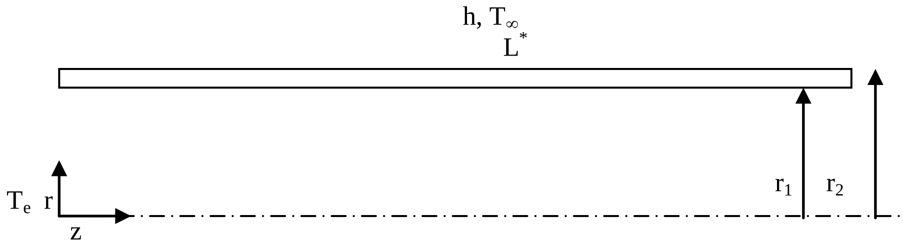

Figure 2. Schematic and coordinates system

Solid region:

$$\frac{\partial^2 T_s(r,z)}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_s(r,z)}{\partial r}\right]=0 \qquad r_1<r<r_2\ ,\ 0<z<L^* \tag{5}$$

$$\frac{\partial T_s(r,0)}{\partial z}=0 \qquad r_1\le r\le r_2 \tag{6}$$

$$\frac{\partial T_s(r,L^*)}{\partial z}=0 \qquad r_1\le r\le r_2 \tag{7}$$

$$k_s\frac{\partial T_s(r,z)}{\partial r}+h(T_s-T_\infty)=0 \qquad r=r_2\ ,\ 0<z<L^* \tag{8}$$

Fluid region:

$$u(r)\frac{\partial T_f(r,z)}{\partial z}=\frac{\alpha_f}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_f(r,z)}{\partial r}\right] \qquad 0<r<r_1\ ,\ z>0 \tag{9}$$

$$T_f(r,0)=T_e \qquad 0\le r\le r_1 \tag{10}$$

$$\frac{\partial T_f(0,z)}{\partial r}=0 \qquad z>0 \tag{11}$$

As for the solid-fluid interface, the usual matching of both the temperature and heat flux is imposed and consequently we have:

$$T_f(r,z)=T_s(r,z) \qquad 0<z<L^* \tag{12}$$

$$k_f\frac{\partial T_f(r,z)}{\partial r}=k_s\frac{\partial T_s(r,z)}{\partial r} \qquad 0<z<L^* \tag{13}$$

Even though strategies for the mathematical solutions of Eqs. (5) to (13) have been devised over the years, a further assumption of a “thermally thin” wall channel is now considered. The basic idea is to admit that in most cases of practical interest, the wall temperature gradients along the radial direction are not as pronounced when compared to those in the “z” coordinate. In other words, the wall effect can be taken into account by simply performing a lumped analysis in the solid region. As discussed in previous contributions (Guedes and Cotta, 1991), the dimensionless version of this problem becomes:

$$W(R)\frac{\partial\theta_f(R,Z)}{\partial Z}=\frac{\partial}{\partial R}\left\{R\frac{\partial\theta_f(R,Z)}{\partial R}\right\} \qquad 0<R<1\ ,\ Z>0 \tag{14}$$

$$\theta_f(R,0)=1 \qquad 0\le R\le 1 \tag{15}$$

$$\frac{\partial\theta_f(0,Z)}{\partial R}=0 \qquad Z>0 \tag{16}$$

$$\frac{\partial\theta_f(1,Z)}{\partial R}+Bi\theta_f(1,Z)=\beta\frac{d^2\theta_w(1,Z)}{dZ^2} \tag{17}$$

where the dimensionless variables are defined in their usual way such as:

$$R=\frac{r}{r_1}\ ,\ Z=\frac{\alpha_f z}{u_m D_h^2}\ ,\ \delta=\frac{r_2}{r_1}\ ,\ L=\frac{\alpha_f L^*}{u_m D_h^2}\ ,\ \hat{k}=\frac{k_f}{k_s}\ ,\ Pe=\frac{D_h u_m}{\alpha_f}$$

$$\theta_f(R,Z)=\frac{T_f(R,Z)-T_\infty}{T_e-T_\infty}\ ,\ U(R)=\frac{u(r)}{u_m}\ ,\ W(R)=\left\{\frac{r_1}{D_h}\right\}^2 U(R)R$$

$$Bi^*=\frac{hr_1}{k_s}\ ,\ Bi=\frac{\delta Bi^*}{\hat{k}}\ ,\ \beta=\frac{\delta^2-1}{8Pe^2\hat{k}}$$

An inspection of relations (14)–(17) illustrates the convenience of the thermally thin wall since the problem is only depended of the modified Biot number Bi and of the wall conjugation parameter β . Also as $\beta\to 0$, the classical Graetz formulation is easily recovered.

Once the mathematical description of the problem has been determined, the dimensionless fluid temperature distribution is expressed in terms of the following eigenfuction expansion:

$$\theta_f(R,Z)=\sum_{i=1}^{\infty}A_i(Z)\psi_i(R) \tag{18}$$

where the eigenfunctions $\psi_i(R)$ are determined from the solution of the regular Sturm-Liouville problem:

$$\frac{d}{dR}\left[R\frac{d\psi(\mu_i,R)}{dR}\right]+\mu_1^2W(R)\psi(\mu_i,R)=0\quad 0<R<1 \tag{19}$$

$$\left.\frac{d\psi(\mu_i,R)}{dR}\right|_{R=0}=0 \tag{20}$$

$$\left.\frac{d\psi(\mu_i,R)}{dR}\right|_{R=1}+Bi\,\psi(\mu_i,1)=0 \tag{21}$$

By utilizing the orthogonality property, the following integral-transform pair is readily identified as:

$$\begin{cases}\overline{\theta_i}(Z)=\dfrac{1}{N_i^{1/2}}\displaystyle\int_0^1 W(R)\psi_i(R)\theta_f(R,Z)dR\\ \theta_f(R,Z)=\displaystyle\sum_{i=1}^{\infty}\frac{1}{N_i^{1/2}}\psi_i(R)\overline{\theta_i}(Z)\end{cases} \tag{22}$$

where the norm N_i associated to the eigenvalue μ_i is defined as:

$$N_i=\int_0^1 W(R)\psi_i^2(R)dR \tag{23}$$

Now we are in a position to perform the integral transformation of the original problem formulation described in Eqs. (14) – (17). Basically, this is accomplished by operating on Eq. (14) with $\int_0^1\frac{\psi_i(R)}{N_i^{1/2}}dR$ and Eq. (19) with $\int_0^1\frac{\theta_f(R,Z)}{N_i^{1/2}}dR$. These results are summed up and the symmetry condition together with the energy equation for the wall channel is invoked. This process yields the following set of equations that govern the transformed fluid temperature field coupled with the temperature of the wall channel:

$$\frac{d\overline{\theta}_i(Z)}{dZ}+\mu_i^2\overline{\theta}_i(Z)=\beta\frac{\psi_i(1)}{N_i^{1/2}}\frac{d^2\theta_W}{dZ^2} \tag{24}$$

$$Bi\,\theta_w(Z)+\left.\frac{\partial\theta_f}{\partial R}\right|_{R=1}=\beta\frac{d^2\theta_w}{dZ^2} \tag{25}$$

The last issue to be addressed is how to express the wall heat flux in terms of the transformed temperature field. As the chosen eigenproblem does not fully incorporate all of the information at $R = 1$, a direct application of the inverse relation is not advisable as it will not yield accurate results at the vicinity of the wall. Therefore, an alternative procedure is to integrate relation (14) over the cross section to find:

$$\left.\frac{\partial\theta}{\partial R}\right|_{R=1}=\sum_{i=1}^{\infty}\overline{f}_i\frac{d\overline{\theta}_i}{dZ} \tag{26}$$

where:

$$\overline{f}_i=\frac{1}{N_i^{1/2}}\int_0^1 W(R)\psi_i(R)dR=\frac{1}{N_i^{1/2}}\frac{Bi\,\psi_1(1)}{\mu_i^2} \tag{27}$$

The above relations can be written in terms of a system of first order differential equations of the form:

$$\begin{cases}\sum\limits_{j=1}^{\infty}\left(\delta_{ij}-\dfrac{\psi_i(1)}{N_i^{1/2}}\overline{f}_j\right)\dfrac{d\overline{\theta}_j}{dZ}=-\mu_i^2\overline{\theta}_i+\dfrac{Bi\,\psi_i(1)}{N_i^{1/2}}\theta_W(Z)\\ \dfrac{d\theta_W(Z)}{dZ}=g_W(Z)\\ \dfrac{dg_W(Z)}{dZ}=\dfrac{Bi}{\beta}\theta_W(Z)+\dfrac{1}{\beta}\sum\limits_{i=1}^{\infty}\overline{f}_i\dfrac{d\overline{\theta}_i}{dZ}\end{cases} \tag{28}$$

and the necessary inlet and boundary conditions are taken as:

$$\overline{\theta}_i(0)=\overline{f}_i\ ,\ \left.\frac{d\theta_W}{dZ}\right|_{Z=0}=0\ ,\ \left.\frac{d\theta_W}{dZ}\right|_{Z=L}=0 \tag{29}$$

By truncating the expansions to a sufficiently large number N, the system can be put in the following matrix form:

$$[A]\{Y'\}=[B]\{Y\} \tag{30}$$

where the solution vector is given by:

$$\{Y\}^T=\{\overline{\theta}_1,\overline{\theta}_2,\overline{\theta}_3,\ldots\overline{\theta}_N,\overline{\theta}_w,g_w\} \tag{31}$$

It is interesting to notice that the above system of ordinary differential equations allows for a fully analytical solution through the following steps. Initially, the inverse of matrix A is determined and therefore system (30) is rewritten as:

$$\{Y'\}=[C]\{Y\},\quad [C]=[A]^{-1}[B] \tag{32}$$

Relation (32) is now solved with the aid of the eigenvalues λ and related eigenvalues ξ of matrix C and therefore, the solution vector is determined as:

$$\{Y(Z)\} = \sum_{i=1}^{N+2} D_i \{\xi^i\} e^{\lambda_i Z} \tag{33}$$

Finally the constants D_i are determined by constraining the solution vector to the relations expressed in Eq. (29), which yields the following $N+2$ algebraic system:

$$\begin{cases} \sum_{i=1}^{N+2} D_i \xi_j^{(i)} = \overline{f}_j ,\ j = 1,\ldots,N \\ \sum_{i=1}^{N+2} D_i \lambda_i \xi_{N+1}^{(i)} = 0 \\ \sum_{i=1}^{N+2} D_i \lambda_i \xi_{N+1}^{(i)} e^{\lambda_i L} = 0 \end{cases} \tag{34}$$

Once the transformed temperature field is obtained, quantities of practical interest can be evaluated such as:

Average fluid temperature distribution:

$$\theta_m(Z) = \frac{\int_0^1 W(R)\theta_f(R,Z)dR}{\int_0^1 W(R)dR} = 8\sum_{i=1}^{N} \overline{f}_i \overline{\theta}_i(Z) \tag{35}$$

Wall temperature distribution:

$$\theta_w(Z) = Y_{N+1} \tag{36}$$

Wall heat flux:

$$\left.\frac{\partial\theta_f(R,Z)}{\partial R}\right|_{R=1} = \sum_{i=1}^{N} \overline{f}_i \frac{d\overline{\theta}_i}{dZ} \tag{37}$$

Local Nusselt number:

$$Nu(Z) = \frac{2\left.\frac{\partial\theta_f(R,Z)}{\partial R}\right|_{R=1}}{\theta_w(Z) - \theta_m(Z)} \tag{38}$$

3. RESULTS AND DISCUSSION

Having derived the solution procedure to the thermally developing slip-flow inside a circular tube with wall effects, in this section we explore the relative merits of the proposed scheme together with the influence of both the modified Biot number and the conjugation parameter β in relevant quantities such as those mentioned at the end of the previous section. Initially, it should be mentioned that the mathematical subroutines DLINRG, DEVCRG and DLSACG from the IMSL package (IMSL, 1980) were employed to respectively determine the inverse of matrix [A], eigenvalues and related eigenvectors of matrix [C] and the solution of the linear system described in Eq. 34. On general grounds, it was found that the convergence rate of the proposed eigenfunction expansion is extremely fast and typically a truncation of 40 terms yields graphically converged results. Therefore, all the results discussed in this contribution were obtained with $N= 40$.

Table 1 shows some common arrangements related to typical applications in fluid flow in microchannels, which are taken from Qu et al. (2000), Tso and Mahulikar (2000) and Rohsenow and Hartnett (1973) and are important to the realistic determination of the conjugation parameter β. A quick inspection of Table 1 indicates that the conjugation parameter is either negligible, for the case of water flowing inside a stainless steel microtube or no greater than $2\ X\ 10^{-1}$ for the N_2 / Silicon arrangement.

Table 1. Parameters for the evaluation of the conjugation parameter β

Fluid/Material Fluid to wall thermal conductivity ratio $\hat{k}$	Hydraulic Diameter D_h (μm)	Wall duct Thickness (μm)	Prandtl number Pr	Reynolds Number Re	Conjugation parameter β	Outer to inner tube radius ratio δ
N_2 / Stainless Steel $\hat{k} = 0.002$	700	300	0.7	1000	3,42 x 10^{-4}	1.857
				400	2.14 x 10^{-3}	
				100	3.42 x 10^{-2}	
H_2O / Stainless Steel $\hat{k} = 0.042$	700	300	5.0	15	1.31 x 10^{-3}	1.857
				50	1.20 x 10^{-4}	
				300	3.28 x 10^{-6}	
H_2O / Silicon $\hat{k} = 0.007$	169	400	5.0	100	2.21 x 10^{-3}	5.734
				300	2.46 x 10^{-4}	
	62			100	1.34 x 10^{-2}	13.903
				300	1.49 x 10^{-3}	
				400	8.38 x 10^{-4}	
N_2 / Silicon $\hat{k} = 0.0003$	700	300	0.7	100	1.98 x 10^{-1}	1.857
				400	1.24 x 10^{-2}	

It is also interesting to compare our results with limiting values, which have been previously published in the open literature of the subject. Table 2 displays the asymptotic Nusselt number obtained by Barron et al. (1997) and those of the present contribution for the specific case of negligible conjugation effects ($\beta \to 0$) and a prescribed wall temperature which is obtained in our analysis by setting $Bi \to \infty$. For the whole range of typical Knudsen numbers ($0 < Kn < 0.12$), an excellent agreement is found between the two solution schemes. Also of interest if the fact that the standard value of 3.657 is recovered for the case of $Kn = 0$ which is the classical Graetz problem with a uniform wall temperature. Moreover, as the slip flow effect becomes more relevant, a steady increase in $Nu(Z \to \infty)$ is observed and for the upper bound of $Kn = 0.12$ the limiting Nusselt number is about 22% higher when compared to the parabolic Graetz flow.

Table 2. Asymptotic Nusselt Number: $Nu(Z \to \infty)$ – Prescribed Wall Temperature and Negligible Wall Effects

Kn	$Nu(Z \to \infty)$ Present solution	$Nu(Z \to \infty)$ Barron et al. (1997)
0.00	3.657	3.657
0.02	3.856	3.855
0.04	4.021	4.020
0.06	4.160	4.160
0.08	4.279	4.228
0.10	4.382	4.380
0.12	4.471	4.471

Due to space limitations, we now present and discuss some few results related to the axial distribution of the average fluid temperature together with the wall temperature and Nusselt number for a fixed $Bi = 1.0$ and for limiting values of both the conjugation parameter β and the Knudsen number Kn. A common trait of all these results is the significant deviation from the standard Graetz problem, especially in the early stages of the thermal entrance region, when both slip flow and wall conjugation effects are taken into account. For example, this trend can be perceived upon the examination of Figure 3 which depicts the average fluid temperature along the flow direction in which the solid line refers to the thermal entry of the Graetz problem. The influence of the variation of the Knudsen number alone does not appear to be very considerable as the Kn = 0 and Kn = 0.12 curves are practically coincident. However, when wall conjugation effects are considered for the limiting case of $\beta = 0.1$, the average fluid temperature becomes clearly smaller when compared to the $\beta = 0$ (Graetz problem) situation and a longer thermal entry region is established.

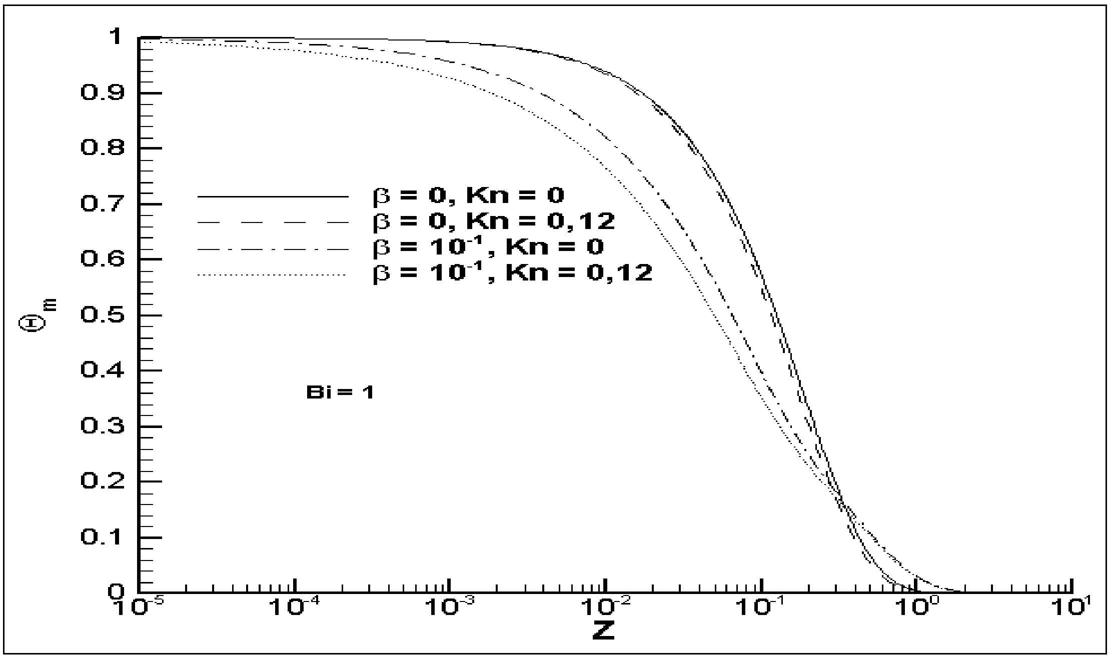

Figure 3. Average fluid temperature distribution as a function of conjugation parameter and Knudsen number

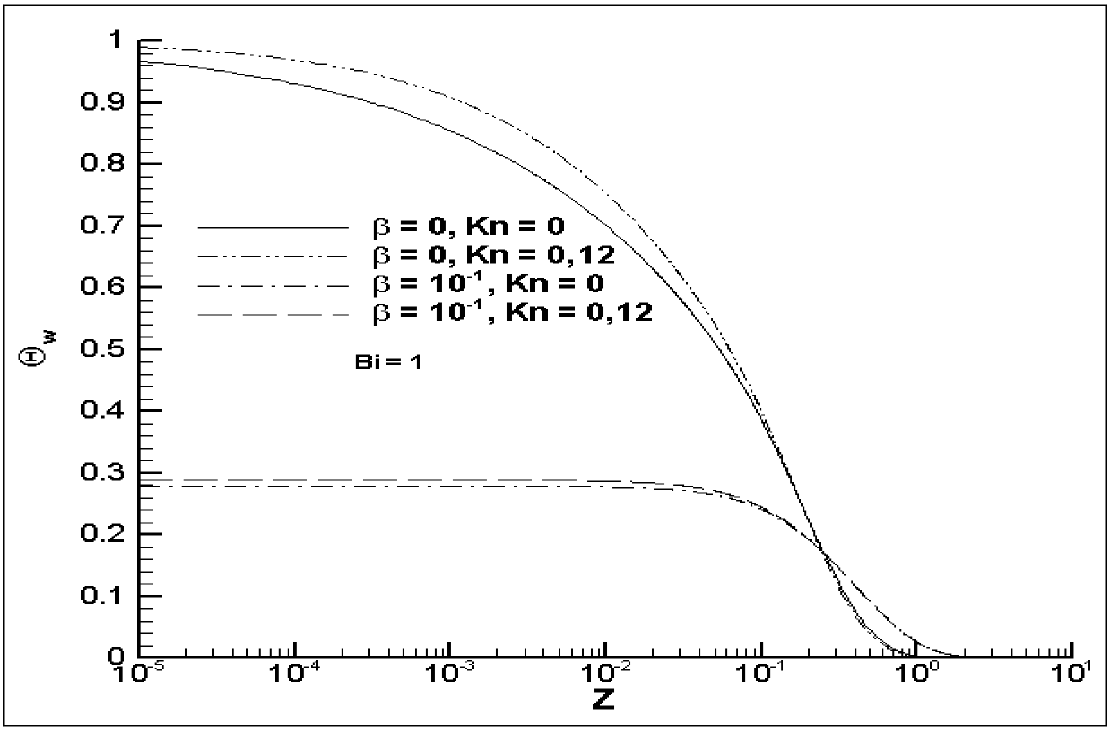

Figure 4. Wall temperature distribution as a function of conjugation parameter and Knudsen number

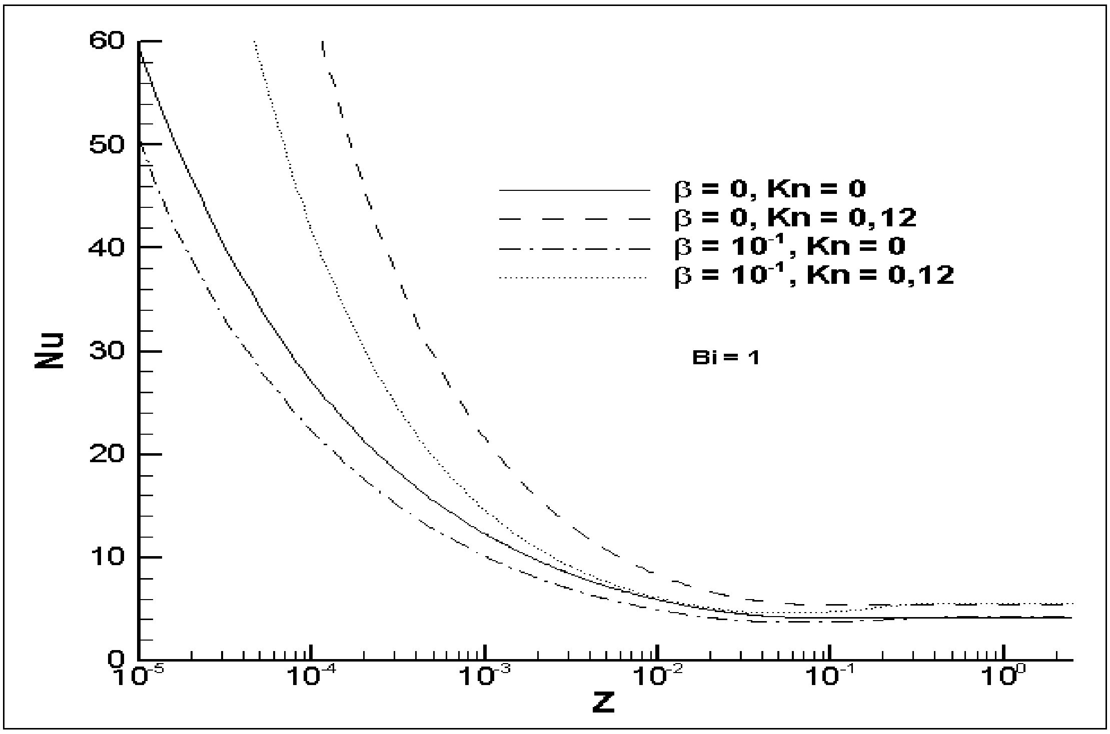

Figure 5. Local Nusselt number distribution as a function of conjugation parameter and Knudsen number

As a matter of fact, the wall effect acts as a barrier delaying the heat transfer between the fluid and the external environment and consequently a longer duct length is needed in order that the average fluid temperature equals to that of the ambient. This effect becomes even more noticeable when the axial distribution of the wall temperature is examined as in Fig. 4. Here, during a long portion of the thermal entry the wall temperature for the $\beta = 0.1$ case is markedly lower than the one for negligible wall heat diffusion. Also perceptible is the fact that the wall temperature becomes slightly higher when slip flow considerations are taken into account. Finally, in Fig. 5 the axial distribution of the local Nusselt number is plotted for the limiting values of the Knudsen number and the conjugation parameter and, once again, considerable deviations from the Graetz solutions are achieved especially at the thermal entry region. From this study it can be observed that ignoring wall effects leads to an overestimate of the local Nusselt number while the inclusion of the slip flow boundary condition enhances the heat exchange to the outer environment leading to higher values of the Nusselt number.

4. CONCLUSION

In summary, this study addresses the thermal entry problem in a circular duct in a slip flow regime including a more generalized boundary condition at the tube wall in order to account for heat diffusion through the solid region. A fully analytical solution scheme is obtained by the Generalized Integral Transform Technique, which allows for the precise and computationally inexpensive evaluation of relevant engineering parameters such as the wall and fluid average temperature fields. It was found that the inclusion of wall heat diffusion effect plays a significant role in this thermal problem by diminishing the wall temperature and the Nusselt number axial distributions due to the “fin effect” caused by the heat conduction in the solid region. Our current research projects in this subject include the evaluation of the viscous dissipation and the temperature jump effects on the overall thermal performance of the microtube.

5. ACKNOWLEDGEMENTS

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