THEORETICAL ANALYSIS OF CONJUGATED HEAT TRANSFER IN MICROCHANNELS WITH A SINGLE DOMAIN FORMULATION AND INTEGRAL TRANSFORMS

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Abstract. *The present work is aimed at developing and validating a methodology for the analytical treatment of conjugated conduction-convection heat transfer in laminar flow within a microchannel. A single domain formulation is proposed for modeling the heat transfer phenomena at both the fluid stream and the microchannel wall regions. By making use of coefficients represented with abrupt variations at the interface fluid- wall, the mathematical model is fed with approximate information concerning the transition of the two domains, unifying the model into a single domain formulation with space variable coefficients. The Generalized Integral Transform Technique (GITT) is then employed in the solution of the resulting convection-diffusion problem with space variable coefficients, employing even fairly simple eigenfunction expansion basis to construct the temperature field analytical representation. A test problem is chosen that still offers an exact solution for validation purposes, based on the extended Graetz problem including transversal conduction only across the channel walls. The excellent agreement between the approximate and exact solutions demonstrates the feasibility of the approach herein presented in handling more involved conjugated heat transfer problems at the micro-scale.*

Keywords*: Conjugated heat transfer, single domain formulation, microchannel, integral transforms, Graetz problem*

1. INTRODUCTION

The miniaturization of mechanical equipment such as heat exchangers is a subject of major interest in recent years, in light of the increasing demand on high performance devices and processes with high thermal efficiency, on sensors with rapid and accurate response, and on the ever growing need of heat dissipation in electronic devices, which tend to be further conceived in even smaller dimensions and with more powerful data processing capacity.

For the conception and design optimization of such equipments, it is of crucial importance to employ reliable mathematical models and solution methodologies capable of describing the physical phenomena that take place in such micro-systems. However, recent contributions have shown significant discrepancies between experimental results and macro-scale correlations and simulations (Morini, 2004; Yener et al., 2005) which may be the result of neglecting terms that are usually not important at the macro-scale, but whose effects may have significant importance in micro-scale heat transfer. In order to achieve simulated results with better agreement against experimental data, a lot of effort is being dispended for the proposition of models and solution methodologies to deal with fluid flow and heat transfer in microchannels, such as the consideration of slip flow in opposition to the classical no-slip condition, the inclusion of terms related to the viscous dissipation and axial diffusion which are often neglected in macro-scale problems (Yu & Ameel, 2001; Tunc & Bayazitoglu, 2001; Tunc & Bayazitoglu, 2002; Mikhailov & Cotta, 2005; Cotta et al., 2005; Castellões & Cotta, 2006; Castellões et al., 2007), besides the investigation of corrugated walls effects in heat transfer enhancement (Castellões & Cotta, 2008; Castellões et al., 2010). Recently, Nunes et al. (2010), motivated by the theoretical conclusions reached by Maranzana et al. (2004), presented some experimental and theoretical results showing the importance of taking into account the heat conduction within the microchannel wall, leading to a conjugated problem which solution yields results in much better agreement with the available experimental data. The theoretical approach then employed was an extension of the work of (Guedes et al., 1991), based on the Generalized Integral Transform Technique (GITT), a well known hybrid numerical-analytical technique for the solution of convection-diffusion problems (Cotta, 1990; Cotta, 1993; Cotta, 1994; Cotta & Mikhailov, 1997; Cotta, 1998; Cotta & Mikhailov, 2006), and accounting for the longitudinal heat conduction along the asymmetric walls.

The present work is thus aimed at progressing into the analysis of conjugated heat transfer in microchannels, developing and validating a methodology for the approximate treatment of the conjugated problem reformulated into a single domain model. Thus, inspired by the well succeeded approach developed by (Naveira Cotta et al., 2009) for the solution of heat conduction problems in heterogeneous media, we propose in this paper the reformulation of conjugated problems as a single region model that accounts for the heat transfer at both the fluid and the microchannel wall regions. By making use of coefficients represented with an abrupt variation at the interface fluid- wall, the mathematical model is fed with the information concerning the two original domains of the problem.

For the solution of the proposed mathematical model we again make use of the Generalized Integral Transform Technique (GITT) (Cotta, 1990; Cotta, 1993; Cotta, 1994; Cotta & Mikhailov, 1997; Cotta, 1998; Cotta & Mikhailov, 2006). This approach is based on extending the classical integral transform method (Mikhailov & Ozisik, 1984) making it sufficiently flexible to handle problems that are not a priori transformable, such as in the case of problems with arbitrarily space-dependent and nonlinear coefficients in either the equation or the boundary conditions. In order to validate the solution of the approximate formulation here proposed, a test problem was chosen based on an extended Graetz problem with transversal conduction across the wall, that provides an exact solution for the conjugated problem achieved with the Classical Integral Transform Technique (CITT) (Mikhailov & Ozisik, 1984), which is then used as a benchmark result.

2. PROBLEM FORMULATION AND SOLUTION METHODOLOGY

The considered problem involves a laminar incompressible internal flow of a Newtonian fluid between parallel plates, in steady-state, and undergoing convective heat transfer due to a prescribed temperature, T_w, at the external face of the channel wall. The microchannel wall is considered to participate on the heat transfer problem through transversal heat conduction, neglecting the longitudinal component of the heat flux within the solid. The fluid flows with a known fully developed velocity profile $u(y)$, and with an inlet temperature T_{in}. Fig. 1 depicts a schematic representation of the application that motivated the present study.

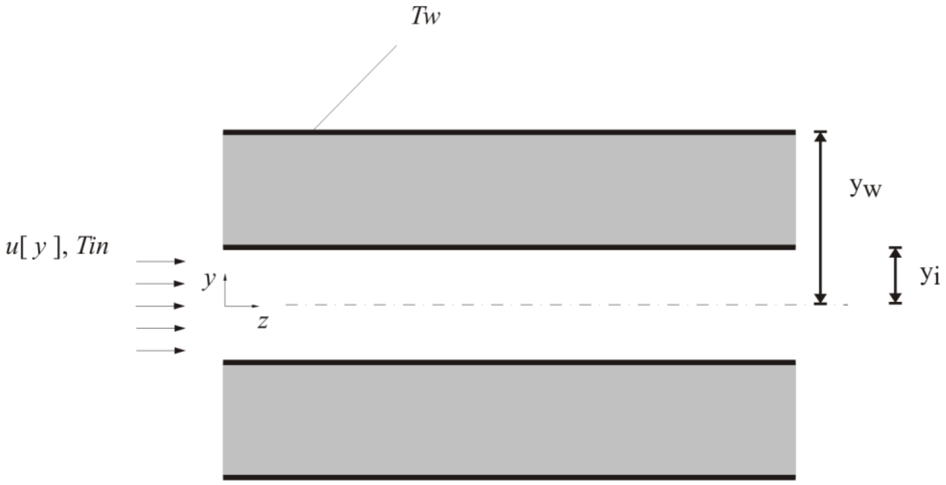


Figure 1. Schematic representation of the conjugated heat transfer problem in a microchannel.

2.1. Approximate solution

We assume that the fluid is thermally developing and neglect axial heat conduction. Then, the formulation of the conjugated problem as a single region model that accounts for the heat transfer phenomena at both the fluid flow and the microchannel solid wall, is achieved by making use of coefficients represented as functions with an abrupt variation at the interface fluid-solid wall. The problem to be here solved is given in the following formulation with space variable coefficients:

$$w(y)\frac{\partial T(y,z)}{\partial z} = \frac{\partial}{\partial y}\left(k(y)\frac{\partial T}{\partial y}\right), \quad 0 < y < y_w, \quad z > 0 \tag{1a}$$

$$T(y, z=0) = T_{in} \tag{1b}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0, \quad T(y = y_w,\, z) = T_w \tag{1c,d}$$

where

$$w(y) = \begin{cases} u(y)\rho c_p, & \text{if } 0 < y < y_i \\ 0, & \text{if } y_i < y < y_w \end{cases} \tag{1e}$$

$$k(y) = \begin{cases} k_f, & \text{if } 0 < y < y_i \\ k_s, & \text{if } y_i < y < y_w \end{cases} \tag{1f}$$

and ρ and c_p are the density and the specific heat of the fluid, respectively, k_s is the thermal conductivity of the microchannel wall, k_f is the thermal conductivity of the fluid, and $u(y)$ is the known parabolic velocity profile of the fully developed flow.

To improve the computational performance of the formal solution derived below, it is recommended to reduce the importance of the boundary source terms, so as to enhance the eigenfunction expansions convergence behavior (Cotta & Mikhailov, 1997). One possible approach for achieving this goal is the proposition of analytical filtering solutions, and in this work the proposed filter is just the temperature at the external wall, as presented in the following expression:

$$T(y,z) = T_w + T^*(y,z) \tag{2}$$

The filtered problem is thus rewritten from Eqs. (1a-d) and (2):

$$w(y)\frac{\partial T^*}{\partial z} = \frac{\partial}{\partial y}\left(k(y)\frac{\partial T^*}{\partial y}\right), \quad 0 < y < y_w, \quad z > 0 \tag{3a}$$

$$T^*(y, z=0) = T_{in} - T_w = T_{in}^{\ *} \tag{3b}$$

$$\left.\frac{\partial T^*}{\partial y}\right|_{y=0} = 0, \quad T^*(y = y_w,\ z) = 0 \tag{3c,d}$$

Following the GITT formalism, the transform/inverse pair is defined as follows:

$$\text{transform}: \quad \bar{T}_n^*(z) = \int_0^{y_w} \tilde{\psi}_n(z) T^*(y,z)dy \tag{4a}$$

$$\text{inverse}: \quad T^*(y,z) = \sum_{n=1}^{\infty} \tilde{\psi}_n(z)\bar{T}_n^*(z) \tag{4b}$$

where

$$\tilde{\psi}_n(y) = \frac{\psi_n(y)}{norm_n}, \text{ normalized eigenfunctions} \tag{4c}$$

$$norm_n = \int_0^{y_w} \psi_n^2(y)dy, \text{ normalization integrals} \tag{4d}$$

where the eigenfunctions $\psi_n(y)$ come from the eigenvalue problem solution, which was here chosen as the simplest possible auxiliary problem to fully demonstrate this flexible solution path:

$$\frac{d^2\psi_n(y)}{dy^2} + \mu_n^{\ 2}\psi_n(y) = 0 \tag{5a}$$

$$\left.\frac{d\psi_n}{dy}\right|_{y=0} = 0 \tag{5b}$$

$$\psi_n(y_i) = 0 \tag{5c}$$

Operating Eq. (3) on with $\int_0^{y_w} \tilde{\psi}_n(y)(\cdot)\,dy$,

$$\begin{aligned}\int_0^{y_w} w(y)\frac{\partial T^*}{\partial z}\tilde{\psi}_n(y)dy &= \int_0^{y_w}\tilde{\psi}_n(y)\frac{\partial}{\partial y}\left(k(y)\frac{\partial T^*}{\partial y}\right)dy = \\ &= \int_0^{y_w}\frac{\partial}{\partial y}\left(\tilde{\psi}_n(y)k(y)\frac{\partial T^*}{\partial y}\right)dy - \int_0^{y_w}\frac{d\tilde{\psi}_n(y)}{dy}k(y)\frac{\partial T^*}{\partial y}dy\end{aligned} \tag{6a}$$

which can be rewritten as:

$$\int_0^{y_w} w(y)\frac{\partial T^*}{\partial z}\tilde{\psi}_n(y)dy = \left.\tilde{\psi}_n(y)k(y)\frac{\partial T^*}{\partial y}\right)_0^{y_w} - \int_0^{y_w}\frac{d\tilde{\psi}_n(y)}{dy}k(y)\frac{\partial T^*}{\partial y}dy \tag{6b}$$

where

$$\tilde{\psi}_n(y)k(y)\frac{\partial T^*}{\partial y}\Bigg)_0^{y_w} = 0 \tag{6c}$$

Thus:

$$\int_0^{y_w} w(y)\frac{\partial T^*}{\partial z}\tilde{\psi}_n(y)dy = -\int_0^{y_w}\frac{d\tilde{\psi}_n(y)}{dy}k(y)\frac{\partial T^*}{\partial y}dy \tag{6d}$$

Using the inverse definition into Eq. (6d), one obtains:

$$\int_0^{y_w} w(y)\left[\sum_{m=1}^{\infty}\frac{d\bar{T}_m^{\,*}}{dz}\tilde{\psi}_m(y)\right]\tilde{\psi}_n(y)dy = -\int_0^{y_w}\frac{d\tilde{\psi}_n(y)}{dy}k(y)\left[\sum_{m=1}^{\infty}\bar{T}_m^{\,*}\frac{d\tilde{\psi}_m(y)}{dy}\right]dy \tag{6e}$$

which can be rewritten as:

$$\sum_{m=1}^{\infty}\frac{d\bar{T}_m^{\,*}}{dz}\int_0^{y_w} w(y)\tilde{\psi}_n(y)\tilde{\psi}_m(y)dy = -\sum_{m=1}^{\infty}\bar{T}_m^{\,*}\int_0^{y_w} k(y)\frac{d\tilde{\psi}_n(y)}{dy}\frac{d\tilde{\psi}_m(y)}{dy}dy \tag{6f}$$

and concisely given in matrix form by:

$$\mathbf{A}\frac{d\underset{\sim}{\bar{T}}^*}{dz} = -\mathbf{B}\,\underset{\sim}{\bar{T}}^* \tag{7a}$$

where: $$A_{nm} = \int_0^{y_w} w(y)\tilde{\psi}_n(y)\tilde{\psi}_m(y)dy, \quad B_{nm} = \int_0^{y_w} k(y)\frac{d\tilde{\psi}_n(y)}{dy}\frac{d\tilde{\psi}_m(y)}{dy}dy \tag{7b,c}$$

The ordinary differential equations (ODE) system (7) can be analytically solved to provide results for the transformed temperatures, upon truncation to a sufficiently large finite order *N*, in terms of the matrix exponential function:

$$\underset{\sim}{\bar{T}}^*(z) = \exp\left(-\mathbf{A}^{-1}\mathbf{B}\,z\right)\underset{\sim}{\bar{T}}^*(0) \tag{8a}$$

where $\underset{\sim}{\bar{T}}^*(0)$ are the transformed initial conditions and are given by:

$$\bar{T}_n^*(0) = \int_0^{y_w}\tilde{\psi}_n(y)T_{\text{in}}^*dy \tag{8b}$$

Once the transformed potentials $\bar{T}_n^*(z)$, with n = 1,2,...,N, have been computed, the inversion formula can be recalled to yield the temperature field $T^*(y,z)$ representation at any desired position y and z. The original temperature field $T(y,z)$ can then be obtained by:

$$T(y,z) = T_w + T^*(y,z) = T_w + \sum_{n=1}^{N}\bar{T}_n^*(z)\tilde{\psi}_n(y) \tag{9}$$

It is always the main interest in convective heat transfer analysis to determine the local heat transfer coefficient, $h(z)$, in general expressed in dimensionless form as the Nusselt number, $Nu(z)$, here computed from both the approximate and the exact solutions. The following expressions for the local Nusselt number and for the bulk temperature, $T_{av}(z)$, are then needed:

$$h(z)=\frac{k_f \left.\frac{\partial T}{\partial y}\right|_{y=y_i}}{T(y_i,z)-T_{av}(z)};\quad Nu(z)=\frac{h(z)D_h}{k_f},\qquad T_{av}(z)=\frac{\int_0^{y_i} u(y)T(y,z)dy}{\int_0^{y_i} u(y)dy} \tag{10a-c}$$

In order to avoid the direct evaluation of the derivative $\partial T/\partial y|_{y=y_i}$ when using the approximate solution, an integral balance alternative expansion is obtained, based on integration of the energy equation (Cotta & Mikhailov, 1997):

$$\begin{aligned} &\int_0^{y_i} u(y)\rho c_p \frac{\partial T}{\partial z}dy=\int_0^{y_i} k_f \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)dy \\ &or,\ \left.\frac{\partial T}{\partial y}\right|_{y=yi}=\frac{1}{k_f}\int_0^{y_i} u(y)\rho c_p \frac{\partial T}{\partial z}dy \end{aligned} \tag{11a,b}$$

The convergence of the approximate solution for the temperature values in the vicinity of the interface can similarly be enhanced, by implementing the integral balance approach also for computing the temperature distribution, via an additional integration of the energy equation along the transversal direction. In this case, the fluid temperature distribution is computed from the following alternative expression:

$$T(y,z)=T(0,z)+\frac{\rho c_p}{k_f}\int_0^{y}\int_0^{y''} u(y')\frac{\partial T(y',z)}{\partial z}dy'dy'' \tag{11c}$$

where the centerline temperature $T(0,z)$ is calculated with the normal inversion formula, Eq. (9).

2.2. Exact solution

For the exact solution of this problem, the heat transfer problem is then modeled as a conduction problem for the solid wall, coupled at the interface $y=y_i$ with the internal convective problem for the fluid, as represented by the following equations for the two domains:

Solid energy equation:

$$0=k_s\frac{\partial^2 T_s(y,z)}{\partial y^2},\ \ y_i<y<y_w,\ \ z>0 \tag{12a}$$

$$T_s(y_w,\ z)=T_w \tag{12b}$$

Fluid energy equation:

$$u(y)\rho c_p\frac{\partial T_f(y,z)}{\partial z}=k_f\frac{\partial^2 T_f}{\partial y^2},\ \ 0<y<y_i,\ \ z>0 \tag{12c}$$

$$T_f(y,z=0)=T_{in} \tag{12d}$$

$$\left.\frac{\partial T_f}{\partial y}\right|_{y=0}=0 \tag{12e}$$

Interface conditions:

$$\begin{cases} k_f\left.\frac{\partial T_f}{\partial y}\right|_{y=y_i}=k_s\left.\frac{\partial T_s}{\partial y}\right|_{y=y_i} \\ T_f(y_i,z)=T_s(y_i,z) \end{cases} \tag{12f,g}$$

For the exact solution of the proposed problem, eqs.(12), we first consider Eq. (12a) and the boundary conditions given by Eqs. (12b) and (12g), which yields the following expression for the solid wall temperature distribution:

$$T_s(y,z) = T_w - \frac{T_w - T_f(y_i,z)}{y_w - y_i} y_w + \frac{T_w - T_f(y_i,z)}{y_w - y_i} y \tag{13}$$

and the boundary condition given by Eq. (12f) can then be rewritten as:

$$k_f \left. \frac{\partial T_f}{\partial y} \right|_{y=y_i} + \frac{k_s}{y_w - y_i} T_f(y_i,z) = k_s \frac{T_w}{y_w - y_i} \tag{14}$$

Thus, the problem for the fluid flow region becomes a Graetz type problem with third kind boundary condition:

$$u(y)\rho c_p \frac{\partial T_f(y,z)}{\partial z} = k_f \frac{\partial^2 T_f}{\partial y^2}, \quad 0 < y < y_i, \quad z > 0 \tag{15a}$$

$$T_f(y, z=0) = T_{\text{in}} \tag{15b}$$

$$\left. \frac{\partial T_f}{\partial y} \right|_{y=0} = 0, \qquad k_f \left. \frac{\partial T_f}{\partial y} \right|_{y=y_i} + \frac{k_s}{y_w - y_i} T_f(y_i,z) = \frac{k_s}{y_w - y_i} T_w \tag{15c,d}$$

Problem (15) has an exact analytical solution readily obtainable by the Classical Integral Transform Technique (Mikhailov & Ozisik, 1984; Cotta, 1993) and then the microchannel wall region temperature distribution, $T_s(y,z)$, can be directly obtained from eq. (13). The exact solution for the fluid flow region, $T_f(y,z)$, will be used later on as a benchmark solution for the validation of the conjugated problem approximate formulation described in the previous section. The exact solution for the fluid flow region is obtained from the solution of the following eigenvalue problem, formulated by directly applying separation of variables to the homogeneous version of problem (15):

$$k_f \frac{d^2\phi(y)}{dy^2} + w_f \lambda^2 \phi(y) = 0 \tag{16a}$$

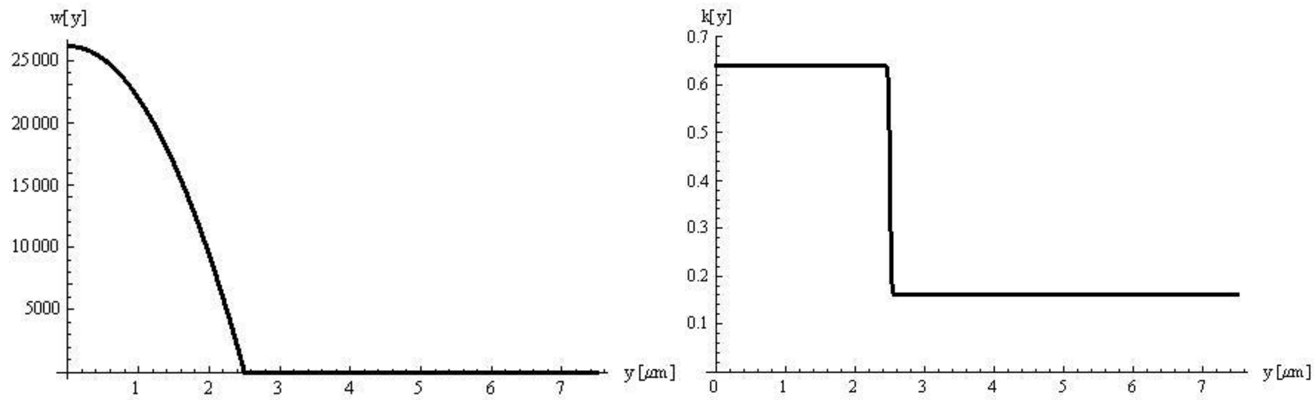
$$\left. \frac{d\phi}{dy} \right|_{y=0} = 0, \ \left. \frac{d\phi}{dy} \right|_{y=y_i} + \frac{k_s}{y_w - y_i} \phi(y_i) = 0 \tag{16b,c}$$

which allows for an analytical solution in terms of hypergeometric functions that can be readily obtained using the routine *DSolve* of the *Mathematica* platform (Mikhailov & Cotta, 2005).

3. RESULTS AND DISCUSSION

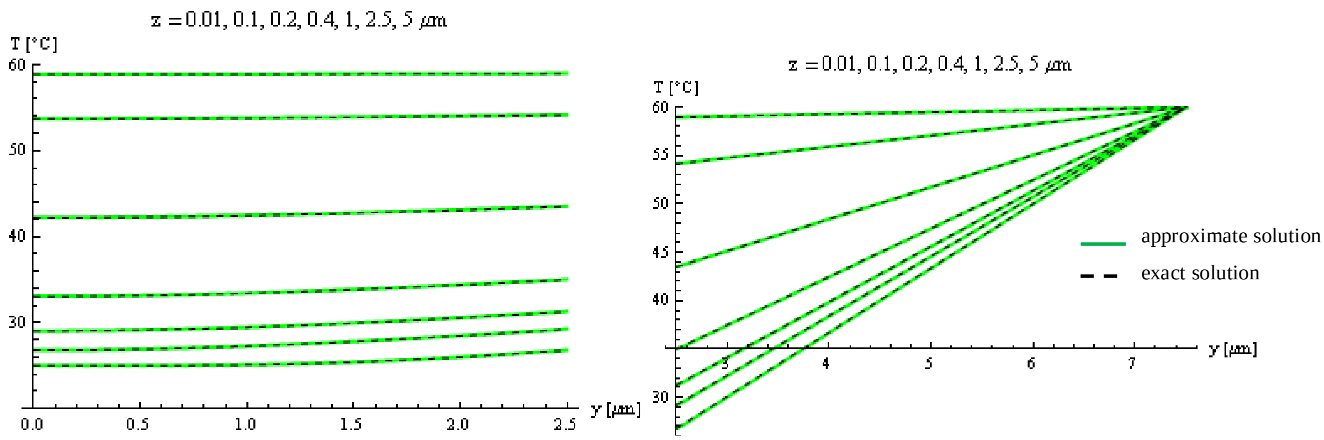
The present work was motivated by an application of water flow with constant mass flow rate, $\dot{m} = 0.25$ mg / min, with a prescribed temperature, $T_w = 60°C$ at the microchannel external wall and with an inlet temperature T_{in} =25°C. The microchannel wall has a $5\mu m$ thickness and is made with polyester resin, and the two parallel plates are kept apart by $5\mu m$. As for the thermophysical properties, we have taken the density and the specific heat of water, respectively, as $989.3\ \text{kg/m}^3$ and $4181\ \text{J/kg°C}$, the thermal conductivity of the microchannel wall as $0.16\ \text{W/m°C}$, the thermal conductivity of water as $0.6396\ \text{W/m°C}$. Figs. 2.a,b below illustrate the behavior of the space variable coefficients that are feeding the single region model in eq. (1.a), $w(y)$ and $k(y)$, as continuous functions with an abrupt variation at the fluid-solid interface.

The conjugated problem presented in this work has been solved using the approximate single region formulation described in Section 2.1 and compared to the exact solution of Section 2.2. Figures 3a and 3b show the temperature transversal profiles for a few different longitudinal positions along the flow, z = 0.01, 0.1, 0.2, 0.4, 1.0, 2.5 and 5.0 μm, for the fluid and the microchannel wall regions, respectively. In these results it can be observed an excellent agreement between the approximate and exact solutions, which are essentially coincident to the graph scale. In Fig. 4 it can be seen the temperature evolution at the centerline of the microchannel (y = 0) for z = 0 up to 5 μm, and it also shows the Graetz problem solution with first kind boundary condition, which is a simplification of this problem when the wall thermal resistance is neglected. It can be concluded that the thermal resistance of the polymeric wall noticeably delays the increase of the fluid temperature along the flow.



(a) (b)

Figure 2. Representation of the space variable coefficients as continuous functions with an abrupt variation at the interface fluid-solid wall, (a)Product velocity and heat capacity, $w(y)$; (b) Thermal conductivity, $k(y)$



(a) (b)

Figure 3. Temperature profiles calculated using the approximate methodology in comparison with the exact solution (a) at the fluid flow region and (b) at the microchannel wall

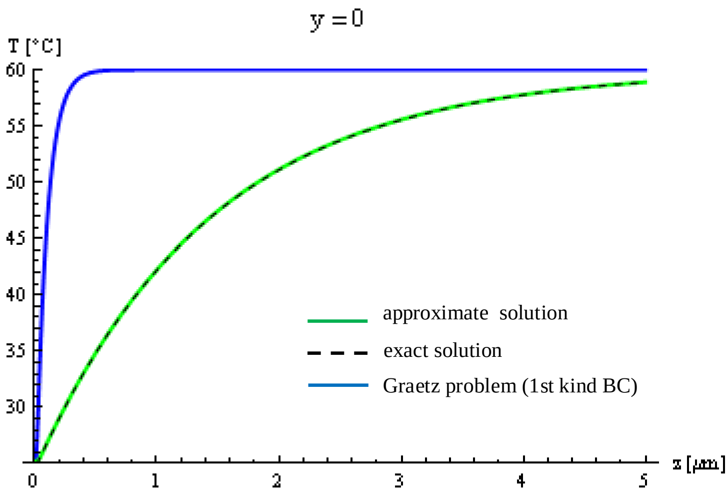


Figure 4. Comparison of the temperature evolution along the centerline of the microchannel (y = 0) for z = 0 up to 5 μm

Tables 1a,b illustrate the convergence behavior of the temperature profile for the approximate solution, respectively at z = 0.1μm and z = 0.5μm, for different positions in the fluid flow region. The results are apparently fully converged to at least 3 digits for N = 50 in all selected positions. The exact solution results are fully converged to all five digits shown, which are achieved to within only five terms in the eigenfunction expansion.

Table 1a. Convergence behavior of the temperature profile for the approximate solution at z = 0.1 μm for fluid flow region

order	y = 1μm	y = 2μm	y = 2.5μm
N = 10	27.257	28.366	29.527
N = 20	27.276	28.508	29.375
N = 30	27.269	28.481	29.313
N = 40	27.268	28.475	29.285
N = 50	27.270	28.486	29.273
Exact solution	**27.265**	**28.473**	**29.229**

Table 1b. Convergence behavior of the temperature profile for the approximate solution at z = 0.5 μm for fluid flow region

order	y = 1μm	y = 2μm	y = 2.5μm
N = 10	35.199	36.039	36.919
N = 20	35.213	36.147	36.804
N = 30	35.208	36.127	36.756
N = 40	35.208	36.122	36.736
N = 50	35.209	36.130	36.726
Exact solution	**35.185**	**36.101**	**36.674**

Tables 2a-b show more closely the direct comparison between the approximate solution as directly computed from the inversion formula, Eq.9, and the exact solution, by presenting the numerical values and the relative error obtained at several positions across the y axes for z = 0.1 and 1.0 μm, respectively. One may observe that in all selected positions the error of the approximate solution with respect to the exact one was smaller than 0.15%. It is also evident that there is a slight increase in the relative error for the positions closer to the interface.

Table 2a. Comparison between the approximate and exact temperatures at z = 0.1 μm

y [μm]	Approximate sol. [°C]	Exact sol. [°C]	$\frac{T_{conj}(y,z)-T_{ex}(y,z)}{T_{ex}(y,z)}\times 100\%$
0.1	26.818	26.814	0.0165%
0.2	26.833	26.828	0.0173%
0.5	26.931	26.926	0.0191%
0.7	27.041	27.036	0.0172%
1.0	27.270	27.265	0.0185%
1.2	27.463	27.457	0.0240%
1.5	27.804	27.797	0.0263%
2.0	28.486	28.473	0.0451%
2.5	29.273	29.229	0.149%

Table 2b. Comparison between the approximate and exact temperatures at z = 1.0 μm

y [μm]	Approximate sol. [°C]	Exact sol. [°C]	$\frac{T_{conj}(y,z)-T_{ex}(y,z)}{T_{ex}(y,z)}\times 100\%$
0.1	42.240	42.205	0.0823%
0.2	42.247	42.213	0.0825%
0.5	42.300	42.265	0.0828%
0.7	42.359	42.324	0.0818%
1.0	42.482	42.447	0.0816%
1.2	42.585	42.550	0.0829%
1.5	42.768	42.732	0.0826%
2.0	43.133	43.095	0.0873%
2.5	43.554	43.501	0.122%

The improved approximate solution here proposed after application of the integral balance, Eq.11c, is also critically examined. The comparison with the exact solution at z = 0.1 μm is shown in Table 3, where it can be noticed that the relative error at the interface (y = 2.5 μm) dropped from about 0.15% to 0.086%. At the other positions the relative error increased, but not significantly, offering an overall improvement in the accuracy of the temperature distribution.

Table 3. Comparison between the approximate solutions with and without the integral balance formulation and the exact solution for the temperature at z = 0.1 μm

y [μm]	Approx. sol. with integral balance [°C]	Approximate solution [°C]	Exact sol.[°C]	Relative error (approx. sol. with integral balance)	Relative error (approx. sol.)
0.1	26.819	26.818	26.814	0.0187%	0.0165%
0.2	26.833	26.833	26.828	0.0193%	0.0173%
0.5	26.932	26.931	26.925	0.0235%	0.0191%
0.7	27.043	27.041	27.035	0.0280%	0.0172%
1.0	27.274	27.270	27.264	0.0369%	0.0185%
1.2	27.468	27.463	27.456	0.0439%	0.0240%
1.5	27.812	27.804	27.796	0.0550%	0.0263%
2.0	28.494	28.486	28.473	0.0728%	0.0451%
2.5	29.254	29.273	29.229	0.0858%	0.149%

Figure 5 below depicts both the approximate and the exact computations of the local Nusselt number, where it can also be observed an excellent agreement. It is also clear that thermal development occurs within a very short length of the microchannel, with the establishment of a fully developed asymptotic Nusselt number, in light of the very low Reynolds numbers achieved in such applications. Nevertheless, the thermal development is noticeably delayed with respect to the classical Graetz problem with prescribed wall temperature and the Nusselt numbers are larger.

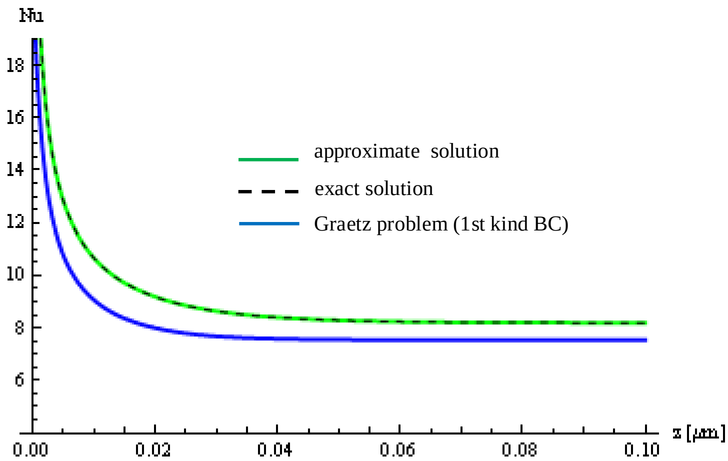


Figure 5. Local Nusselt number calculated from the approximate and the exact solutions and critically compared to the classical Graetz problem with prescribed wall temperature

4. CONCLUSIONS

In this work we have developed and validated a methodology for the approximate treatment of the conjugated heat transfer problem for laminar flow in microchannels, by proposing a single domain formulation for modeling the heat transfer phenomena at both the fluid flow and the microchannel wall regions. By making use of coefficients represented as continuous functions presenting an abrupt variation at the interface fluid-microchannel wall, the mathematical model is fed with the information concerning the two original domains of the problem. A test problem is selected involving an extension to the classical Graetz problem, with wall participation via transversal conduction only, which allows for an exact solution to be obtained for benchmarking purposes. The approximate formulation of the conjugated problem is then tackled with the Generalized Integral Transform Technique (GITT) and an excellent agreement between the two analytical solutions was obtained, demonstrating the feasibility of the approach herein proposed.

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