BAYESIAN ESTIMATION OF THERMAL PROPERTIES OF OVERHEAD ELETRIC POWER CABLES

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Abstract. *In this work we apply a Bayesian approach to the simultaneous estimation of the thermo properties appearing in the heat conduction problem involving overhead power cables. The cables used for the transportation of electric energy are composed by layers of stranded metallic wires, resulting in a heterogeneous media (wires and air voids). The inner core is made of steel, over which three aluminum wires layers are winded to form the conductor. The direct problem is formulated in terms of transient heat conduction, considering two homogeneous cylindrical regions. The heat generation inside the cable is given as a function of current density, electrical resistivity and its effective thermal conductivity. The Markov Chain Monte Carlo (MCMC) method is used in order to estimate the effective radial thermal conductivities and the volumetric heat capacities of both regions (steel core and aluminum conductor), and the thermal emissivity of the conductor surface. Simulated temperature measurements are used in the inverse analysis*

Keywords*: Bayesian techniques, Thermal properties, Power Cables, MCMC method.*

1. INTRODUCTION

In recent years, the Bayesian estimation has been used in different research areas, becoming a useful tool in the estimation of physical parameters and functions. Though the Bayesian theory was proposed many years ago, the technique has been becoming popular with the advent of powerful computers. Such a statistical technique was especially adopted in the heat conduction problem (Nicholas Zabaras et al, 2004; Orlande et al, 2007; Naveira-Cotta, 2009) for which, satisfactory results were obtained even for complex physical models (Orlande, 2010) The estimative of physical parameters have also been applied in practical problems (Paez, 2009) being suitable for the heat conduction problem of power cables used in transmission lines.

Most of the overhead conductors used in transmission lines are mainly composed of two materials, the most common type being ACSR (*Aluminum Conductor Steel Reinforced.*) The inner core is made of steel wires, providing most of the mechanical resistance. Over the core, three (two or one) aluminum wires layers are winded to form the conductor, carrying approximately 98% of the current (Morgan, 1991). Such a configuration results in an heterogeneous cross sectional area composed by wire and air voids. Morgan (1990) analyzed the thermal behavior of overhead conductors, considering the heterogeneous cross sectional area. The author shows experimental results in terms of the effective thermal conductivity of the total conductor area (aluminum conductor an air voids). From those results, Morgan conclude the effective thermal conductivity is approximately a hundred times lower than the material conductor.

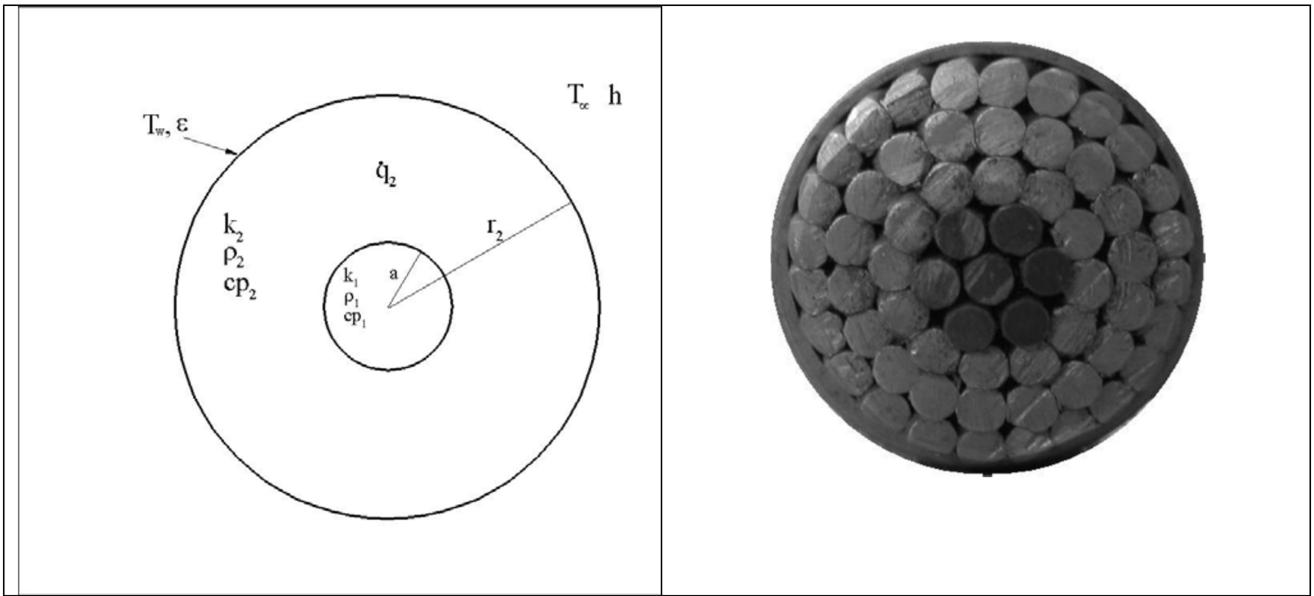
In the transport of electric energy, the analysis of the thermal response of overhead power cables is crucial in the line transmission project. The capability of the energy transport is closely dependent on its' temperature distribution. Also, economical aspects as Joule losses (electric energy converted to heat energy) and life time of the cables are related to the temperature achieved during its operation. Therefore, the knowledge of thermal parameters of high voltage conductors is important, in order to preview accurately the thermal behavior of power cables under several work conditions. The estimation of these physical parameters through a Bayesian approach, allow us to take into consideration the uncertainly of the measurements and of the unknown parameters.

A Bayesian estimator (J. Kaipio, 2005) is basically concerned with the analysis of the posterior probability density, which is the conditional probability of the parameters given the measurements, while the likelihood is the conditional probability of the measurements given the parameters. If we assume the parameters and the measurement errors to be independent Gaussian random variables, with known means and covariance matrices, and that the measurement errors are additive, a closed form expression can be derived for the posterior probability density. In this case, the estimator that maximizes the posterior probability density can be recast in the form of a minimization problem involving the maximum a posteriori objective function. On the other hand, if different prior probability densities are assumed for the parameters, the Posterior Probability Distribution does not allow an analytical treatment. In this case, Markov Chain Monte Carlo (MCMC) methods are used to draw samples of all possible parameters, so that inference on the posterior probability becomes inference on the samples.

In this paper constant thermal parameters are estimated by employing a Markov Chain Monte Carlo (MCMC) method through the implementation of the Metropolis-Hastings algorithm (Gamerman and Lopes 2006, Lee 2004, Migon and Gamerman 1999, Orlande *et al*. 2008). The physical problem of interest consists of the heat conduction problem of overhead power cables used in the transmission of electrical energy. The numerical solution for the direct problem was based on the finite differences method. Simulated experimental data for the temperature distribution inside the cable are used in the inverse analysis, in order to show the capabilities of the proposed approach.

2. DIRECT PROBLEM – HEAT EQUATION

In the statistical approach presented in this work, the direct problem has to be solved in each iteration of the process, resulting in a high computational cost. For this reason, the direct problem is stated with some simplifications. The bimetallic cable is modeled as two concentric cylinders. It is considered that the current flux passes through the aluminum conductor, i.e., the heat generation takes place in the conductor.



a) Simplified model of an ACSR cable | b) Real ACSR cable composed by wires.

Figure 1. Cross sectional area of a ACSR cable.

The general equation, which describes the heat conduction problem in high voltage cables, is written in terms of the transient one dimensional heat equation. The equation for the core region is given:

$$\frac{1}{\alpha_1}\frac{\partial T_1}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right) \quad \text{at} \quad 0 < r < a, \quad \text{and} \quad t > 0, \tag{2.1.a}$$

$$\frac{\partial T_1}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad \text{and} \quad t > 0, \tag{2.1.b}$$

$$-k_1\frac{\partial T_1}{\partial r} = h_c(T_2 - T_1) \quad \text{at} \quad r = a \quad \text{and} \quad t > 0, \tag{2.1.c}$$

$$T_1(r,0) = T_0 \quad \text{at} \quad t = 0 \quad \text{and} \quad 0 \le r \le a. \tag{2.1.d}$$

The equation for the conductor with heat generation is given:

$$\frac{1}{\alpha_2}\frac{\partial T_2}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T_2}{\partial r})+\frac{g(r,t;T_2)}{k_2} \quad \text{at} \quad a<r<r_2, \quad \text{and} \quad t>0. \tag{2.2.a}$$

Considering the convective and thermal radiation losses on the surface conductor, the boundary conditions are given:

$$k_2\frac{\partial T_2}{\partial r}=k_1\frac{\partial T_1}{\partial r} \quad \text{at} \quad r=a \quad \text{and} \quad t>0, \tag{2.2.b}$$

$$-k_2\frac{\partial T_2}{\partial r}=h\ (T_w-T_\infty)+\varepsilon\sigma_{SB}(T_w{}^4-T_\infty{}^4)\approx h_t(T_w-T_\infty) \quad \text{at} \quad r=r_2, \quad \text{and} \quad t>0 \tag{2.2.c}$$

$$T_2(r,0)=T_0 \quad \text{at} \quad t=0, \quad \text{and} \quad a\le r\le r_2. \tag{2.2.d}$$

Where k_1 and k_2 are the thermal conductivities, α_1 and α_2 the thermal diffusivities of the core and the conductor, respectively. The parameter h_c is the contact resistance between both materials, σ_{SB} is the Stefan-Boltzman constant, ε is the thermal emissivity, T_∞ is the surrounding temperature, T_1 and T_2 are the temperatures of the core and the conductor, respectively. The total heat transfer coefficient is defined as $h_t = h + 4\varepsilon\ \sigma_{SB}\ T_\infty^3$, where h is the convective heat transfer coefficient. With the assumption of the uniform distribution current, the heat generation function can be written as:

$$g(r,t;T)=\frac{I^2R(T)}{A_{ef}} \tag{2.3}$$

Where I is the total current and A_{ef} is the effective conductor area. The electrical resistance is defined as $R(T)=R_{ref}\left[1+z(T-T_{ref})\right]$, where R_{ref} is the electrical resistance reference measured at temperature T_{ref}, and z is a correction factor of the aluminum, which is considered constant.

3. INVERSE PROBLEM – BAYESIAN INFERENCE

The inverse problem of interest for this work involves the simultaneous estimation of the thermal parameters showed in the heat equations (2.1.a-d; 2.2.1-d) The parameters to be estimated are the effective thermal conductivities k_1 and k_2, the volumetric thermal capacities C_1 and C_2 and the total heat transfer coefficient h_t, which involves the convective and radiative losses. The thermal properties are considered constants.

The vector notation of the parameters to be estimated is:

$$\mathbf{P}^T \equiv [P_1, P_2, ...P_N] \tag{3.1}$$

The vector of available measurements

$$\mathbf{Y}^T = \left(\vec{Y}_1, \vec{Y}_2, ..., \vec{Y}_I\right) \tag{3.2}$$

Where $\vec{Y}_i$ can be expressed as:

$$\vec{Y}_i = (Y_{i1}, Y_{i2}, ..., Y_{iM}) \quad \text{for i = 1,2,...,I} \tag{3.3}$$

Where *I* is the number of parameters and *M* is the number of measurements

3.1. Sensitivity coefficients

The sensitivity coefficients were explored in order to obtain some information about the parameters to be estimated. The analysis of these coefficients provides important information about the ill-posedness of the problem. Thus, we are capable to identify the magnitudes and the linear dependency between them. The sensitivity coefficients represent a sensitivity measure of the temperature with respect to the parameter variation. Therefore, small magnitudes of the coefficients indicate that high variations of the unknown parameter could produce small temperature variation. For this reason, high magnitudes of the coefficients are desired. The sensitivity coefficients are defined as:

$$\frac{\partial \vec{T}_i^T}{\partial P_j} = \begin{bmatrix} \dfrac{\partial T_{i1}}{\partial P_j} \\ \dfrac{\partial T_{i2}}{\partial P_j} \\ \vdots \\ \dfrac{\partial T_{iM}}{\partial P_j} \end{bmatrix} \quad \text{for } i = 1, 2, \ldots, \mathrm{I} \quad \text{and} \quad j = 1, 2, \ldots, \mathrm{N}. \tag{3.4}$$

Where the subscript I is the number of observations by the sensor, M is the number of sensors and N is the number of unknown parameters.

3.2. MCMC – Metropolis Hastings Algorithm

In the Bayesian approach to statistics, an attempt is made to utilize all available information in order to reduce the amount of uncertainty present in an inferential or decision making problem. As new information is obtained, it is combined with any previous information to form the basis for statistical procedures. The formal mechanism used to combine the new information with the previously available information is known as Bayes' theorem (R. Winkler, 2003). Therefore, the term Bayesian is often used to describe the so-called statistical inversion approach, which is based on the following principles (Migon and Gamerman, 1999):

1. All variables included in the model are modeled as random variables.
2. The randomness describes the degree of information concerning their realizations.
3. The degree of information concerning these values is coded in probability distributions.
4. The solution of the inverse problem is the posterior probability distribution.

Bayes' theorem can then be stated as (J. Kaipio, 2005):

$$\pi_{\text{posterior}}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi(\mathbf{Y}|\mathbf{P})\pi_{\text{prior}}(\mathbf{P})}{\pi(\mathbf{Y})} \tag{3.5}$$

where $\pi_{\text{posterior}}(\mathbf{P})$ is the posterior probability density, that is, the conditional probability of the parameters **P** given the measurements **Y**; $\pi_{\text{prior}}(\mathbf{P})$ is the prior density, that is, the coded information about the parameters prior to the measurements; $\pi(\mathbf{Y}|\mathbf{P})$ is the likelihood function, which expresses the likelihood of different measurement outcomes **Y** with **P** given; and $\pi(\mathbf{Y})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant. In practice, such normalizing constant is difficult to compute and numerical techniques, like Markov Chain Monte Carlo Methods, are required in order to obtain samples that represent accurately the posterior probability density. The numerical method most used to explore the space of states of the posteriori distribution is the Monte Carlo simulation. The Monte Carlo simulation is based on a large sample of the probability density function (in this case, the function of the posterior probability density $\pi(\mathbf{P}|\mathbf{Y})$).

An important practical question is how the initial values influence the behavior of the chain. The idea is that as the number of iterations increases, the chain gradually converges to an equilibrium distribution. Thus, generally the initial states are discarded until the chain reaches equilibrium. The problem then is to build algorithms that generate the Markov chain whose distribution converges to the distribution of interest. One of the most commonly used MCMC method is the Metropolis-Hastings algorithm (Gamerman and Lopes 2006, Migon and Gamerman 1999).

In order to implement the MCMC, a density $q(\mathbf{P}^*,\mathbf{P}^{(t-1)})$ is required, which gives the probability of moving from the current state in the chain $\mathbf{P}^{(t-1)}$ to a new state $\mathbf{P}^*$. The Metropolis-Hastings algorithm (Gamerman and Lopes

2006, J. Kaipio 2005, Migon and Gamerman 1999) used in this work to implement the MCMC method can be summarized in the following steps:

1. Sample a candidate point **P*** from a jumping distribution $q(\mathbf{P}^*,\mathbf{P}^{(t-1)})$.

2. Calculate:

$$\alpha(\mathbf{P}^{(t-1)},\mathbf{P}^*) = \min\left[1, \frac{\pi(\mathbf{P}^* \mid \mathbf{Y})q(\mathbf{P}^{(t-1)} \mid \mathbf{P}^*)}{\pi(\mathbf{P}^{(t-1)} \mid \mathbf{Y})q(\mathbf{P}^* \mid \mathbf{P}^{(t-1)})}\right] \tag{3.6}$$

3. Generate a random value U which is uniformly distributed on (0,1).

4. If $U \le \alpha$ define $\mathbf{P}^{(t)}=\mathbf{P}^*$; otherwise, define $\mathbf{P}^{(t)}= \mathbf{P}^{(t-1)}$.
5. Return to step 1 in order to generate the sequence $\{\mathbf{P}^{(1)}, \mathbf{P}^{(2)}, \ldots, \mathbf{P}^{(n)}\}$.

The success of the method depends on the acceptance rate and on proposals that are easy to simulate. The method replaces a difficult to generate $\pi(\mathbf{P}|\mathbf{Y})$ by several generations of the proposal q.
In this study we have chosen to adopt symmetrical chains, and for the Metropolis-Hastings algorithm the notion of symmetric chain is applied to the proposed transition q. Thus, q defines a single transition around the earlier position in the chain, i.e., $q(\mathbf{P}^*,\mathbf{P}^{(t-1)}) = q(\mathbf{P}^{(t-1)},\mathbf{P}^*)$ for all $(\mathbf{P}^*, \mathbf{P}^{(t-1)})$. In this case, Eq. (9) does not depend on q. For more details on theoretical aspects of the Metropolis-Hastings algorithm and MCMC methods, the reader should consult references (Gamerman and Lopes 2006, J. Kaipio 2005, Migon and Gamerman 1999).

We assume in this work that the measurement errors were additive, uncorrelated, normally distributed, with zero mean and a constant standard deviation and independent of the unknown parameters. The simulated measurements were then generated with Eq. (3.7), where ω is a random number with normal distribution and unitary standard deviation, and $\sigma = 0.01\ T^{\max}$, that is 1% of the maximum exact temperature obtained from the solution of the direct problem

$$Y_m = T_m^{\text{exact}} + \omega\sigma \tag{3.7}$$

Hence, the likelihood equation is given by (Gamerman and Lopes 2006, J. Kaipio 2005, J. V. Beck and K. Arnold 1977, Migon and Gamerman 1999):

$$\pi(\mathbf{Y} \mid \mathbf{P}) = (2\pi)^{-1/2}|\mathbf{W}|^{-1/2}\exp\left[(\mathbf{Y}-\mathbf{T}(\mathbf{P}))^T\mathbf{W}^{-1}(\mathbf{Y}-\mathbf{T}(\mathbf{P}))\right] \tag{3.8}$$

where ***T*** is the estimated temperature, obtained from the solution of the direct problem with estimates for the vector of unknown parameters **P**, **Y** is the vector of measured temperatures and **W** is the inverse of the covariance matrix of the measurements.

4. RESULTS AND DISCUSSION

In order to examine the accuracy and robustness of the proposed inverse analysis approach, we make use of simulated measured temperature data, which could be obtained through thermocouples inserted between the layers conductor. The results presented below from the inverse analysis have employed the parameter values: k_{al} = 2 W/m ^{0}C, k_{st} = 1 W/m ^{0}C, which are the effective thermal conductivities of the conductor and the core, respectively. The volumetric thermal capacity C_{al} = 2425.472 KJ/m^3 ^{0}C, C_{st} = 3728.750 KJ/m^3 ^{0}C , and the total thermal coefficient h_t = 10.5 W/m^2 ^{o}C.

The ACSR cable used in the simulation has the steel core diameter $d = 0.00675\ m$ and the total cable diameter $D = 0.027\ m$ The electrical resistance reference of the aluminum conductor $R_{ref} = 8.76 \times 10^{-5}\ \Omega/m$ at the reference temperature $T_{ref} = 75\ ^oC$, the aluminum correction factor $z = 0.00404$ and the contact resistance $h_c = 2 \times 10^4$ W/m^2 ^{o}C. It is important to mention that the numerical model was compared with the analytic solution, considering a monometallic aluminum conductor when a current of 800 A was applied. The compared rise temperature of both solutions reveals a quite a good agreement.

Figure 2 shows the normalized sensitivity coefficients of the unknown thermal parameters: k_{al}, k_{st}, C_{al}, C_{st}, h_t. The time evolution of such coefficients was numerical calculated with the differences finite method. In this figure, we note the high magnitudes of C_{al} e h_t, and the linear dependency between the volumetric heat capacities C_{al} e C_{st}. The magnitudes of the sensitivity coefficient related to the thermal conductivities, k_{al}, k_{st}, are small as a consequence of the small temperature gradients in both regions (core and conductor), however they are linearly independent. The choice of the prior information, needed in the Bayesian inversion process, is influenced from the analysis of the curves showed in Fig 2. Hence, coefficients with high magnitudes could adopt non informative prior distributions, as uniform distribution with wide boundary limits. On the other hand, with small magnitudes (or linearly dependent), it is recommended to choose informative prior distributions, as a Gaussian distribution with small variance.

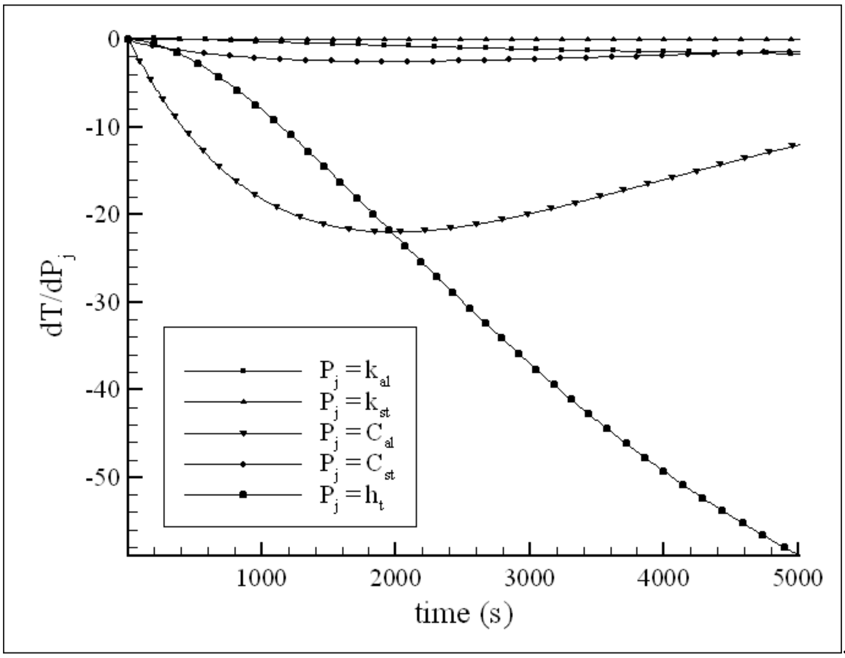
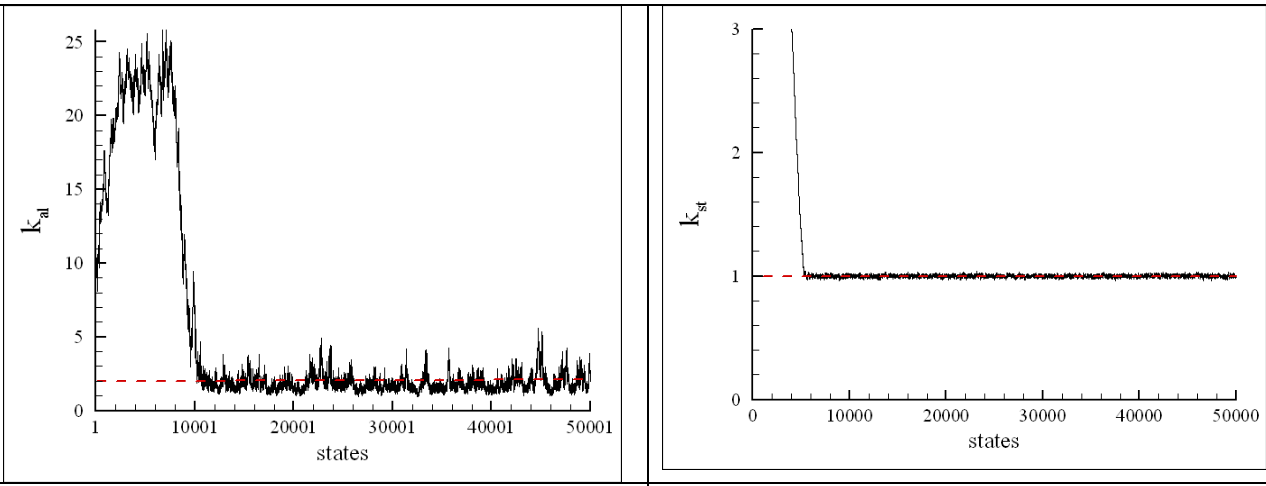


Figure 2. Curves of the sensitivity coefficients of the unknown thermal parameters.

Figure 3.a shows the estimative of the conductor effective thermal conductivity, which is presented in terms of its Markov chain evolution along a total 50,000 states. The simulated measured temperature was considered to be taken at $r = a$, in the boundary region between the core and the conductor. The prior distribution adopted in this case was an uniform distribution (k_{min} , k_{max}), with k_{min} = 0 W/m^2 and k_{max} = 200 W/m^2. The initial guess was 10 W/m^2. The candidate parameters were obtained from a uniform distribution (0,1), with step search Δp_{kal} = 0.05 $k_{al,Max}$. The convergence of the Markov chain was instable with respect to Δp_{kal} variations. The equilibrium distribution was reached after 10,000 iterations.

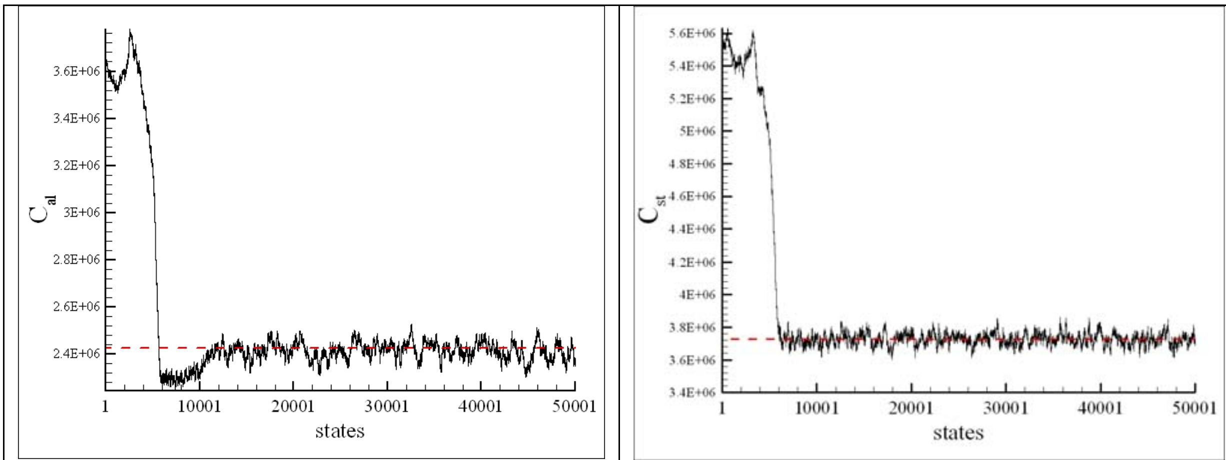
Figure 3.b shows the Markov chain evolution of the effective thermal conductivity of the steel core. The initial guess was 10 W/m^2. The prior information was given in terms of a Gaussian distribution with mean $k_{st,\ mean}$ = 1 W/m ^{o}C and standard deviation between 1% and 5%. The step search was Δp_{kst} = 0.005 k_{st}. The evolution of the Markov chain shows a stable convergence to the theoretical value (red dashed line), reaching its equilibrium distribution after 5,000 states.



a. Effective thermal conductivity of aluminum conductor	b. Effective thermal conductivity of steel core

Figure 3. Markov chain evolution of the effective thermal conductivities of the power cable.

Figure 4.a and 4.b show the Markov chain evolution of the volumetric heat capacities of the aluminum conductor and the steel core, respectively. From the analysis of the sensitivity coefficients, the prior information for the volumetric heat capacity of aluminum was given by a uniform distribution ($C_{al,min}$, $C_{al,max}$), with $C_{al,min}$ = 0 J/m^3 ^{o}C e $C_{al,max}$ = 1.5 C_{al} J/m^3 ^{o}C. In the case of steel, the prior information was given by a Gaussian distribution, with mean $C_{st,médio}$ = C_{al} and standard deviation between 1% e 5%. The initial guess had not influenced the Markov chain convergence and the search step used for the candidate parameters generation were Δp_{Cal} = 0.005 C_{al} e Δp_{Cst} = 0.005 C_{st}. For both cases the samples do not exhibit low-frequency oscillations and generally reach equilibrium within the number of states used for the simulation.



a. Volumetric heat capacity of the aluminum	b. Volumetric heat capacity of steel

Figure 4. Markov chain evolutions of the volumetric heat capacities of power cable.

Figure 5 shows the Markov chain evolution along a total of 50,000 states for the total heat transfer coefficient, h_t. The prior information used in this case was a uniform distribution ($h_{t,\ min}$, $h_{t,max}$), with $h_{t,\ min}$ = 0 W/m^2 e $h_{t,max}$ = 20 W/m^2. The candidate parameters were generated with the search step Δp_{ht} = 0.005 h_t. The value chosen for the initial guess was irrelevant and the equilibrium distribution was achieved with more accuracy. For this parameter, the behavior of Markov chain evolution shows a good convergence to the theoretical value (red dashed line).

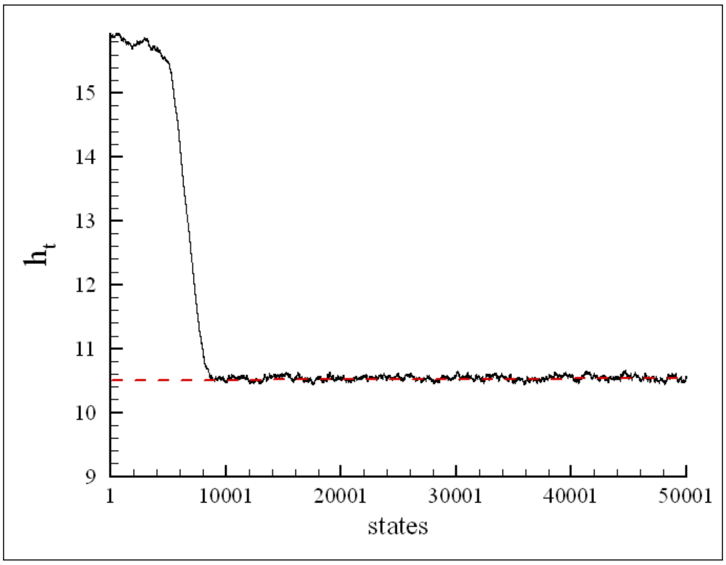


Figure 5. Markov chain evolution of the total heat transfer coefficient.

Finally, Table 1 shows the mean values for the unknown parameters. The limits of the 95% confidence intervals for such parameters are also illustrated in the table. The table shows an excellent agreement between the exact parameters and their corresponding estimated means.

Table 1 –Estimations for the unknown parameters

Parameter	Exact	MCMC	Min. 95%	Max. 95%
k_{AL}	2	1.88	1.19	3.4
k_{ST}	1	0.99	0.98	1.02
C_{Al}	2.42547×10^6	2.41403×10^6	2.33939×10^6	2.48328×10^6
C_{St}	3.72875×10^6	3.72787×10^6	3.65485×10^6	3.80411×10^6
h_t	10.50	10.54	10.47	10.61

5. CONCLUSIONS

In this paper we applied a Bayesian approach to the estimation of the thermal parameters related to electrical power cables. The physical problem is formulated in terms of one dimensional heat conduction problem, stated separately for both regions of the cable. The Metropolis-Hastings algorithm of the Markov Chain Monte Carlo method is used in this work.

The results obtained in this work for a heat conduction problem, involving the simultaneous estimation of several parameters, reveal the accuracy and robustness of the present approach for the solution of the inverse problem. Such a statistical approach is suitable since the uncertainties of the measurements and the unknown parameters are taking into account.

6. ACKNONWLEDGMENT

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