DYNAMICAL ANALYSIS OF A COUPLED OSCILLATORS SYSTEM USED FOR HUMAN GAIT SIMULATION

Armando Carlos de Pina Filho, armando@poli.ufrj.br
Universidade Federal do Rio de Janeiro, Urban Engineering Program, PEU/POLI, Technology Center, 21941-909, Rio de Janeiro - RJ, Brazil.

Max Suell Dutra, max@mecanica.coppe.ufrj.br
Universidade Federal do Rio de Janeiro, Mechanical Engineering Program, PEM/COPPE, Technology Center, 21945-970, Rio de Janeiro - RJ, Brazil.

Abstract. *The human being has a system, in spinal marrow, capable to control, in part, the human gait. This system, called central pattern generator (CPG), is responsible for the production of rhythmic movements. Modeling of this CPG can be made by means of coupled oscillators. This system generates patterns similar to human CPG, becoming possible the human gait simulation. In order to create an adequate system, a set of mutually coupled nonlinear oscillators was used. From a model of two-dimensional locomotor, oscillators with integer relation of frequency were used for simulating the behavior of the hip angle and of the knees angles. Each oscillator has its own parameters and the link to the other oscillators is made through coupling terms. The objective of this work is to analyze the dynamics of this coupled oscillators system by means of bifurcation diagrams and Poincaré maps. From the analysis and graphs generated in MATLAB, it was possible to evaluate some characteristics of the system, such as: sensitivity to the initial conditions, presence of strange attractors and other phenomena of the chaos. In the course of tests, we verify that diagrams did not clearly present bifurcation as simple curves, which normally happens in the dynamical analysis of systems. In fact, we have a cloud of points. Considering the complexity involved in the analysis of systems with coupled oscillators, this fact can be explained by two points of view: one is related to the coupling terms and their relationship, and other one is related to the quasi-periodic response of the system. Both of them will be presented and analyzed in this work. Based on the results of the study, we conclude that although the use of coupled oscillators represents an excellent way for generating pattern signals of locomotion, its application in the control of a bipedal locomotor will only be possible with the correct choice of parameters, which must be done from the data provided by the analysis of bifurcation and chaos.*

Keywords*: CPG, Chaos, Gait, Oscillators.*

1. INTRODUCTION

In the course of many years, the man tries to copy or to imitate some systems of the human body. It is the case, for example, of the central pattern generator (CPG), a system in the human spinal marrow, responsible for the production of rhythmic movements. The spinal marrow is constituted by nerves, and its forepart contains the motoneurons, which transmit information to the muscles and stimulate the movement. The posterior part and the lateral parts contain the sensitive nerves, receiving information from the skin, joints, muscles and viscera (see Fig. 1).

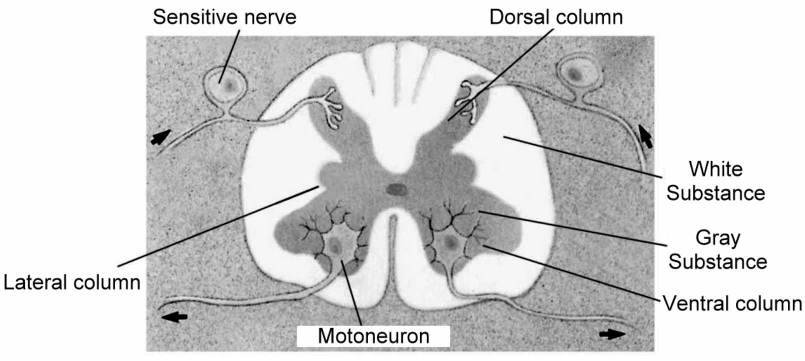

Figure 1. Human spinal marrow.

The human gait requires a coordination of the muscular activity between the two legs, which is made by a flexible neural coupling to the level of the spinal marrow. Thus, in the course of the locomotion, a disturbance in one of the legs leads to a pattern of propositional reply of the spinal marrow, characterising the existence of the so-called central pattern generator. Some works about this subject were presented by Calancie *et al.* (1994), Dimitrijevic *et al.* (1998), Mackay-Lyons (2002), and Dietz (2003).

In relation to the control system of the human locomotion, Fig. 2 presents a scheme, showing the central nervous system, where the central pattern generator supplies a series of pattern curves for each part of the locomotor. This information is passed to the muscles by means of a network of motoneurons, and the conjoined muscular activity performs the locomotion. Sensorial information about the conditions of the environment or some disturbance are supplied as feedback of the system, providing a fast action proceeding from the central pattern generator, which adapts the gait to the new situation.

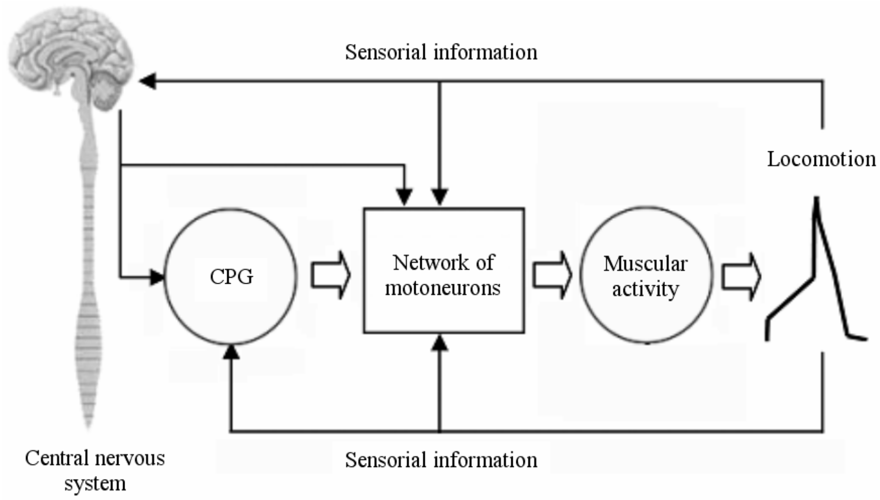

Figure 2. Control system of the human locomotion.

CPG system can be simulate by means of coupled oscillators, generating patterns similar to human CPG. From this artificial system, it is possible to perform the human gait simulation, using a locomotor model. CPG system generates data related to the hip and knee angles. Each oscillator used in the system has its own parameters and the link to the other oscillators is made through coupling terms. We intend to evaluate a system with coupled van der Pol oscillators. Some previous works about CPGs using nonlinear oscillators, applied in the human gait simulation, can be seen in Bay and Hemami (1987), Zielinska (1996), Dutra *et al.* (2003), and Pina Filho (2005).

From the CPG system, we achieve the dynamical analysis by means of bifurcation diagrams and Poincaré maps. With graphs generated in MATLAB, it was possible to evaluate some characteristics of the system, such as: sensitivity to the initial conditions, presence of strange attractors and other phenomena of the chaos.

2. LOCOMOTION PATTERNS

Despite the people not walk in completely identical way, some characteristics in the gait can be considered universal, and these similar points serve as base for description of patterns of the kinematics, dynamics and muscular activity in the locomotion. In this work, our interest is related to the patterns of the hip and knee angles, more specifically, in sagittal plane (see Fig. 3).

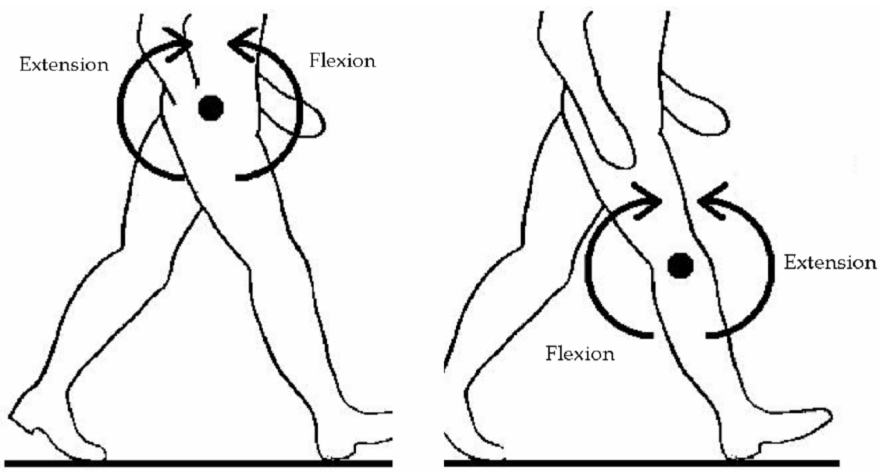

Figure 3. Movements of hip and knee in sagittal plane.

Figure 4 presents the graphs of angular displacement and phase space of the hip, while Fig. 5 presents the graphs of angular displacement and phase space of the knee. These figures were extracted from Pina Filho *et al.* (2006).

From the knowledge of these patterns of behaviour, a CPG using the system of coupled oscillators was created. This system will be applied in a biped locomotor model to simulate the human gait.

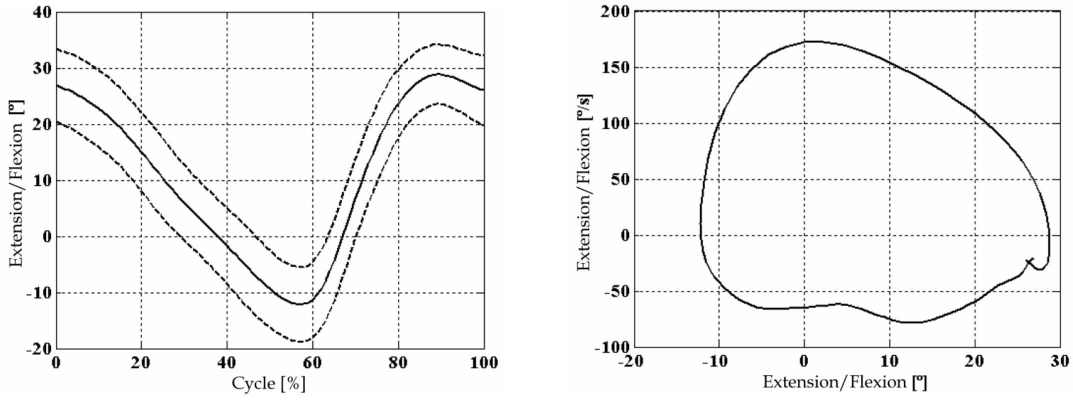

Figure 4. Angular displacement and phase space of the hip (mean ± deviation).

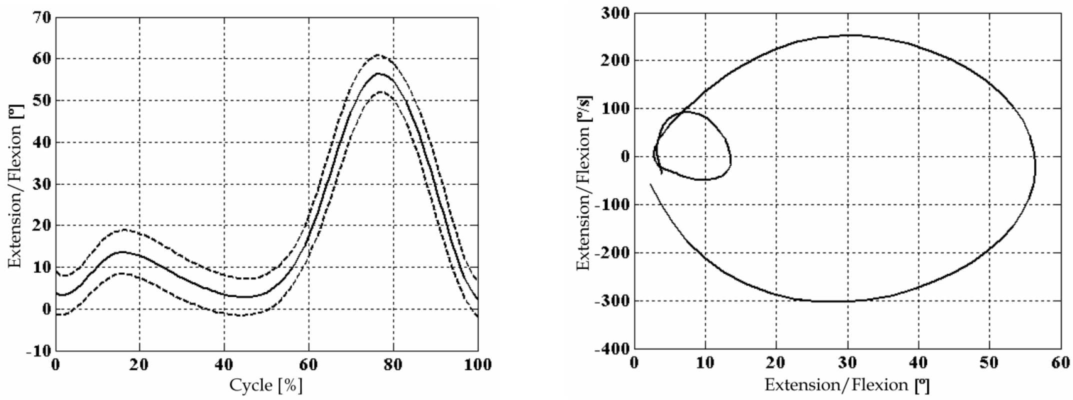

Figure 5. Angular displacement and phase space of the knee (mean ± deviation).

3. CPG SYSTEM

Coupled oscillators systems have been extensively used in studies of physiological and biochemical modelling. Since the years of 1960, many researchers have studied the case of coupling between two oscillators, because this study is the basis to understand the phenomenon in a great number of coupled oscillators. One of the types of oscillators that can be used in coupled systems is the auto-excited ones, which have a stable limit cycle without external forces. The van der Pol oscillator is an example of this type of oscillator, and it will be used in this work. Then, considering a system of *n* coupled van der Pol oscillators, from van der Pol equation:

$$\ddot{x}-\varepsilon\left(1-p\left(x-x_0\right)^2\right)\dot{x}+\Omega^2\left(x-x_0\right)=0 \qquad \varepsilon, p \geq 0 \tag{1}$$

where ε, p and Ω correspond to the parameters of the oscillator, and adding coupling terms that relate the oscillators velocities, we have:

$$\ddot{\theta}_i-\varepsilon_i\left[1-p_i\left(\theta_i-\theta_{io}\right)^2\right]\dot{\theta}_i+\Omega_i^2\left(\theta_i-\theta_{io}\right)-\sum_{j=1}^{n}c_{i,j}\left(\dot{\theta}_i-\dot{\theta}_j\right)=0 \qquad i=1,2,\ldots,n \tag{2}$$

which represents coupling between oscillators with the same frequency, where θ corresponds to the system degrees of freedom (see more details in Pina Filho (2005)). In the case of coupling between oscillators with integer relation of frequency, the equation would be:

$$\ddot{\theta}_h - \varepsilon_h\left[1 - p_h(\theta_h - \theta_{ho})^2\right]\dot{\theta}_h + \Omega_h^2(\theta_h - \theta_{ho}) - \sum_{i=1}^{m} c_{h,i}\left[\dot{\theta}_i(\theta_i - \theta_{io})\right] - \sum_{k=1}^{n} c_{h,k}\left(\dot{\theta}_h - \dot{\theta}_k\right) = 0 \quad (3)$$

where $c_{h,i}[\dot{\theta}_i(\theta_i - \theta_{io})]$ is responsible for the coupling between oscillators with different frequencies, while $c_{h,k}\left(\dot{\theta}_h - \dot{\theta}_k\right)$, also seen in Eq. (2), effects the coupling between oscillators with the same frequency. Both terms were determined by Dutra (1995).

To generate the angles θ_3, θ_4 and θ_5 as a periodic attractor of a nonlinear net, three coupled van der Pol oscillators were used. These oscillators are mutually coupled by terms that determine the influence of one oscillator on the others (see Fig. 6).

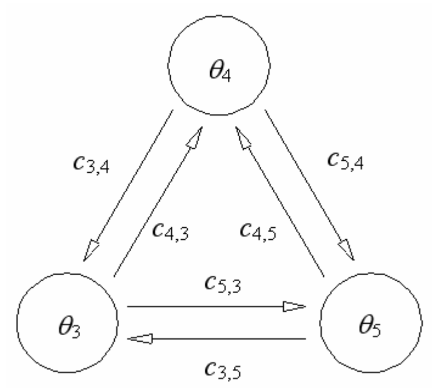

Figure 6. Structure of coupling oscillators.

Applying Eq. (2) and (3) to the proposed problem, knowing that the frequency of θ_3 and θ_5 (knee angles) is double of θ_4 (hip angle), we have the following equations:

$$\ddot{\theta}_3 - \varepsilon_3[1 - p_3(\theta_3 - \theta_{3o})^2]\,\dot{\theta}_3 + \Omega_3^2(\theta_3 - \theta_{3o}) - c_{3,4}\left[\dot{\theta}_4(\theta_4 - \theta_{4o})\right] - c_{3,5}\left(\dot{\theta}_3 - \dot{\theta}_5\right) = 0 \quad (4)$$

$$\ddot{\theta}_4 - \varepsilon_4[1 - p_4(\theta_4 - \theta_{4o})^2]\,\dot{\theta}_4 + \Omega_4^2(\theta_4 - \theta_{4o}) - c_{4,3}\left[\dot{\theta}_3(\theta_3 - \theta_{3o})\right] - c_{4,5}\left[\dot{\theta}_5(\theta_5 - \theta_{5o})\right] = 0 \quad (5)$$

$$\ddot{\theta}_5 - \varepsilon_5[1 - p_5(\theta_5 - \theta_{5o})^2]\,\dot{\theta}_5 + \Omega_5^2(\theta_5 - \theta_{5o}) - c_{5,4}\left[\dot{\theta}_4(\theta_4 - \theta_{4o})\right] - c_{5,3}\left(\dot{\theta}_5 - \dot{\theta}_3\right) = 0 \quad (6)$$

From Eq. (4)-(6), using the parameters shown in Tab. 1 together with values supplied by Pina Filho (2005), the graphs were generated in MATLAB as shown in Fig. 7, which present, respectively, the behaviour of θ_3, θ_4 and θ_5 as a function of time, and trajectories in the phase space.

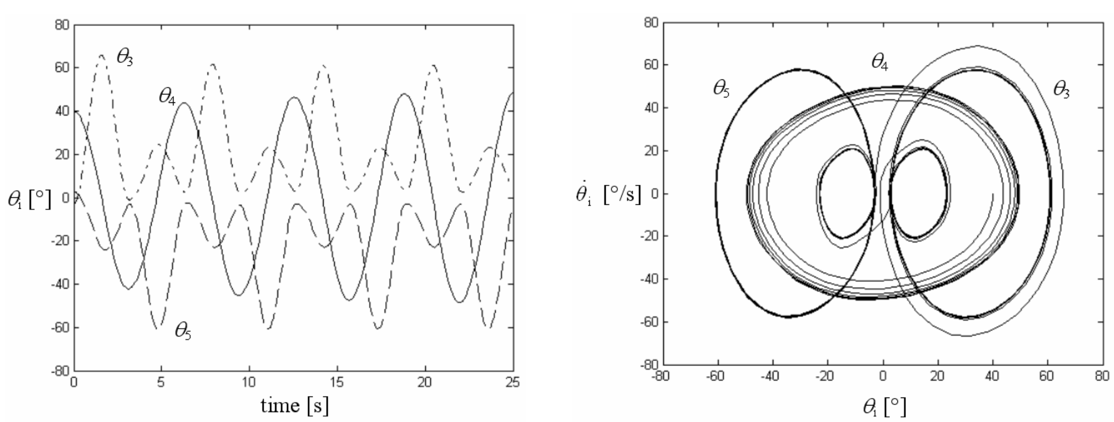

Figure 7. Angles as a function of time, and trajectories in the phase space.

Table 1. Parameters of van der Pol oscillators.

$c_{3,4}$	$c_{4,3}$	$c_{3,5}$	$c_{5,3}$	$c_{4,5}$	$c_{5,4}$	ε_3	ε_4	ε_5
0.001	0.001	0.1	0.1	0.001	0.001	0.01	0.1	0.01

Comparing Fig. 7 with the experimental results presented in Fig. 4 and 5, it is verified that the coupling system supplies similar results, what confirms the possibility of use of mutually coupled van der Pol oscillators in the modelling of the CPG.

4. HUMAN GAIT SIMULATION

In order to specify a locomotor model, we consider some particularities of the human locomotion (see Saunders *et al.,* 1953, and McMahon, 1984), such as the determinants of gait. The model presented here characterises the three most important determinants of gait: the compass gait that is performed with stiff legs like an inverted pendulum (the pathway of the centre of gravity is a series of arcs); the knee flexion of the stance leg, which combined with pelvic rotation and pelvic tilt achieve minimal vertical displacement of the centre of gravity; and the plantar flexion of the stance ankle, where the effects of the arcs of foot and knee rotation smooth out the abrupt inflexions at the intersection of the arcs of translation of the centre of gravity.

Figure 8 presents the biped model with its angles and lengths, where: ℓ_s is the length of foot responsible for the support (toes), ℓ_p is the length of foot that raises up the ground (sole), ℓ_t is the length of tibia, and ℓ_f is the length of femur. The angle of the hip θ_4 and the angles of the knees θ_3 and θ_5 will be determined by the CPG system, while the other angles are calculated by the kinematical analysis of the mechanism. In this work we will not present details of this analysis, which can be seen in Pina Filho (2005).

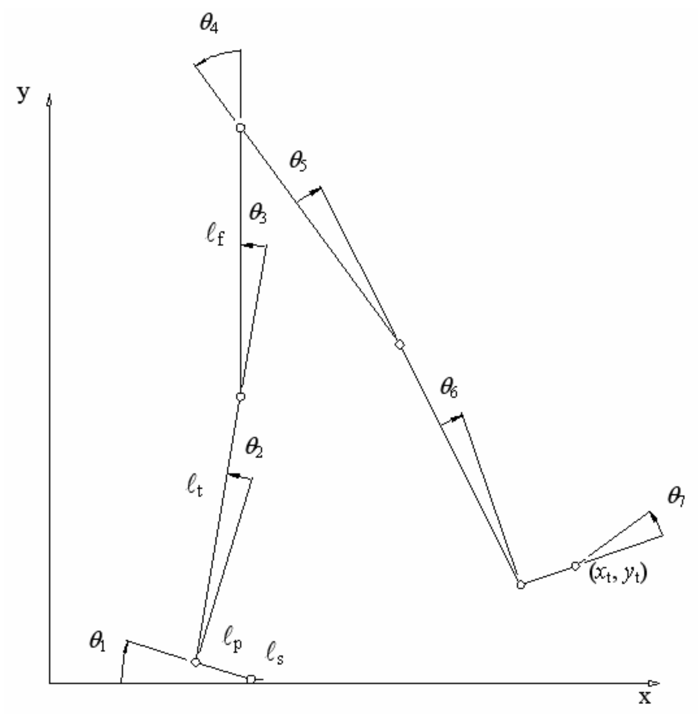

Figure 8. Biped model with the determinants of gait, angles and lengths.

Applying the presented CPG system to this model, we have, in Fig. 9, a stick figure showing the gait with a step length of 0.5 m. Dimensions used in the model can be seen in Tab. 2.

Table 2. Model dimensions.

Thumb	Foot	Leg (below the knee)	Thigh
0.03 m	0.11 m	0.37 m	0.37 m

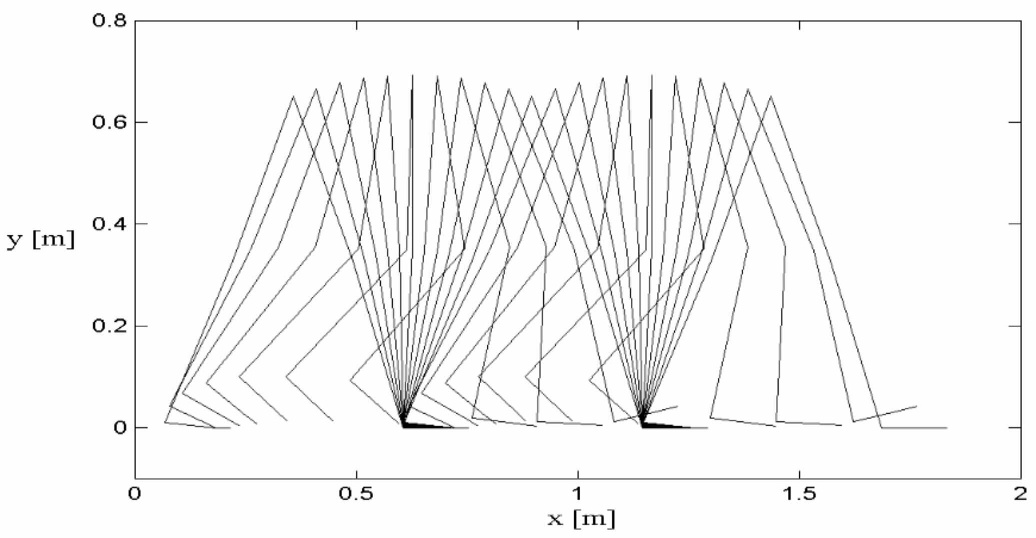

Figure 9. Stick figure showing the gait with a step length of 0.5 m.

5. DYNAMICAL ANALYSIS

The dynamical analysis of the system presented here will help the choice of some optimal parameters, to simulate the human gait. We intend to avoid the chaotic behaviour of the system, which in biped models mean unpredictable locomotion patterns. Thus, we need to study the presence of this chaotic configuration.

Two conditions must be satisfied to make possible that a system presents chaotic behaviour: the equations of motion must include a nonlinear term; and the system must have at least three independent dynamic variables. The main consequence associated with the chaos is the sensitivity to the initial conditions. In chaotic systems, a small change in the initial conditions results in a drastic change in the system behaviour.

Also, in chaotic systems, we observe the bifurcation phenomenon, which represent the stroboscopic distribution of the system response from slow variation of a parameter. This method was applied here, which implies to simulate different parameter values that we want to analyze, evaluating the response in bifurcation diagrams and Poincaré maps.

More details about the Chaos theory and its characteristics can be found in Thompsom and Stewart (1986), Strogatz (1994), and Baker and Gollub (1996).

Then, considering different values for the parameters ε_3, ε_4 and ε_5, the tests have been performed using MATLAB to generate the bifurcation diagrams and Poincaré maps. In principle, keeping values of $\varepsilon_4 = 0.1$ and $\varepsilon_5 = 0.01$, the value of ε_3 was varied from 0 to 2. Other values of the system have been kept. Figure 10 presents the bifurcation diagram showing the behaviour of knee oscillator θ_3 with variation of parameter ε_3, which represents the damping term related with this oscillator. Also, we have the strange attractor generated in the analysis of knee oscillator θ_3, observed in a Poincaré map.

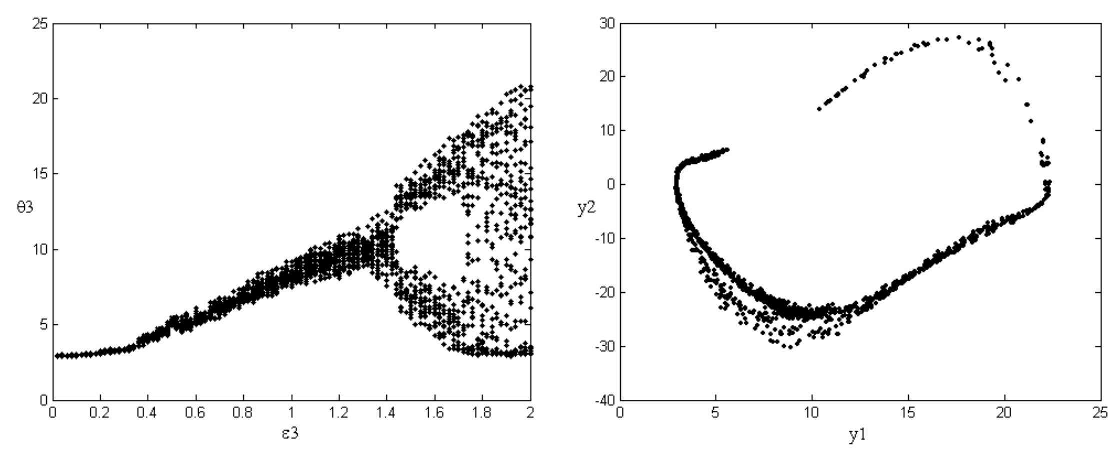

Figure 10. Bifurcation diagram for θ_3 with variation of ε_3, and strange attractor for θ_3.

This diagram does not represent the bifurcation as simple curves, which normally happens in dynamical analysis of a system, but with a cloud of points. Considering the complexity of coupled oscillators system, this fact can be explained by relation between coupling terms or by quasiperiodic response of the system.

A great variation between coupling terms, with one of them approaching to zero, makes the system presents practically a unidirectional coupling, and consequently the response in bifurcation diagram is represented by a cloud of points, characterizing not only the presence of periodic and chaotic orbits, but also pseudo-trajectories (Grebogi *et al.*, 2002, and Santos *et al.*, 2004).

In relation to system behaviour, with small values of damping term, below 0.1, the system presents a periodic response. With the increase of damping term, the system starts to present a quasiperiodic response, and later chaotic response, when $\varepsilon_3 = 2$ (see Fig. 11).

More details about transition between quasiperiodic and chaotic response are presented in Yoshinaga and Kawakami (1994), Yang (2000), and Pazó *et al.* (2001).

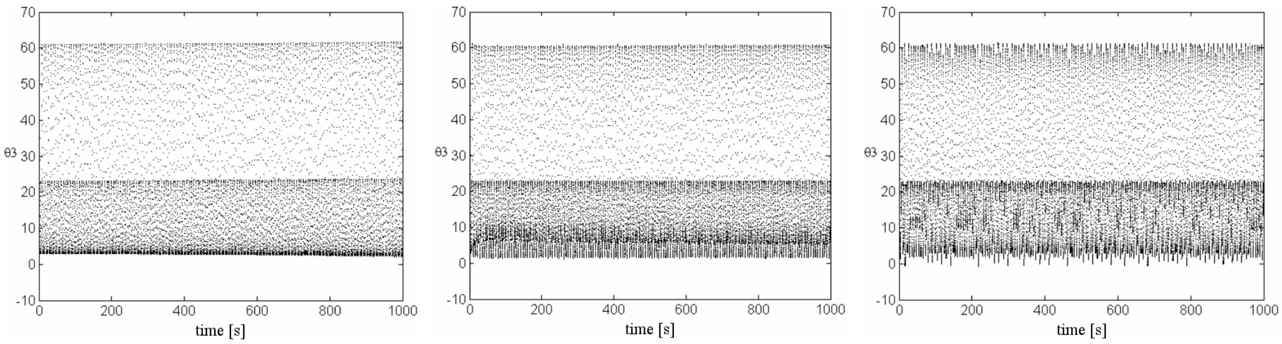

Figure 11. Periodic response: $\varepsilon_3 = 0.01$, Quasiperiodic response: $\varepsilon_3 = 1$, and Chaotic response: $\varepsilon_3 = 2$.

Sensitivity to the initial conditions can be verified considering two simulations with different conditions, for example, with $\varepsilon_3 = 3$ (chaotic regime, similar to seen in Fig. 11, for $\varepsilon_3 = 2$), choosing initial values for the angles: $\theta_3 = 3º$, $\theta_4 = 50º$, $\theta_5 = -3º$, and changing $\theta_3 = 3.001º$, we observed the influence of initial conditions in the system response (Fig. 12).

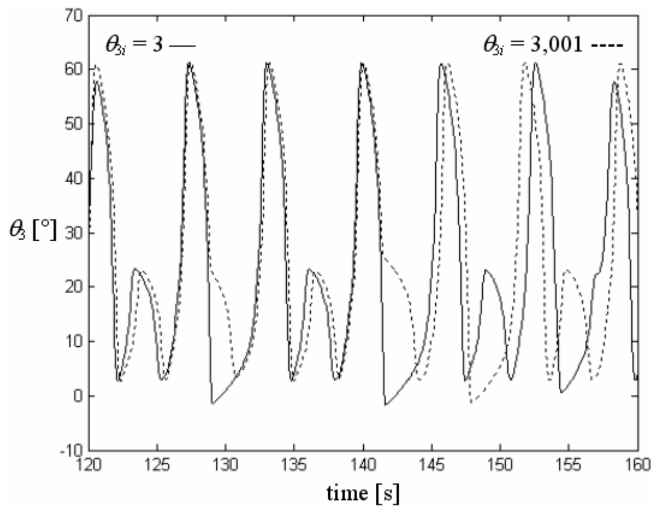

Figure 12. Sensitivity to the initial conditions in chaotic response.

Considering the coupling oscillators, the degree of influence between them is defined by the coupling term. Then, a change of oscillator parameters must influence the behaviour of other oscillators. In the case of the hip angle, the influence of knee oscillator θ_3 on the hip is small, therefore the behaviour of θ_4 does not show many alterations. This occurs due to small value adopted for the coupling term between the oscillators ($c_{34} = c_{43} = 0.001$). In relation to the knees, the coupling term is greater ($c_{35} = c_{53} = 0.1$), configuring a more significant influence.

Similarly to analysis of ε_3, the system response can be analyzed by varying the values of ε_4 (from 0 to 2) and keeping the other values fixed. Figure 13 presents the bifurcation diagram showing the behaviour of hip oscillator θ_4 with variation of parameter ε_4, which represents the damping term related with this oscillator. Also, we have the strange attractor generated in the analysis of this oscillator.

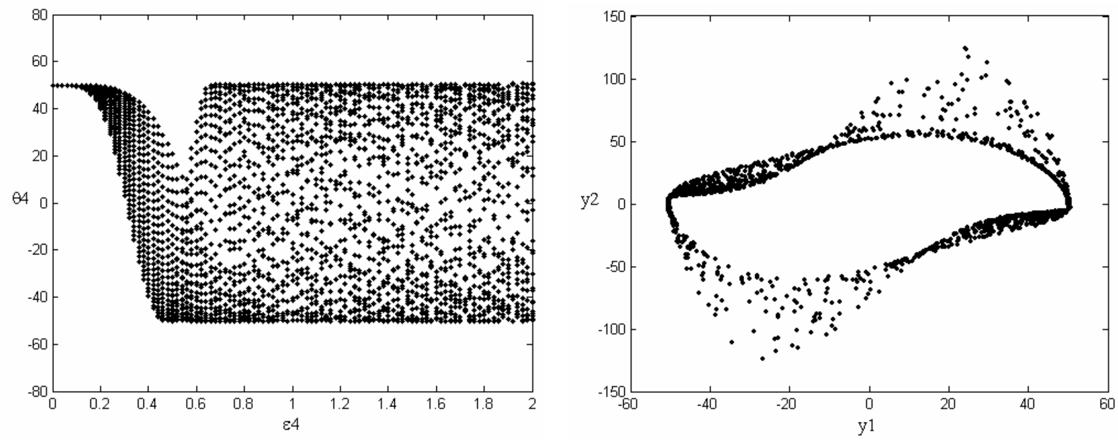

Figure 13. Bifurcation diagram for θ_4 with variation of ε_4, and strange attractor for θ_4.

As seen previously in the analysis of ε_3, the influence of hip on the knees is small, then a variation of ε_4 does not cause great changes in θ_3 and θ_5.

Finally, the system response can be analyzed by varying the values of ε_5 (from 0 to 2) and keeping the other system values fixed. Figure 14 presents the bifurcation diagram showing the behaviour of knee oscillator θ_5 with variation of the parameter ε_5, which represents the damping term related with this oscillator. Also, we have the strange attractor generated in the analysis of this oscillator.

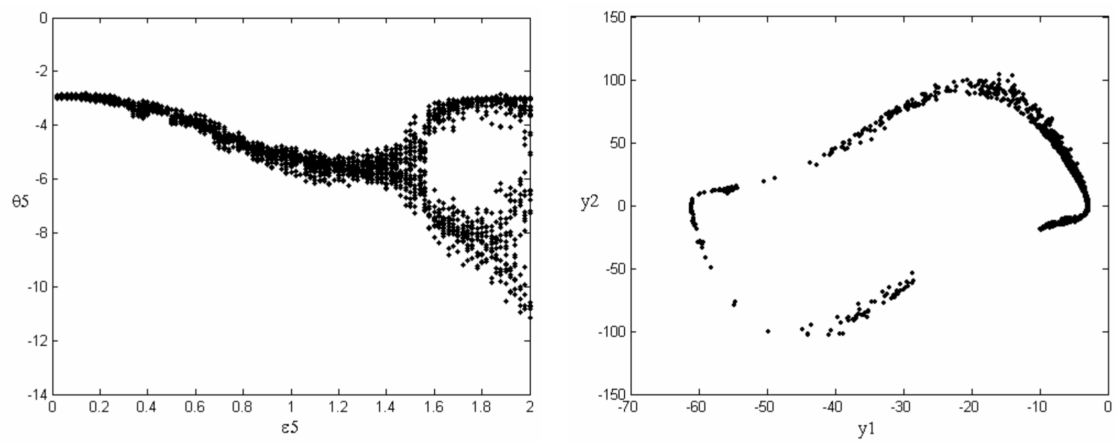

Figure 14. Bifurcation diagram for θ_5 with variation of ε_5, and strange attractor for θ_5.

6. CONCLUSION

In this work, we present the study of a biped locomotor with a CPG formed by a system of coupled van der Pol oscillators. A biped locomotor model with three of the six most important determinants of human gait was used in the analyses. After the modelling of the oscillators system, a dynamical analysis was performed to verify the performance of the system, in particular, aspects related to the chaos. From presented results and discussion, we come to the following conclusions: the use of mutually coupled nonlinear oscillators of van der Pol can represent an excellent way to generate locomotion pattern signals, allowing its application for the control of a biped by the synchronization and coordination of the legs, once the choice of parameters is correct, which must be made from the data supplied by the analysis of bifurcation and chaos. Through the dynamical analysis it was possible to evidence a weak point of coupling systems. The influence of the knee oscillators on the hip, and vice versa, is very small, what can harm the functionality of the system. The solution for this problem seems immediate: to increase the value of the coupling term between the hip and knees. However, this can make the system unstable. Then, it is necessary a more refined study of the problem, which will be made in future works, as well as a study of the behaviour of the ankles, and simulation of the hip and knees in the other anatomical planes, increasing the network of coupled oscillators, and consequently simulating with more details the human locomotion CPG. This study has great application in the project of autonomous robots and in the rehabilitation technology, not only in the project of prosthesis and orthesis, but also in the searching of procedures that help to recuperate motor functions of human beings.

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