EXPERIMENTAL INVESTIGATION OF AN IMPACT SYSTEM WITH SHAPE MEMORY ALLOY ELEMENT

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Abstract. *Nonsmooth systems appear in many physical situations as rotordynamics, oil drilling and machining. The vibration control is an interesting application of nonsmooth systems, especially the ones that employ smart materials. The use of shape memory alloys (SMAs) may explore their high dissipation capacity related to hysteretic behavior. This contribution explores the SMA thermomechanical behavior in a nonsmooth system represented by a single-degree of freedom system with discontinuous support. This main aspects of nonsmooth systems are captured allowing the understanding of this kind of system. Basically, an experimental set up is constructed by considering a car free to move over a rail. The system is monitored to obtain all variables. Two different supports are of concern: linear elastic and SMA. Results from linear elastic system are compared with numerical simulations. Afterward, this model is employed in the analysis of the SMA support system, trying to evaluate the SMA influence on the response. The system presents a rich dynamical behavior that includes bifurcations and chaos.*

Keywords: *Non-smooth, non-linear dynamics, smart materials, shape memory alloy, control.*

1. INTRODUCTION

Nonlinearities appear in many kinds of engineering systems and also in everyday life. These systems can induce different kinds of responses that make difficult the use of traditional approaches of engineering analysis. One of the sources of nonlinearities is called nonsmoothness that is generally related to friction and intermittent contact with some part of the system.

The idea that nonsmooth systems can be considered as continuous in a finite number of subspaces and the system parameters do not change abruptly inspires some researchers to try to describe nonsmooth systems in a smooth way. Wiercigroch *et al.* (1998), Wiercigroch (2000), Leine (2000), Leine *et al.* (2000), Leine & Van Campen (1998, 2002), Divenyi *et al.* (2006) and Savi *et al.* (2007) are some efforts where interesting approaches are proposed to deal with mathematical discontinuities. Several nonsmooth systems have been treated in literature. Wiercigroch *et al.* (1998) analyzed the effects of discontinuous stiffness due to gaps in the contact between shaft and bearing in rotating machinery, highlighting the importance of analysis for the design of efficient and reliable machines. Divenyi *et al.* (2008) presented an experimental investigation treating the influence of different aspects in the nonlinear dynamics of a single-degree of freedom oscillator with discontinuous support.

Smart materials and structures have an increasing importance nowadays and applications are related to several areas that include vibration control. Shape memory alloys are metallic materials that can recover its original shape by imposing a temperature and/or stress field due to the phase transformations occurring in the material. These materials present a complex thermomechanical behavior associated with different physical processes. The most common phenomena are pseudoelasticity, shape memory effect, two-way shape memory effect and phase transformation due to temperature variation. The hysteretic response of SMAs is one of their essential characteristics being related to the martensitic phase transformation. The hysteresis loop may be observed either in stress-strain curves or in strain-temperature curves (Monteiro *et al.*, 2009; Paiva *et al.*, 2005). The unique properties of SMAs have several applications varying from biomedical to aerospace engineering (Lagoudas, 2008; Machado & Savi, 2003; Paiva & Savi, 2006).

The use of SMA in vibration control and vibration absorbers is an interesting approach that considers either stiffness change due to temperature variations or the energy dissipation due to hysteretic behavior. In this regard, Willians *et al.* (2002, 2005a, 2005b) investigated the use of SMAs in vibration absorbers with the aim of adjusting the natural frequency of the device. Savi *et al.* (2011) presented an investigation of SMA vibration absorbers discussing different aspects of the system dynamics including the change in tuned frequency and dynamical jumps. Recently, SMAs have motivated their use in nonsmooth system for vibration reduction (Santos & Savi, 2009; Sitnikova *et al.*, 2010).

This paper deals with the dynamics of a single degree of freedom oscillator with discontinuous support. Basically, two systems are of concern: linear elastic support and SMA support. The system with linear elastic support is treated numerically and experimentally. The SMA system, on the other hand, is treated only experimentally. Results of both systems are compared providing a comprehension of the influence of the SMA in the system dynamics. The SMA spring is in the austenitic state at room temperature and, therefore, is expected to observe pseudoelastic behavior.

2. MATHEMATICAL MODEL OF THE SYSTEM WITH LINEAR ELASTIC SUPPORT

A single degree of freedom system with discontinuous support, shown in Fig. 1, is now of concern. The oscillator is composed by a mass m free to move over a rail. The mass is connected by two linear springs with stiffness k_1 and k_2. The dissipation process is modeled by dry friction with coefficient μ and also by a viscous damping with coefficient c. The support is massless, having a linear spring with coefficient k_s and the dissipation process is represented by a linear viscous damping element with coefficient c_s. The mass displacement is denoted by x, relative to the equilibrium position, while the support displacement is denoted by y. The distance between the mass and the support is defined by a gap g. Therefore, the system has two possible modes of operation: with contact and without contact. Denoting the contact force between the mass and the support by f_s, these two situations may be represented as follows:

$$\begin{cases} x < g \quad \text{and} \quad f_s = 0, & \text{without contact} \\ x \geq g \quad \text{and} \quad f_s = -\left(k_s y - c_c \dot{y}\right) < 0, & \text{with contact.} \end{cases} \tag{1}$$

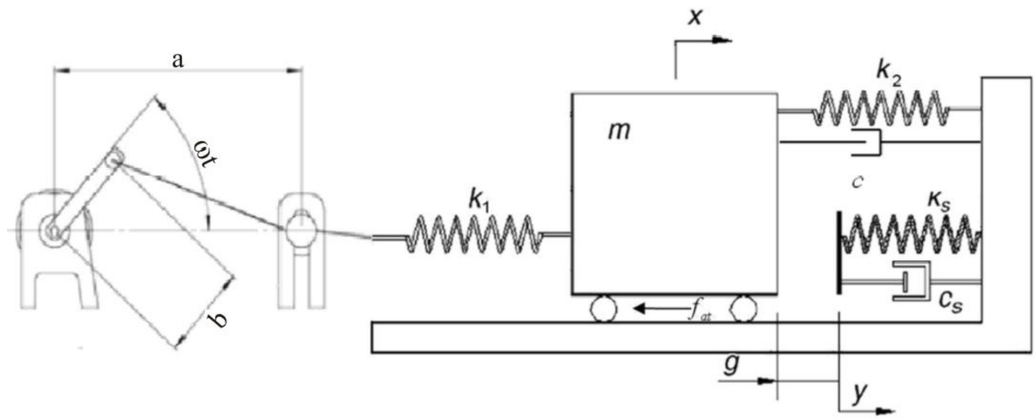


Figure 1. Nonsmooth system with discontinuous support.

The support relaxes to the equilibrium state when there is no contact between the mass and the support. By assuming that the support relaxation time is much smaller than the time between two contact events, the support dynamics can be neglected. This assumption reduces the system dynamics to a second-order differential equation. Therefore, the governing equations may be defined by two different equations, representing situations with and without contact. The system is subjected to a harmonic excitation provided by the DC motor and a spring-string system. The variation of string length, Δl, depends on the rotor arm length and the angular velocity of the rotor, ω, which result in an excitation given by:

$$F(t) = k_1 \Delta l = k_1 b\left[\cos(\omega t + \pi) + 1\right] \tag{2}$$

Thus, the equations of motion are:

$$\begin{cases} m\ddot{x} + \mu \operatorname{sgn}(\dot{x}) + c\dot{x} + kx = F(t) & \text{without contact} \\ m\ddot{x} + \mu \operatorname{sgn}(\dot{x}) + c\dot{x} + c_s\dot{x} + kx + k_s(x - g) = F(t) & \text{with contact} \end{cases} \tag{3}$$

Note that the system dissipation is represented by a combination of dry friction and viscous damping. The dry friction, μ sgn($\dot{x}$), is nonsmooth and can be smoothen by considering a function as follows (Figure 2) (De Paula *et al.*, 2005):

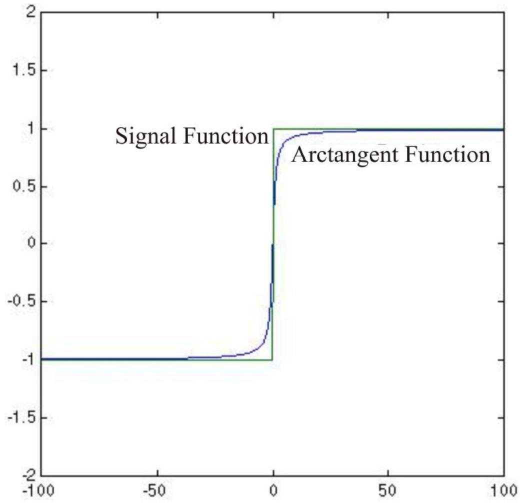


Figure 2. Description of the dry friction dissipation: signal function, $\mathrm{sgn}(\dot{x})$, and $2/\pi\,\arctan(\dot{x})$.

$$\mu\,\mathrm{sgn}(\dot{x}) \cong \mu \frac{2}{\pi} \arctan(q\dot{x}) \tag{4}$$

These equations of motion are classified as piecewise linear and can be transformed into a set of first-order ordinary differential equations. Filippov theory can be applied to treat this dynamical system and here, the fourth order Runge-Kutta method is employed to solve this set of equations.

3. EXPERIMENTAL APPARATUS

The experimental analysis is performed by considering the apparatus presented in Figure 3. It is a single degree of freedom oscillator composed by a car free to move over a rail, connected to an excitation system composed of springs, strings and DC motor (PASCO ME-8750 with 0-12V and 0-0.3A). The discontinuous support is fixed in the rail providing the system discontinuity. The system is monitored by two rotating sensors PASCO CI-6538, which has accuracy +/−0.25 degrees, a maximum velocity of 30rev/s and a maximum sampling frequency of 1000Hz. One of the sensors monitors the position of the rotor arm and the other monitors the position and velocity of the car.

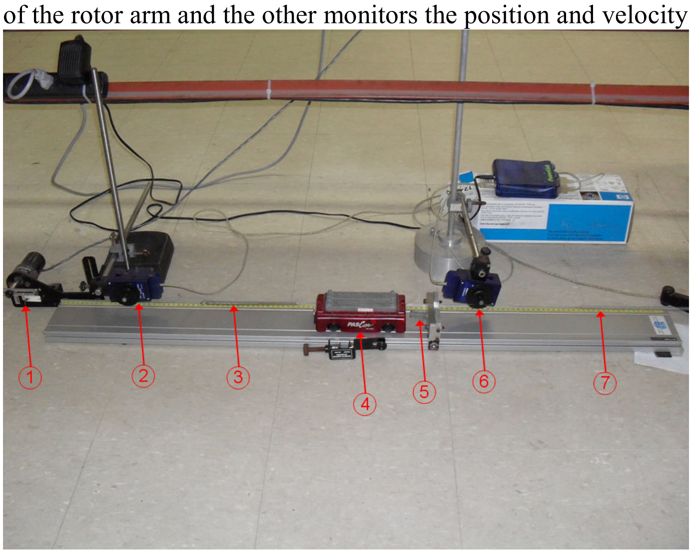
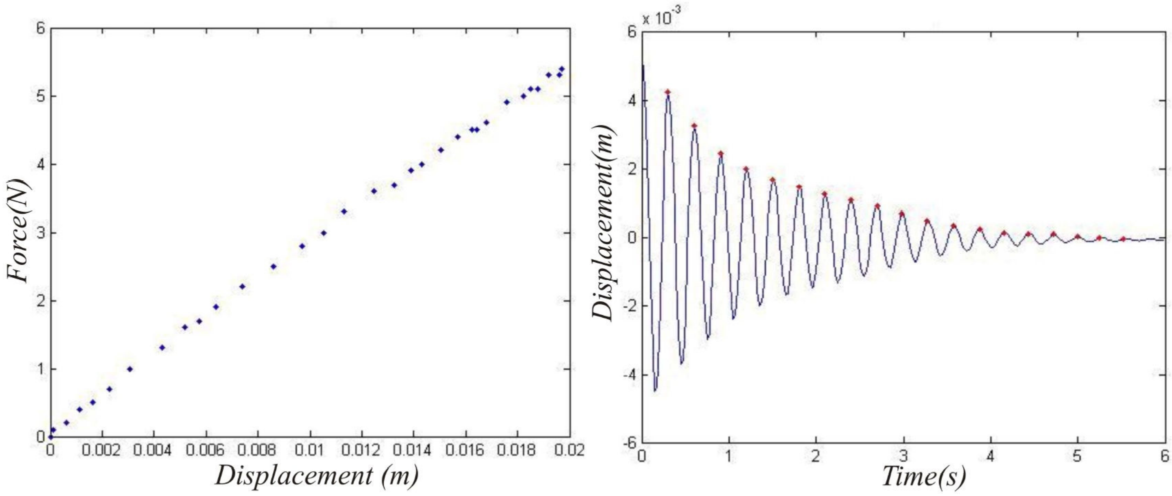


Figure 3. Experimental apparatus: (1) rotor, (2) rotating sensor, (3) spring k_1, (4) car, (5) discontinuous support, (6) rotating sensor and (7) spring k_2.

The mass of the system is measured with a weight scale, establishing a value of m=0.759kg. The stiffness of the springs are estimated by evaluating the slope of a force-displacement curve generated with the aid of two sensors: the rotary sensor and the force transducer. The force transducer is PS-2104 with full scale of 750N, accuracy of 1% and resolution of 0.003N, is employed to analyze the stiffness of the springs. This analysis establishes: k_1=5.327N/m and k_2=8.381N/m; the equivalent stiffness of the system is, therefore, k=13.708N/m. Similar procedure identifies the support stiffness as k_s=269.7N/m (Figure 4a). The support dissipation is identified connecting the support to a mass and analyzing its free response. By using a logarithmic decrement procedure (Meirovich, 2001), the viscous damping coefficient is estimated as c_s=1.35Ns/m. This technique evaluates the ratio between two consecutive displacement amplitudes (Figure 4b).



(a) (b)

Figure 4. (a) Force-displacement curve of the elastic support, (b) displacement time history.

The energy dissipation of the system has a dry friction characteristic due to the linear decrement of the free vibration response (Figure 5). The dry friction coefficient is proportional to the stiffness of the system and the mean of consecutive amplitudes from free response (De Paula *et al.*, 2005). Therefore, assuming that the viscous coefficient can be neglected, the value of the mean dry friction coefficient is μ=0.022.

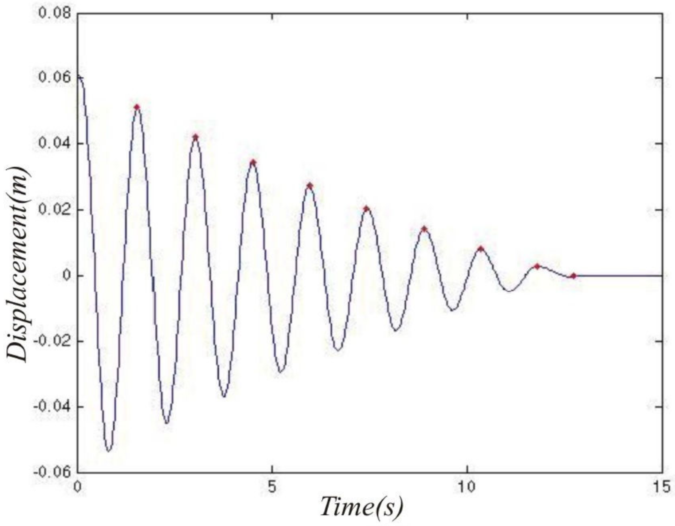


Figure 5. Free response of the system without discontinuous support.

4. DYNAMICAL ANALYSIS OF THE SYSTEM WITH LINEAR ELASTIC SUPPORT

The system with linear elastic support is now of concern establishing a comparison between numerical and experimental results. Initially, free response analysis is focused on. This is accomplished by placing a nonzero value for the initial conditions and a large value for the gap g. Under this condition, the support does not have any influence in the system response. Numerical response (green curve) is presented in Figure 6 together with experimental data (blue curve), presenting a good agreement.

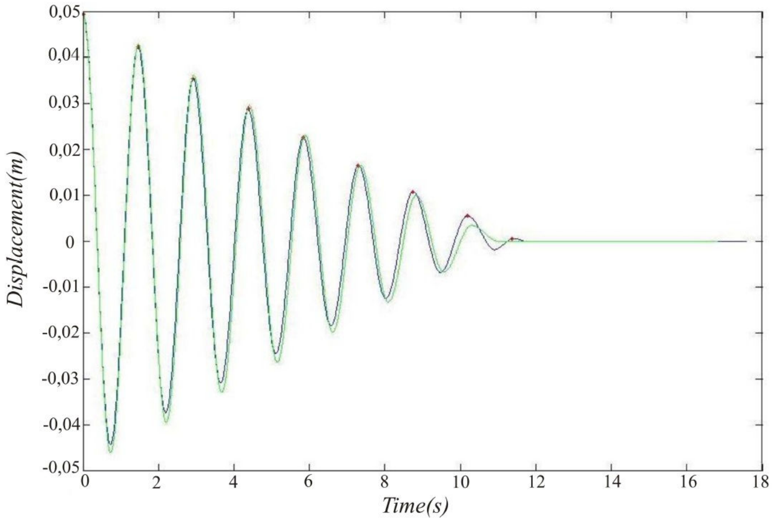


Figure 6. Comparison between numerical (green) and experimental (blue) models.

The forced response is now of concern presenting bifurcation diagrams. Basically, we evaluate the Poincaré map of the system under the slow quasi-static variation of the excitation frequency. Figure 7 presents the bifurcation diagram of the system with a gap g=7mm. Both experimental (red dots) and numerical (blue dots) results are presented where it is possible to note periodic and chaotic behaviors. Periodic response is characterized by a finite number of points while chaos is characterized by a cloud of points. Once again, it is noticeable a good agreement between numerical and experimental results.

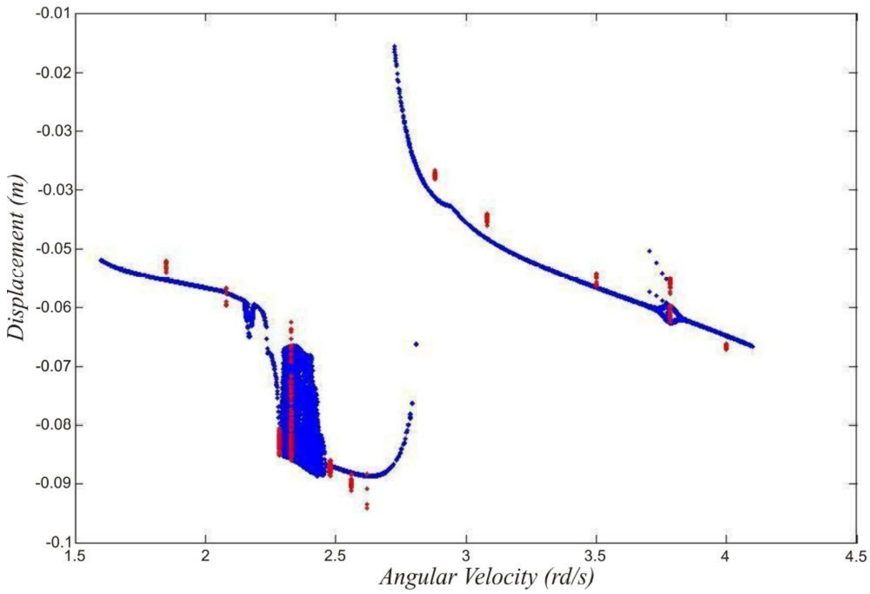


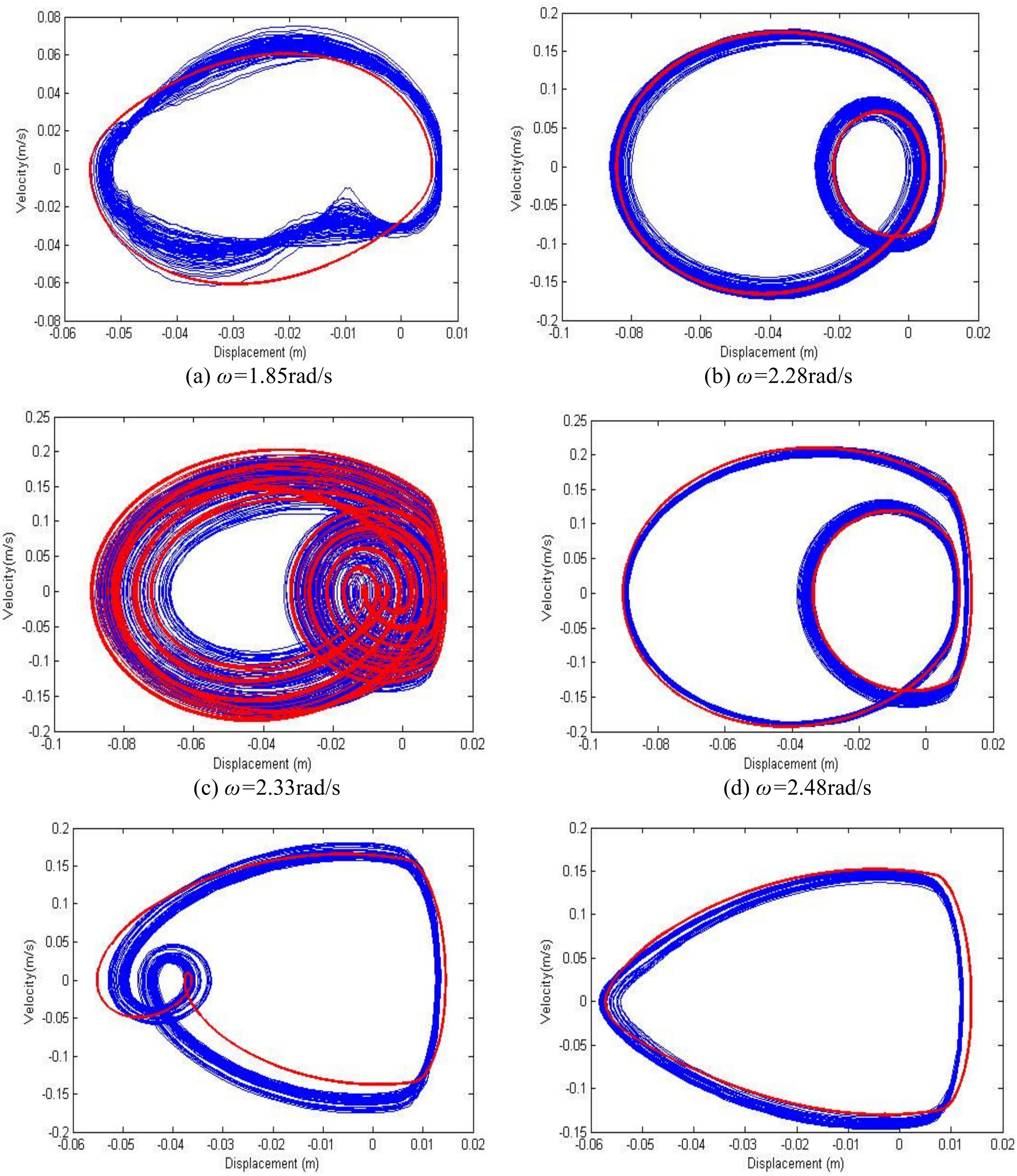
Figure 7. Bifurcation diagram: numerical (blue) and experimental (red) responses.

Different kinds of responses are now in focus by considering distinct values of the excitation frequency. Figure 8 presents phase spaces for each one of the responses, presenting a comparison between numerical (red curve) and experimental (blue curve) results. When ω=1.85 rad/s the system presents a period-1 response. Then, increasing the frequency to ω=2.28rad/s, a period-2 response occurs. By increasing the frequency to ω=2.33rad/s, a chaotic-like response occurs. Two different kinds of period-2 occurs when ω=2.48rad/s and ω=2.89rad/s. And finally, a period-1 behavior occurs when ω=3.50rad/s. All these results confirm the good agreement between numerical and experimental data.

5. SYSTEM WITH SMA SUPPORT

This section investigates the dynamical response of the system with an SMA support. Experimental tests are performed and compared with numerical results of the system with linear elastic support. The idea is to observe how the

SMA influences the system dynamics, exploring the main characteristics of the SMA thermomechanical behavior in order to promote vibration reduction.



(a) ω=1.85rad/s (b) ω=2.28rad/s

(c) ω=2.33rad/s (d) ω=2.48rad/s

(e)ω=2.89rad/s (f) ω=3.50rad/s

Figure 8. Phase space of different responses of the system with an elastic discontinuous support. Experimental (blue) and numerical (red) results.

5.1. SMA Characterization

The SMA spring is made from a Ni-Ti wire in austenitic state at room temperature and its diameter is 0.8mm. Its manufacturing is to roll up and fasten the wire in a screw and take them to an oven at 520ºC for 30 minutes. After that, the wire is cooled to room temperature in air. Phase transformation temperatures are experimentally identified using a scanning calorimeter DSC (Differential Scanning Calorimeter), Netzch DSC-200 F3, in a sample of 8.8mg of the wire. Figure 9 shows the DSC test performed in the spring sample. Note that the cooling process (black dots) establishes the start and finish temperatures of the martensitic transformation, respectively, M_s=19.0ºC and M_f=9.0ºC. During the heating (red dots) the start and finish temperatures of austenite transformation are, respectively, A_s=14.9ºC and A_f=*30*.0ºC. Based on these results, it is possible to say that this spring has a pseudoelastic behavior at room temperature (*T*=22ºC).

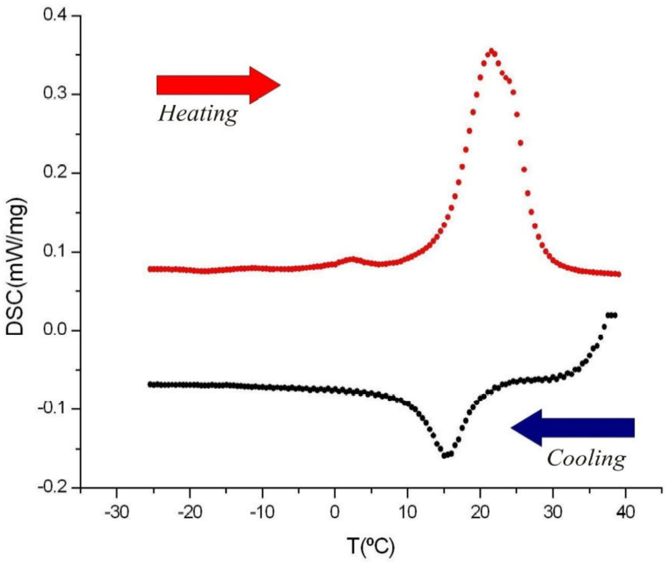


Figure 9. DSC test performed in the spring sample.

Figure 10 presents the force-displacement curve showing the pseudoelastic behavior of this spring. The analysis of the elastic response of the spring before the phase transformation establishes a stiffness k_{SMA}=23N/m. This value is used in order to analyze an equivalent linear elastic support. Besides this, it is assumed that all dissipation process different from the hysteretic one is provided by a viscous damping with coefficient c_{SMA} = 0.13Ns/m.

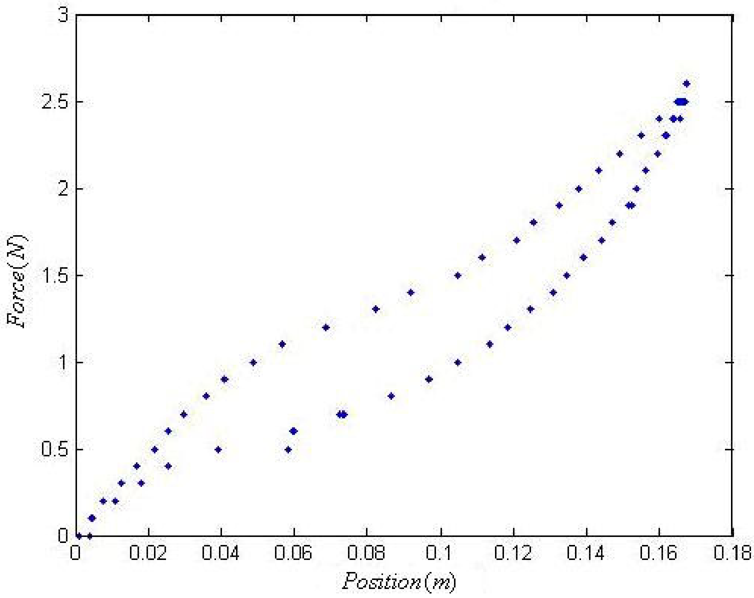
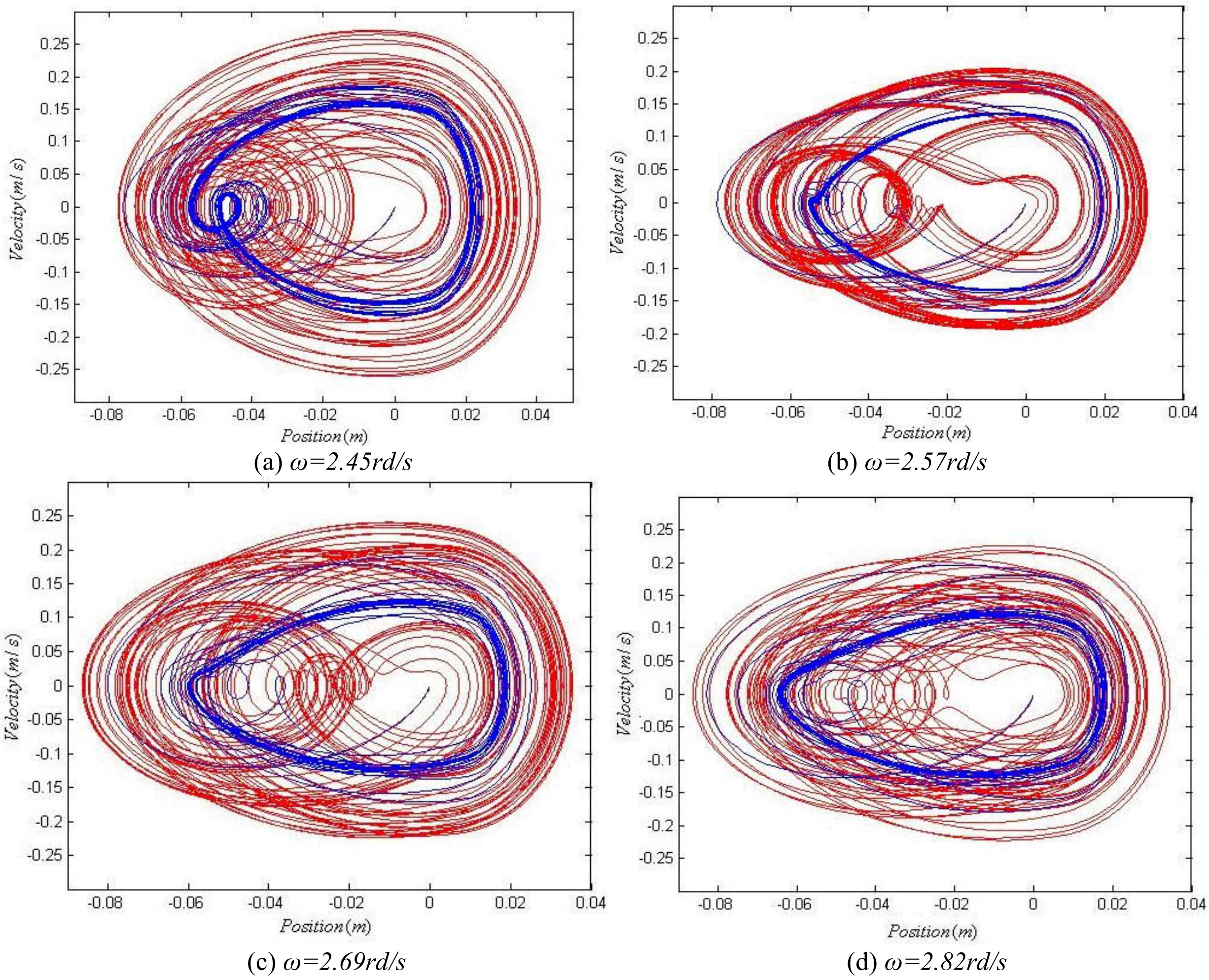


Figure 10. Force-position diagram for the SMA spring used in the support.

5.2. Dynamical Analysis of the System with SMA Support

This section investigates the influence of the SMA support on the dynamic response of the system by establishing a comparison between the system response with the one with a linear elastic support. It is expected that the high dissipative capacity of the SMA can produce less complex behaviors when compared with a linear elastic support (Santos & Savi, 2009, Sitnikova *et al.*, 2010). Once again, a gap *g*=7mm is considered with different excitation frequency.

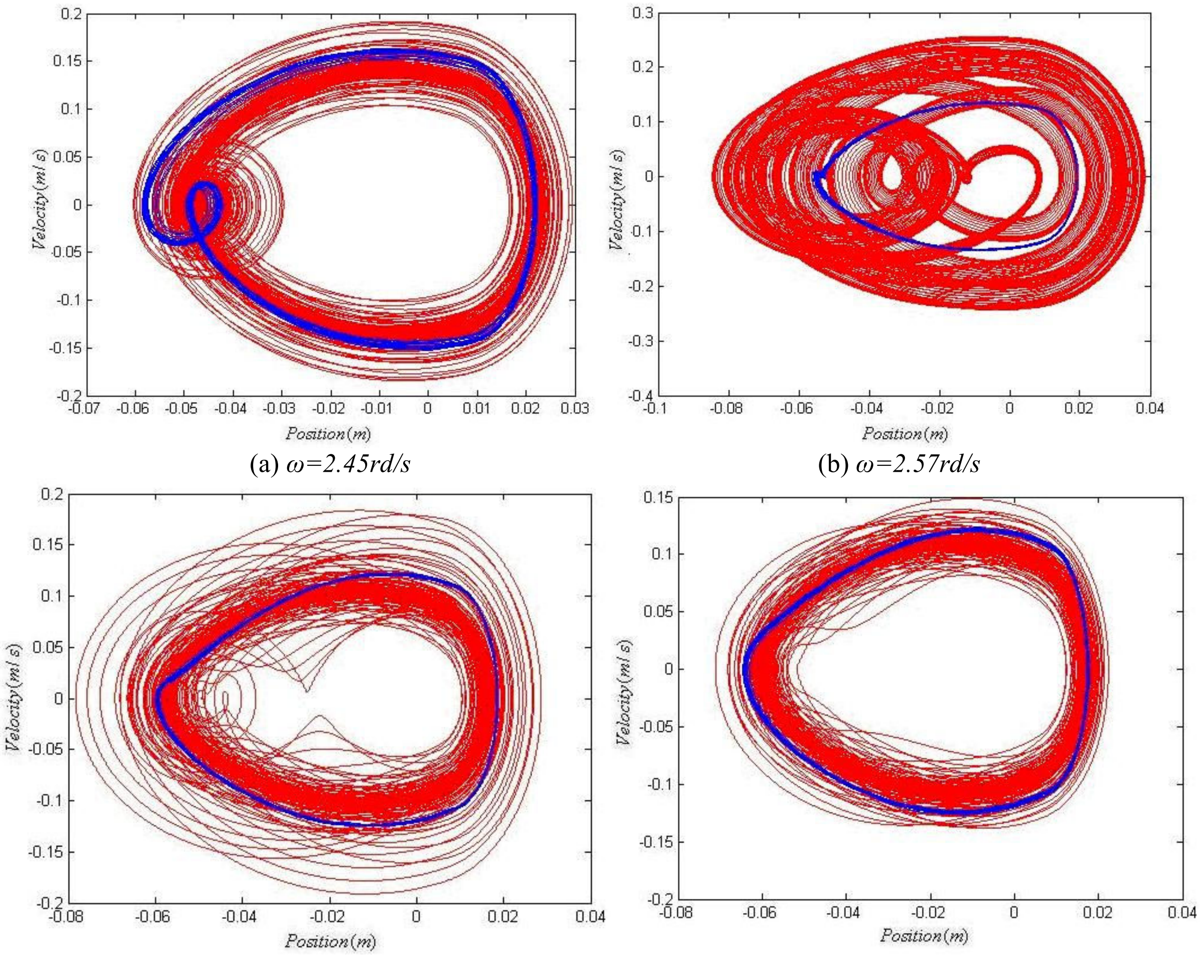
Figure 11 shows the phase space for the first 100 seconds. The response of the system with linear elastic support is obtained from numerical simulation with stiffness k_{linear}=23N/m and damping with coefficient c_{linear} = 0.13Ns/m (red curve). On the other hand, the response of the system with SMA support is obtained from experimental tests (blue curve). Note that both responses has the same behavior until the impact of the car with the SMA spring induces phase transformation. Thereafter, responses are not the same and, actually, huge changes occur. Figure 12 presents the steady-state responses of the same situations. It should be pointed out that the SMA support provides a less complex behavior, with lower amplitudes and lower transients as well. It is important to highlight an emblematic situation when ω=2.57rad/s. The linear elastic support system presents a chaotic-like response while the system with SMA support presents a period-1 behavior.



(a) ω=2.45rd/s (b) ω=2.57rd/s

(c) ω=2.69rd/s (d) ω=2.82rd/s

Figure 11. Phase spaces for the transient response: system with elastic support (red) and system with SMA support (blue).



(a) *ω=2.45rd/s* (b) *ω=2.57rd/s*

(c) *ω=2.69rd/s* (d) *ω=2.82rd/s*

Figure 12. Phase spaces for the steady state response: system with elastic support (red) and system with SMA support (blue).

6. CONCLUSIONS

This contribution deals with the analysis of a nonsmooth system with discontinuous support. Two different supports are of concern: linear elastic and SMA. Numerical and experimental approaches are treated. A single degree of freedom oscillator is analyzed as an application of this kind of system. Basically, the experimental apparatus consists of an oscillator composed by a car free to move over a rail and connected to an excitation system. The apparatus is instrumented to obtain system state variables, making possible to compare experimental results with those obtained by numerical simulations. Based on results of the linear elastic support system, it is possible to conclude that numerical and experimental tests are in close agreement. Afterward, the SMA support system is treated comparing experimental results with those obtained by numerical simulations of the linear elastic system. In general, SMA system presents a less complex behavior when compared with linear elastic system. This is due to the high dissipation capacity of SMAs. The SMA system response presents a reduction in the transient time and less amplitude behaviors. Moreover, it should be highlighted situations where chaotic response is suppressed. This behavior can be exploited in different applications including rotordynamic systems.

7. ACKNOWLEDGEMENTS

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