# Dynamical model of cargo offshore operations, vessel-crane-cargo.

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**Abstract.** Different kind of offshore operations are considered activities with high impact in the economy; container cargo ships, offshore petroleum platforms or wind farms, are some examples of possible scenarios where the offshore operations has an important role. This offshore operations has a specific process called load and unload cargo operations.

The cargo operations, especially in the area of oil and gas require high levels of accuracy and security in various stages of exploration and exploitation. Nowadays a big portion of cargo operations are controlled by human operators, unfortunately, this operators are not prepared to handled several wind or wave conditions.

In order to improve this cargo operations, is necessary to define and study the problem of the processes. Consequently to develop these technologies is mandatory to know the dynamical behavior of these cargo operations, therefore, in this work, the authors present the study carried through on the dynamics of a suspended load that is connected to a vessel's cargo manipulator.

The paper presents the complete dynamic model of the vessel-crane-cargo systems. Moreover some simulations under specific sea conditions were presented and show the complex response motion of the load due the visibly nonlinear behavior of the ship and the crane motions.

Keywords: Offshore operations; Dynamic models; Multibody System.

# 1. INTRODUCTION

Different kind of offshore operations are considered activities with high impact in the economy; container cargo ships, offshore petroleum platforms or wind farms, are some examples of possible scenarios where the offshore operations has an important role. This offshore operations has a specific process called load and unload cargo operations.

In order to improve this cargo operations, is necessary to define and study the problem of the processes. This work is a study carried through on the dynamics of a suspended load that is connected to a vessel's cargo manipulator.

The paper presents the complete dynamic model of the vessel-crane-cargo systems. Besides, the authors shows the structure and the methodology to handled with the large systems of equations in order define the complete dynamical model of the problem. Moreover some simulations under specific sea conditions were presented and show the complex response motion of the load due the visibly nonlinear behavior of the ship and the crane motions.

# 2. MATHEMATICAL MODEL

In order to construct the mathematical model was necessary to define three systems of reference, the first one was the inertial frame defined by  $X_I, Y_I$  and  $Z_I$ , the system of reference of the ship  $X_S, Y_S$  and  $Z_S$  is the ship's system of reference and finally the system of reference of the crane is defined by  $X_C, Y_C$  and  $Z_C$ . All these systems of reference can be seen in the Fig 1.



Figura 1. Systems of references

### 2.1 Ship's frame

The first step is the definition of degrees of freedom of the ship as done by Schaub (2008) and recommended by Amerongen and Breedveld (2003). For the ship has six degrees of freedom, three translational named sway(yn), heave(zn) and surge(xn) and three rotational degrees of freedom called  $pitch(\theta_{yn})$ ,  $yaw(\theta_{zn})$  and  $roll(\theta_{xn})$ Journée (2003). Figure 2 shows the degrees of freedom of the ship.



Figura 2. Ship Degrees of freedom

For further simplicity in reading and interpretation, the degrees of freedom of the ship are expressed as vector components of the vector  $Dof_N$  as presented in equation (1).

$$\mathbf{Dof_N} = \begin{bmatrix} xn \\ yn \\ zn \\ \theta_{xn} \\ \theta_{yn} \\ \theta_{zn} \end{bmatrix}$$
(1)

In order to representation and calculation of the different references used are implemented corresponding transformation matrices, that is based on homogeneous transformation matrices Barrientos (2007).

Multiplication of matrices, in the order presented in Equ. (2) where  $T_x$  and  $R_x$  means translation and rotation on x respectively, allows to transform the inertial reference frame in the local frame of the ship.

$$\mathbf{Ref}_2 = \mathbf{T}_{\mathbf{x}} * \mathbf{T}_{\mathbf{y}} * \mathbf{T}_{\mathbf{z}} * \mathbf{R}_{\mathbf{x}} * \mathbf{R}_{\mathbf{y}} * \mathbf{R}_{\mathbf{z}}$$
(2)

The elements in matrix  $Ref_2$  are presented at Equ. (3).

$$\mathbf{Ref_2} = \begin{bmatrix} C_{\theta_{yn}} * C_{\theta_{zn}} & -C_{\theta_{yn}} * S_{\theta_{zn}} & S_{\theta_{yn}} & xn \\ C_{\theta_{xn}} * S_{\theta_{zn}} + C_{\theta_{zn}} * S_{\theta_{xn}} * S_{\theta_{yn}} & C_{\theta_{xn}} * C_{\theta_{zn}} - S_{\theta_{xn}} * S_{\theta_{yn}} * S_{\theta_{zn}} & -C_{\theta_{yn}} * S_{\theta_{xn}} & yn \\ S_{\theta_{xn}} * S_{\theta_{zn}} - C_{\theta_{xn}} * C_{\theta_{zn}} * S_{\theta_{yn}} & C_{\theta_{zn}} * S_{\theta_{xn}} + C_{\theta_{xn}} * S_{\theta_{yn}} * S_{\theta_{zn}} & C_{\theta_{xn}} * C_{\theta_{yn}} & zn \\ 0 & 0 & 1 \end{bmatrix}$$
(3)

#### 2.2 Crane's Frame.

The second reference system is defined by the spatial position of the shaft of the crane, (xp,yp,zp), for the rotation of the rope in x axis and y axis  $((\theta_1,\theta_2))$  and for the length of the rope  $(L_c)$ . The vector representation of the degrees of freedom of the crane  $(Dof_{gc})$  are presented in Equ. (4).

$$\mathbf{Dof_{gc}} = \begin{bmatrix} xp \\ yp \\ zp \\ \theta_1 \\ \theta_2 \\ L_c \end{bmatrix}$$
(4)

Multiplication of the matrices in the order presented in Equ (5) allows to transform the inertial reference system in the local coordinate system of the ship.

$$Ref_{3} = T_{x2} * T_{y2} * T_{z2} * R_{x2} * R_{y2} * T_{z3}$$
(5)

The elements of the matrix  $Ref_3$  are presented in Equ. (6).

$$\mathbf{Ref_3} = \begin{bmatrix} C_{\theta_2} & 0 & S_{\theta_2} & xp - L_c * S_{\theta_2} \\ S_{\theta_1} * S_{\theta_2} & C_{\theta_1} & -C_{\theta_2} * S_{\theta_1} & yp + L_c * C_{\theta_2} * S_{\theta_1} \\ -C_{\theta_1} * S_{\theta_2} & S_{\theta_1} & C_{\theta_1} * C_{\theta_2} & zp - L_c * C_{\theta_1} * C_{\theta_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(6)

Finally the transformation matrix that takes the reference from  $Ref_3$  to the inertial frame is defined by Equ. (7).

$$\mathbf{Ref}_4 = \mathbf{Ref}_2 * \mathbf{Ref}_3 \tag{7}$$

$$\mathbf{Ref_4} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix}$$
(8)

The elements of the matrix  $Ref_4$  are presented in Equ. (9) until Equ. (24).

$$c_{11} = C_{\theta_{yn}} * C_{\theta_{zn}} * C_{\theta_2} - C_{\theta_1} * S_{\theta_{yn}} * S_{\theta_2} - C_{\theta_{yn}} * S_{\theta_{zn}} * S_{\theta_1} * S_{\theta_2}$$
(9)

$$c_{12} = S_{\theta_{yn}} * S_{\theta_1} - C_{\theta_{yn}} * C_{\theta_1} * S_{\theta_{zn}}$$

$$(10)$$

$$c_{13} = C_{\theta_{yn}} * C_{\theta_{zn}} * S_{\theta_2} + C_{\theta_1} * C_{\theta_2} * S_{\theta_{yn}} + C_{\theta_{yn}} * C_{\theta_2} * S_{\theta_{zn}} * S_{\theta_1}$$
(11)  
$$c_{14} = rn + S_{\theta_1} * (h_{24}) + C_{\theta_2} * C_{\theta_2} * (h_{24}) - C_{\theta_2} * S_{\theta_2} * (h_{24})$$
(12)

$$c_{14} = xn + S_{\theta_{yn}} * (o_{34}) + C_{\theta_{yn}} * C_{\theta_{zn}} * (o_{14}) - C_{\theta_{yn}} * S_{\theta_{zn}} * (o_{24})$$
(12)  
$$c_{24} = C_{24} * (o_{24}) + S_{24} * S_{24} * (o_{24}) + C_{24} * S_{24} * S_{24} * S_{24}$$
(13)

$$c_{21} = C_{\theta_2} * (a_{21}) + S_{\theta_1} * S_{\theta_2} * (a_{22}) + C_{\theta_{yn}} * C_{\theta_1} * S_{\theta_{xn}} * S_{\theta_2}$$
(13)

$$c_{22} = C_{\theta_1} * (a_{22}) - C_{\theta_{y_n}} * S_{\theta_{x_n}} * S_{\theta_1}$$
(14)

$$c_{23} = S_{\theta_2} * (a_{21}) - C_{\theta_2} * S_{\theta_1} * (a_{22} - C_{\theta_{yn}} * C_{\theta_1} * C_{\theta_2} * S_{\theta_{xn}}$$
(15)

$$c_{24} = yn + (b_{24}) * (a_{22}) + (b_{14}) * (a_{21}) - C_{\theta_{yn}} * S_{\theta_{xn}} * (b_{34})$$
(16)

$$c_{31} = C_{\theta_2} * (a_{31}) + S_{\theta_1} * S_{\theta_2} * (a_{31}) - C_{\theta_{xn}} * C_{\theta_{yn}} * C_{\theta_1} * S_{\theta_2}$$

$$(17)$$

$$c_{32} = C_{\theta_1} * (a_{31}) + C_{\theta_{xn}} * C_{\theta_{yn}} * S_{\theta_1}$$
(18)
$$c_{32} = S_0 * (a_{21}) - C_0 * S_0 * (a_{22}) + C_0 * C_0 * C_0 * C_0$$
(19)

$$c_{33} = S_{\theta_2} * (a_{31}) - C_{\theta_2} * S_{\theta_1} * (a_{31}) + C_{\theta_{xn}} * C_{\theta_{yn}} * C_{\theta_1} * C_{\theta_2}$$

$$c_{34} = zn + (b_{24}) * (a_{31}) + (b_{14}) * (a_{31}) + C_{\theta} * C_{\theta} * (b_{24})$$

$$(19)$$

$$(20)$$

$$c_{34} = 2n + (b_{24}) * (a_{31}) + (b_{14}) * (a_{31}) + C_{\theta_{xn}} * C_{\theta_{yn}} * (b_{34})$$

$$c_{41} = 0$$
(21)

$$c_{42} = 0$$
 (22)  
 $c_{43} = 0$  (23)

$$c_{43} = 0$$

$$b_{44} = 1$$
 (24)

The position of center of mass of the load is defined by the vector  $p^*$  defined in Equ. (25).

$$\mathbf{p}^* = [c_{14}, c_{24}, c_{34}] \tag{25}$$

Parameter	Value
Simulation time	10 s
Mass	10 kg
$Dof_N$	$[0 \ 0 \ 20 \ 0 \ 0 \ 0]$
$Dof_N$	$[0 \ 0 \ 0 \ 0 \ 0 \ 0]$
$Dof_{gc}$	$[0 \ 0 \ 10 \ \theta_1 \ \theta_2 \ 10]'$
$\dot{Dof}_{gc}$	$[0 \ 0 \ 0 \ \dot{\theta_1} \ \dot{\theta_2} \ 0]'$

Cuadro 1. Simulations condition's

After calculated the vector  $p^*$  is possible to find the values of kinetic and potential energy of the system. The kinetic energy of the body  $(K^C)$  can be calculated by the Equ. (26) Tenenbaum (2006) where  $v^*$  is the velocity of the center body mass, **I** the inertia tensor of the body and  $w^c$  the angular velocity of the body.

Likewise, the potential energy can be calculated using Equ. (27) where g represents the acceleration of gravity and h the distance from the inertial coordinate system to the body.

$$K^{C} = \frac{1}{2}m\mathbf{v}^{*} \cdot \mathbf{v}^{+} \frac{1}{2}\mathbf{w}^{c} \cdot \mathbf{I} \cdot \mathbf{w}^{c}$$
(26)

$$\Phi^C = mgh \tag{27}$$

After calculated the kinetic and potential energy of the body is calculated the Lagrangian (La). It is described by Equ. (28)Meirovithc (1970) where  $K^S$  represents Kinetic energy of the system and  $\Phi^S$  is the Potential energy

$$La = K^S - \Phi^S \tag{28}$$

Finally the Lagrangian of the load hanging on the crane which is installed on a ship is represented by the Equ. (29).

$$La_c = Bk_{19}/2 - Bk_{31}/2 - Bk_{34}/2 - Bk_{37}/2 - g * m * Bk_{22}$$
<sup>(29)</sup>

Where  $La_c$  represents the Lagrangian of the load hanging on a Cartesian manipulator installed on a ship.

## 2.2.1 Model Validation

The model presented in the previous section, the system for ship-loading crane, can be evaluated using a model previously known and studied by Baier (2008). In this case it is possible, imposing some restrictions, to compare the dynamics of the model with the dynamics of a spherical pendulum. The comparison can be made when fixing the degrees of freedom of the ship and the crane, so the only dynamics involved in proposed model is corresponding to the load.

Table 1 present the initial condition of the simulation used to study the model, in the same way, Tab. 2 present the initial values of  $\theta_1$  and  $\theta_2$  for the various tests for evaluating the model.

Number	Initial condition	
of test	$\theta_1(\text{Degree})$	$\theta_2$ (Degree)
1	0	0
2	0	80
3	0	180
4	80	80
Cuadro 2. Test specification		

One way to condense the simulations is to perform the phase diagram, this diagram one can observe the behavior of the charge degrees of freedom as well as the speeds of the same. Figure 7 presents simulations to



initial  $\theta_1$  equal to zero and variations of initial  $\theta_2$  in from 20 to 20 degrees. Figure 8 presents simulations to initial  $\theta_2$  equal to zero and variations of initial  $\theta_1$  from 20 to 20 degrees.

Finally are presented, in Fig. 9, phase diagrams for initial conditions of  $\theta_1$  equal to 10 degrees and variations of  $\theta_2$  from 20 em 20 degrees.

Was performed a second comparison of the model presented in this section against a model of spherical pendulum. For this evaluation the spherical pendulum model was developed in SimMechanics. The comparison



(b)  $\theta_2$  vs  $\dot{\theta_2}$ 

Figura 8. Phase diagram 2

of the dynamics of the two models is done through simulations with equal conditions. The conditions of simulation for the comparison were the same ones used in Tab. 2 The simulation result, with the conditions of test 4, are presented in Fig. 10.

To evaluate the influence of degrees of freedom of the ship  $(\mathbf{Dof_N})$  was implemented a new simulation. For this simulation the rotational degree of vessel  $\theta_{zn}$  was set at 10 degrees. Figure 11 presents the diagram of phase and is possible to see the displacement of the equilibrium point to 10 degrees, confirming the effect of changing value of  $\theta_{zn}$  in the dynamics of the load hanging.



Figura 9. Phase diagram 3



Figura 10. Model comparition



Figura 11. Phase diagram

# 3. CONCLUSIONS

The paper presents the complete dynamic model of the vessel-crane-cargo systems using an Lagrangian approach. Additionally are presented simulations that validate the model using external softwares, like Sim-Mechanics, showing physical concordance.

The Model presented is used to generate different phase diagram that shows the influence of  $\theta_1$  in the equilibrium position of the cargo as is presented in Fig.11.

The authors shows the structure and the methodology to handled with the large systems of equations in order define the complete dynamical model of the problem. Moreover some simulations under specific sea conditions were presented and show the complex response motion of the load due the visibly nonlinear behavior of the ship and the crane motions.

For future works, the authors will implement different kinds of control algorithms in order to guarantee the stability during the process of load transfer.

## 4. ACKNOWLEDGEMENTS

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