OPTIMIZATION OF A PIEZOELECTRIC ENERGY HARVESTING SYSTEM

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Abstract. *The search for alternative sources of energy is increasing. This has occurred for many reasons, among them the most important being the necessity to develop new sources of clean energy due to environment problems and the problem of exhaustible sources of energy in the face of a growing demand. In this context, a sector that has attracted great interest is the proposal of new efficient devices that are able to convert other types of energy into electrical energy. This area is named Power or Energy Harvesting and is based on transducers that provide changes in the energy type. The most frequenctly used devies include magnetic, electrostatic and piezoelectric transducers. This paper investigates the modelling and optimization of one of these devices that uses a piezoelectric element to convert mechanical vibration energy in electrical energy. However, when an electrical circuit is coupled to the transducer the mechanical system is strongly influenced by it. The paper proposes a methodology to model the energy harvesting considering this interaction between the mechanical and electrical system. The optmazation process uses Genetic Algorithms in order to find the optimal parameters for the device. The structure modeled of interest is a piezobeam with free-sliding boundary conditions. The results illustrated that the generated power can be maximized if some optimal conditions are set in the mechanical and in the electrical system simultaneously.*

Keywords: *Energy Harvesting, Piezoelectric Transducer, Genetic Algorithms*

1. INTRODUCTION

Energy Harvesting, Power Harvesting or Energy Scavenging concerns the conversion of ambient energy into electrical energy (electrical power). In every case, if no action were taken, this environmental energy would be wasted. Normally, the converted electrical energy is stored in a battery to be used later but sometimes the energy is used at the same time that is generated.

Energy Harvesting may be a solution for the generation of energy in remote, inaccessible or hostile environments and in applications where connection with the electrical energy network is difficult. For instance, these could be small autonomous devices are used in wearable electronics and wireless sensor networks.

The external source can be solar, wind, thermal, salinity gradients and kinetic. In this paper the source is kinetic; specifically, vibration sources. These sources could be the small vibrations of a machine, the motion of walking, and even the motion of blood circulation. Transducers mostly used for energy harvesting include electromagnetic, electrostatic and piezoelectric systems. A large discussion about principles and state-of-art in motion-driven miniature energy harvesters can be finding in Mitcheson *et al.* (2008). In this work, the harvesting of energy is through a piezoelectric transducer.

Piezoelectric transducers have the ability to convert applied strain into electrical charge. According to Cook-Chennault *et al*. (2008) it happens because when a load is applied in the material it causes a molecular deformation in the structure that causes a separation of the positive and negative gravity centers, resulting in the macroscopic polarization of the material.

Many researches have been performing studies to provide technical improvements in the area. Between them is Sodano *et al.* (2002) that performed a study to investigate the amount of power generated through the vibration of a piezoelectric plate, as well as methods of power storage. Lesieutre *et al.* (2002) that investigated the damping added to a structure due to the removal of electrical energy from the system during power harvesting. Leffeuvre *et al.* (2005) that constructed an electromechanical structure, trying to optimize the power flow of vibration-based piezoelectric energy-harvesters. Ertruk and Inman (2008) showed important characteristics of a coupled distributed parameter system, such as modal electromechanical coupling and dependence of the electrical outputs on the locations of the electrodes. Yang and Tang (2009) present a circuit model for energy harvesting, which bridges structural modeling and electrical simulation.

One problem occurs when an electric circuit is connected in the transducer because there is an interaction between the electrical and mechanical systems. This paper applies the approach described by Nakano *et al.* (2007) to capture this interaction. The model is based on a two-port network model of the transducer and is applied to a piezoelectric transducer connected to a resistive load. The dynamics of the system are modeled analytically and a Genetic Algorithm is used to determine the optimum parameters of the structure to obtain the maximum power harvested.

The section 2 presents the theory concepts adopted in this work. In the sections 3 can be find the parameters used on simulations. The section 4 and 5 show the results and the conclusions, respectively.

2. THEORY CONCEPTS REVIEW

This section present the basics description for the two-port network modeling that can be used to different structures and transducers, the piezoelectric transducer equations to be used for the case studied, the structure model used on analyses that was a piezobeam free-sliding and the Genetic Algorithm principles for the optimization process.

2.1. Two-port network model

The model of the harvesting system involves a two-port network model of the transducer connected to Thévenin equivalent systems for the vibrating structure and the electric load as shown in Fig. 1 (Nakano *et al.*, 2007).

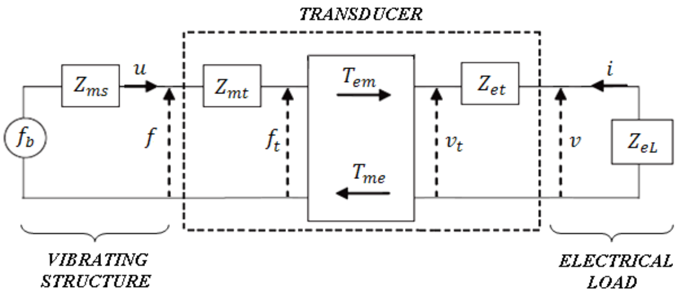


Figure 1. Two-port network model

In this figure, f_b is the blocked force, Z_{ms} is the mechanical impedance of the system, u is the velocity, f is the force acting on the transducer, Z_{mt} is the mechanical impedance of the transducer, Z_{et} is the electrical impedance of the transducer, Z_{eL} is the impedance of the external load, i is the current, v is the voltage applied to load, and T_{em} and T_{me} are the transduction coefficients. The term T_{em} represents the voltage generated per unit velocity and the term T_{me} describes the force produced per unit electric current. For the transducer, the relationship between the mechanical and electrical variables is given by

$$\begin{Bmatrix} f \\ v \end{Bmatrix} = \begin{bmatrix} Z_{mt} & T_{me} \\ T_{em} & Z_{et} \end{bmatrix} \begin{Bmatrix} u \\ i \end{Bmatrix} \tag{1}$$

where $|T_{me}| = |T_{em}|$. The voltage across the external load is given by:

$$v = -Z_{eL} i \tag{2}$$

Substituting Eq. (2) into Eq. (1) gives the current as function of velocity as

$$i = -\frac{T_{em}}{Z_{et} + Z_{eL}} u \tag{3}$$

Substituting Eq. (3) into Eq (1) gives the force as a function of velocity as

$$f = \left(Z_{mt} - \frac{T_{em} T_{me}}{Z_{et} + Z_{eL}} \right) u \tag{4}$$

The force applied to the transducer is related to the blocked force by

$$f = f_b - Z_{ms} u \tag{5}$$

Now, combining Eq. (4) and (5) gives the expression for the velocity in terms of the blocked force

$$u = \frac{f_b}{\left(Z_{mt} + Z_{ms} - \frac{T_{me} T_{em}}{Z_{et} + Z_{eL}} \right)} \tag{6}$$

Here, the power harvested is considered as the power dissipated in the electric load. Under harmonic excitation this is given by

$$P_h = \frac{1}{2} \mathrm{Re}\left\{ -i v^* \right\} \tag{7}$$

where * denotes the complex conjugate. Substituting for voltage and current from Eq. (2) and (3) into Eq.(7) results in

$$P_h = \frac{1}{2} |u|^2 \left| \frac{-T_{em}}{Z_{et} + Z_{eL}} \right|^2 \mathrm{Re}\left\{ Z_{eL} \right\} \tag{8}$$

which can be used to determine the power generated for a given input velocity.

2.2. Piezoelectric transducer

The piezoelectric transducer model showed in this work is based on previous studies by Preumont (2006) and Nakano *et al.* (2007). This model is used to find the mechanical and electrical impedances and also the transduction coefficients of the transducer. Figure 2 shows the piezoelectric transducer.

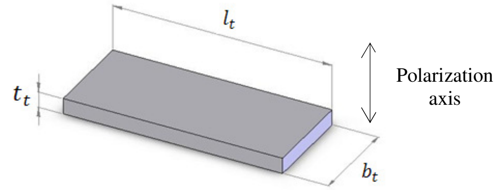


Figure 2. Piezoelectric transducer

In Fig. 2, l_t, b_t and t_t are length, width and thickness of transducer respectively. This transducer can be modeled as a uniaxial element with the constitutive equations given by (Leo, 2008)

$$\begin{Bmatrix} Q \\ S \end{Bmatrix} = \begin{bmatrix} \varepsilon^T & d_{31} \\ d_{31} & s^E \end{bmatrix} \begin{Bmatrix} E_f \\ T \end{Bmatrix} \tag{9}$$

where Q is electrical displacement, S is Strain, E_f is Electric Field, T is Stress, s^E is the compliance of the material under constant electric field, d_{31} is a piezoelectric constant, ε^T is the permittivity when the stress is constant. Assuming a harmonic force, the constitutive equation can be transformed to

$$\begin{Bmatrix} f \\ v \end{Bmatrix} = \begin{bmatrix} \frac{1}{C_{mt}} & -D_{31} \\ -D_{31} & \frac{1}{C_{et}} \end{bmatrix} \begin{Bmatrix} q_{mt} \\ q_{et} \end{Bmatrix} \tag{10}$$

where q_{mt} is the mechanical deflection and q_{et} is the electrical charge; C_{mt} is the mechanical compliance with open electrodes ($q_{et} = 0$) and C_{et} is the capacitance of the transducer for a fixed geometry ($q_{mt} = 0$) given by

$$\frac{1}{C_{mt}} = \frac{K_a}{1-\kappa^2} \tag{11}$$

$$\frac{1}{C_{et}} = \frac{1}{C\left(1-\kappa^2\right)} \tag{12}$$

in which κ is the coupling coefficient of the transducer given by:

$$\kappa = \frac{\left|d_{31}\right|}{\sqrt{s^E \varepsilon^T}} \tag{13}$$

The piezoelectric coefficient D_{31} is given by

$$D_{31} = \frac{d_{31}\, K_a}{C\left(1-\kappa^2\right)} \tag{14}$$

in which $C = \dfrac{\varepsilon^T A}{l}$ is the capacitance of the transducer with no external load $\left(f_t = 0\right)$, and $K_a = \dfrac{A}{s^E\, l}$ is the stiffness of the transducer with short-circuited electrodes ($v_t = 0$) and $A = t_t b_t$ is the cross section area. Finally, the mechanical and electrical impedances and the transduction coefficients are given, respectively, by:

$$Z_{mt} = \frac{1}{j\omega C_{mt}}(1 + j\eta_{mt}) \tag{15}$$

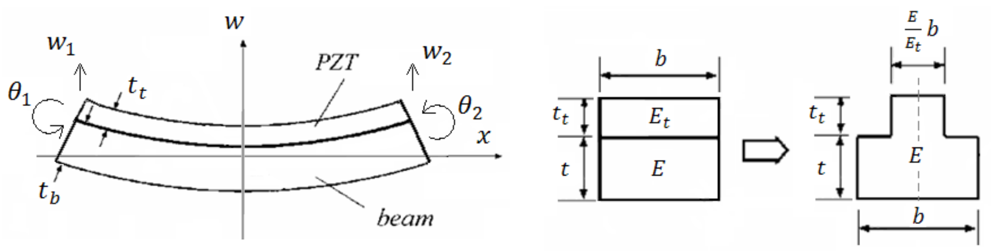
$$Z_{et} = \frac{1}{j\omega C_{et}}(1 + j\eta_{et}) \tag{16}$$

$$T_{me} = T_{em} = \frac{D_{31}}{j\omega} \tag{17}$$

where η_{mt} and η_{et} are the loss factors in the mechanical and electrical compliances.

2.3. Finite piezobeam

The system investigated in this work is a Euler-Bernoulli finite beam with a piezoelectric patch bonded on one surface. For this system it is necessary to find the uniform equivalent beam for ease of modeling. Figure 3 shows the beam and its cross-section before and after the determination of the equivalent beam.



(a) (b)

Figure 3. Piezobeam; (a) Finite element; (b) cross-section of beam and equivalent beam

2.4. Genetic algorithm

Optimization problems involve the variables determination so that an objective function reaches an extreme value (maximum or minimum) subject to some constraints. Algorithms to solve these problems type are often classified in two groups:

- classic methods based on the gradient values computation (derivatives) - provide the search direction of the algorithm (Luenberguer and Ye, 2008);
- heuristic methods - changes the optimization parameters based on random decisions (Tebaldi *et al.*, 2006).

Although the popularity of classical methods, it is often not possible to ensure that the final solution found by these strategies is actually the global optimum. This is dependent on the level of problem optimization complexity of (Levin and Lieven, 1997). In this case is usual to apply heuristic algorithms as: genetic algorithms, simulated annealing, particle swarm, and others.

Genetic algorithms are methods for search and optimization that simulate the natural process of evolution, by means of the species natural selection described by Charles Darwin (Goldberg, 1989 and Michalewicz, 1996). These algorithms are robust methods and applicable to various problems. In the general procedure of a basic genetic algorithm, in the first step, the initial population of chromosomes or individuals is created. It represents possible solutions for the problem and is codified, usually in binary code. In the next step, each chromosome is evaluated by a quality measure called fitness that is related to the objective function of the problem. After this, crossover and mutation operators are applied generating a new population of chromosomes. This process is iteratively repeated until a pre-defined stop criterion is reached or until a maximum number of generations have been reached. The flowchart of the basic genetic algorithm is showed in Fig. 4.

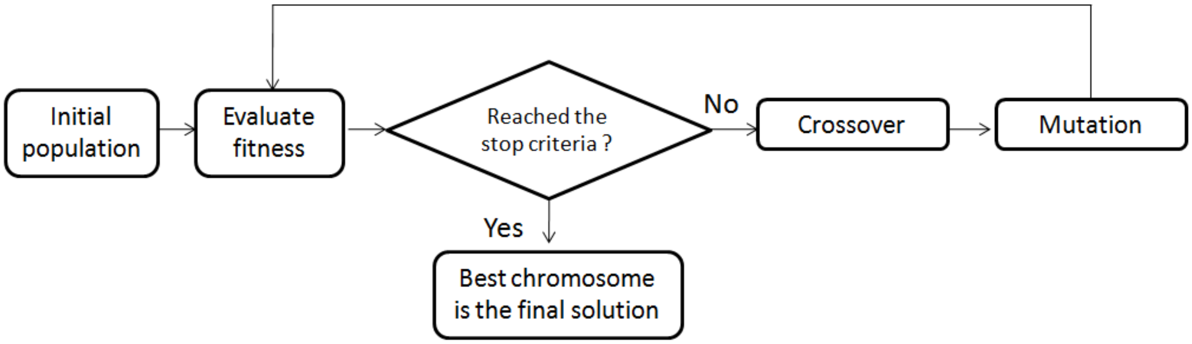


Figure 4. Flowchart of the genetic algorithm used for the parameters optimization

3. CONSIDERATIONS FOR THE OPTIMIZATION PROCESS

Of interest in this paper is the response of the piezobeam shown in Fig. 5. The input in this case is considered to be a displacement input at the root of the beam next to the sliding boundary.

Figure 5. Piezobeam free-sliding with harmonic excitation

The Frequency Response Function (FRF) of harmonic displacement W of the beam at point x_j due to a harmonic force of amplitude F being applied at point x_i can be determined by applying the boundary conditions following the procedure given in (Gardonio and Brennan, 2004) and is given by

$$\frac{W_j}{F_i} = \sum_{n=1}^{\infty} \frac{\psi_n(x_i)\psi_n(x_j)}{\rho S l\ (\omega_n^2(1+j\eta)-\omega^2)} \tag{19}$$

where $\psi_n(x)$ is the n-th natural mode, ω_n is the natural frequency for the n-th natural mode, l is the length of the finite beam and η is the loss factor for the material of the beam. The natural modes can be obtained in many text books. Here those given by Gonçalves et al (2007) are used. The analytical model is used to find the displacement in the piezobeam which is related with the power harvested give by Eq. (8).

Genetic algorithms were implemented to find the optimum parameters that give a maximum power harvested. For this analysis four parameters were chosen to the optimization process: thickness of transducer (t_t), thickness of beam (t_b), mechanical loss factor of beam (η) and electrical impedance (Z_{el}). These values were codified in binary vectors, which represent the genetic code of each chromosome. An elitist model was used which saves the best chromosomes for each generation (elite chromosome) (Michalewicz, 1996). This ensures that the best fitness of each generation remain the same or evolves over generations. The objective function in this problem, which is also the fitness function of the genetic algorithm, is given by

$$max\,J\left(p\right)=P_h\left(p\right) \tag{20}$$

where J is the objective function, p is a optimization parameters vector and P_h is the power harvested given by Eq.(8). It should be noted that the parameter vector is restricted in a range of values connected to physical meaning.

Table 1 gives the properties of system and of piezoelectric material. The parameters intervals are also shown in bold on the table. The load connected was a resistance representing a battery. The genetic algorithm used to solve this problem has the configurations showed in Tab. 2.

Table 1. Properties of the system (the range of optimization parameters are in bold).

Descriptions	**Symbols**	**Values**
Length of the beam and transducer	e	0.1 [m]
Width of the beam and transducer	e	0.02 [m]
Thickness of the transducer	t_t	**0.00127- 0.00026 [m]**
Thickness of the beam	T	**0.001 - 0.005 [m]**
Piezoelectric constant of material	d_{31}	-320 x10^{-12} [C/N]
Young's modulus of the transducer	$1/s^E$	62 [GPa]
Dielectric constant of the transducer	ε^T	3.36452x10^{-8} [F/m]
Electrical loss factor of the transducer	η_{et}	0.003
Mechanical loss factor of the transducer	η_{mt}	0.000056
Density of the transducer	ρ_t	7600 [m^3/kg]
Density of the beam	ρ	2700 [Ns/m]
Young's modulus of the beam	E	70 [GPa]
Mechanical loss factor of the beam	η	**0.001 - 0.005**
Electrical Impedance	Z_{el}	**0 - 5000 [Ω]**

Table 2: Genetic algorithm configuration

Type of selection	**Type of crossover**	**Population size**	**Participants of tournament**	**Number of iterations (generations)**	**Crossover tax**	**Mutation tax**
Tournament	One point	100	5	50	0.8	0.01

4. RESULTS

Table 3 presents the best chromosome found from five simulations using genetic algorithms in the configuration showed in Tab. 2.

Table 3: The best chromosome found (optimized parameter).

t_t[m]	t_b[m]	η	$Z_{el}\,[\Omega]$	Fitness $[W]$
0.00026	0.001	0.001	713	0.0891

The optimum beam thickness (t_b) obtained was expected because is the smallest interval value that represent more flexibility for the beam. Similar conclusions can be obtained from the parameter thickness of transducer.

Figure 6 shows the objective function for the first and the last chromosomes generations obtained from the five simulations.

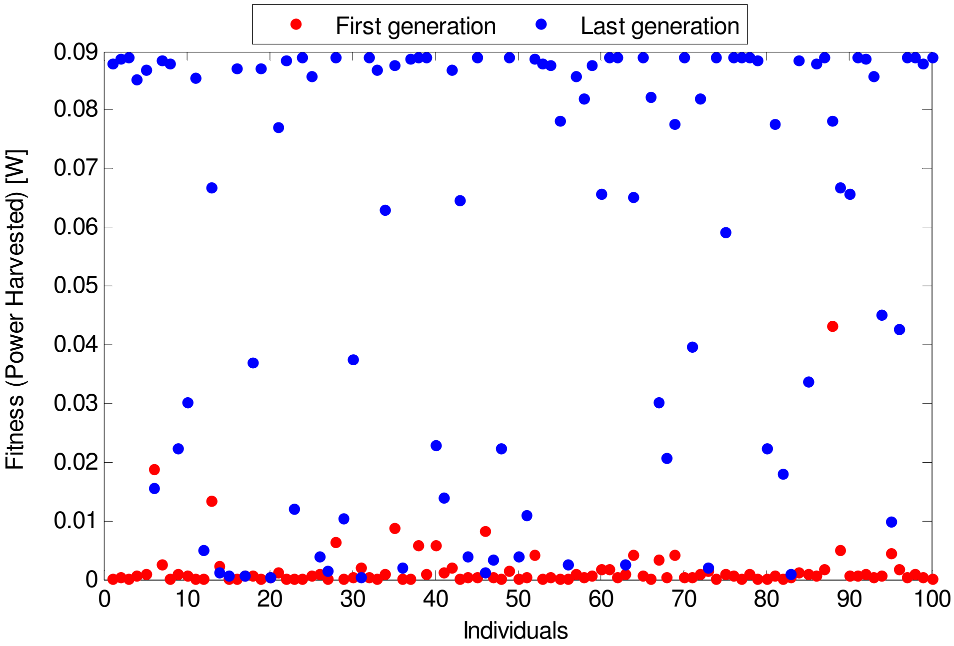


Figure 6. Evolution of populations fitness

Observing the Fig. 6 it is clear note than although the most part of first fitness generation have presented small values the genetic algorithm is able to improve the chromosomes fitness to higher values, even for fitness bigger than the biggest first generation fitness value.

Figure 7 shows the population fitness evolution over generations for the case shown in Fig. 6.

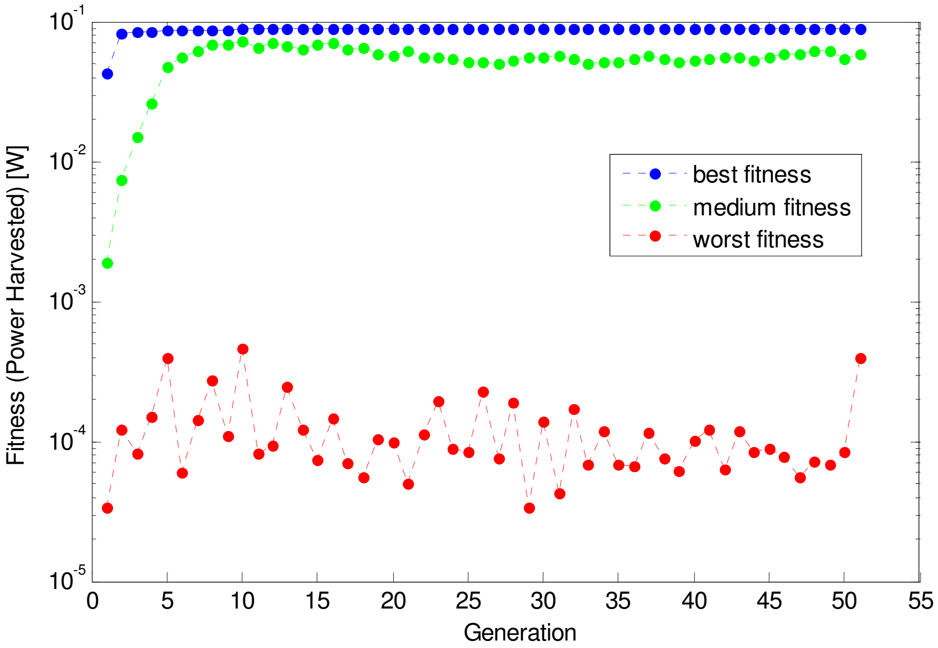


Figure 7. Evolution of Population

The blue line shows the elite chromosome fitness for each generation showing the elitist model characteristic of sustainable growth for the best chromosome fitness. Examining figure 7, it can be noted that the best chromosome of the simulation was found by the algorithm before five generations

5. CONCLUSIONS

This work aims the methodology using Genetic Algorithm to optimize the parameters a piezobeam free-sliding connected to a load (resistance) for obtain the maximum power harvested. The interaction of the mechanical and electrical system was performed using a two-port network model.

The optimum parameters for the mechanical structure and the load impedance (resistance) were determined in five different genetic algorithms examples. The simulations were performed searching five parameters optimization. The optimization results from genetic algorithm had good coherence with the analytic optimization presented by Nakano *et al* (2007). However, the relevant point is that the optimization methodology using genetic algorithm can be extended for more complex cases, including multi-objective function, since some parameters are conflicting.

The Genetic Algorithm have proved be effective for this application. The Tab. 3 shows the optimum parameters determined to this algorithm.

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