MODELS OF FAILURE AND STATE OBSERVERS IN FRAME STRUCTURES

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Abstract. *In recent years, there has been a great interest of industries in developing new techniques for detection and location of faults, because there is an increasing need to worry about security, so there is the need for supervision and monitoring systems for failures been avoided or remedied as soon as possible. Certain parameters in real systems such as mass, stiffness and damping can vary due to some failures to own or wear and tear of components. A failure generated by character cans cause economic loss or even lead to dangerous situations with abrupt stopping of machines and/or equipments. Through theoretical models methods well defined for parameter identification, state's observers and decision aid it was possible to develop a methodology to detect and locate cracks in frame structures, emphasizing the three-dimensional. It could also detect and locate the cracks already in their early stages and monitor their spread to a possible shutdown. In this paper it was used the methodology of the state's observer that cans reconstruct the unmeasured states of values from points of difficult access in the system.*

Keywords*: state observer, model of fault, detection and fault location, three dimensional frame structures.*

1. INTRODUCTION

Currently it has a growing concern about the safety of mechanical systems and need better supervision and monitoring, which requires an investment of industries growing in developing new techniques for detecting and locating faults for which these are avoided or remedied as soon as possible.

Certain parameters in real systems such as mass, stiffness and damping can vary due to some failures to own or wear and tear of components. And the appearance of cracks can cause economic loss or even lead to dangerous situations with abrupt stopping of machines and equipment.

There are great difficulties in predicting the dynamic behavior of certain structural components and diagnose faults, is the inaccuracy of the theoretical model or by the difficulty of measuring some variables of the system. But through the aid of theoretical models well defined methods for parameter identification, state observers and decision aid is possible to develop a methodology to detect and locate faults in frame structures.

It is known that in mechanical systems, the occurrence of cracks in some components may lead to unplanned downtime resulting in financial losses or even dangerous situations. By this it is necessary to identify the occurrence of cracks and analyze the involvement of the structure under study. The position of the crack and its size can be detected by changes in natural frequency and vibration modes, because when a beam is subjected to dynamic situations the crack opens and closes alternately depending on the direction of vibration causing variations of the physical parameters of the system such as stiffness.

The methodology of state observers can reconstruct the unmeasured states or values from points of difficult access in the system. This methodology is to develop a model system for analyzing and comparing the estimated output with the output measure. The difference between these two signals will produce a residue, which is used for analysis. The idea is to build a bank of monitors to oversee the process, where each observer is dedicated only to a physical parameter of the system.

2. FINITE ELEMENT METHOD

According to Meirovitch (2001), in a finite element modeling to a bar of a truss, it has that truss are structures built from sets of axial vibration rods. Through the potential energy of each element of a conservative system, considering the element of the truss as a uniform rod subjected to elastic vibration in the axial direction and the rigid body motion in the transverse direction, it's found the stiffness matrix of each element and from the kinetics energy is obtained the mass matrix.

According to Meirovitch (2001), for the system, is made a sum of each energy, made for each element of the truss and the total number of members of the truss, K is the stiffness matrix of the system, M the mass matrix of the system and a is the system displacement vector.

$$\Pi_{\max} = \sum_{i=1}^{N} \Pi_{\max,i} = \frac{1}{2}\sum_{i=1}^{N} \vec{\bar{a}}_i^T \, \underset{\sim}{\bar{K}}_i \, \vec{\bar{a}}_i = \frac{1}{2}\vec{a}^T \, \underset{\sim}{K} \, \vec{a} \tag{1}$$

$$T_{ref} = \sum_{i=1}^{N} T_{ref,i} = \frac{1}{2}\sum_{i=1}^{N} \vec{a}_i^{T} \underset{\sim}{\overline{M}}_i \vec{a}_i = \frac{1}{2}\vec{a}^{T} \underset{\sim}{M} \vec{a} \tag{2}$$

3. STATE OBSERVERS

The state observers were initially proposed and developed by Luenberger (1964), the simplicity of its design and its resolution of the observer made an attractive component of the overall project, mainly due to the fact reconstruct unmeasured states. His theory is closely related to fundamental concepts of controllability and observability.

The state of a system is a mathematical structure consisting of a set of n variables $x_1(t)$, $x_2(t)$,... , $x_n(t)$, called *state variables*, such that the initial values $x_i(t_0)$ of this set and excitation system $u_i(t)$ are sufficient to uniquely describe the system's response to all $t \geq t_0$.

A system is called completely controllable if, whatever the initial instant t_0, it can transfer any initial state $\{x(t_0)\}$ for any final state $\{x(t)\}$ in a finite time $t_f>t_0$ by a vector excitation $\{u(t)\}$ subject to no restriction (D'AZZO; HOUPIS, 1988). And a system is called completely observable if every initial state $\{x(t_0)\}$ can be accurately determined from measurements of the response $\{y(t)\}$ for a finite time interval $t_0 \leq t \leq t_f$ (D'AZZI; HOUPIS, 1988).

For a linear time-invariant, it considers the following representation of the state variable of the system for describing the state observer:

$$\{\dot{x}(t)\} = [A]\ \{x(t)\} + [B]\ \{u(t)\} \tag{3}$$

$$\{y(t)\} = [C_{me}]\ \{x(t)\} + [D]\ \{u(t)\} \tag{4}$$

where $\{x(t)\} \in R^{nx1}$ is the state vector, $\{u(t)\} \in R^{px1}$ is the input vector (excitation vector), $\{y(t)\} \in R^{kx1}$ is the output vector, $[A] \in R^{nxn}$ is the dynamic matrix of the system, $[B] \in R^{nxp}$ is the distribution matrix, $[C_{me}] \in R^{kxn}$ the array of measures and e $[D] \in R^{kxp}$ is a constant matrix, and n the order of the system, p is the number input $\{u(t)\}$, and k the number of outputs $\{y(t)\}$.

One advantage of this type of representation is that the state vector $\{x(t)\}$ contains enough information to completely summarize the past behavior of the system, and future behavior is governed by a simple first order differential equation.

According to Meirovitch (1990), a state observer for the system of Eq. (3) is given as follows:

$$\{\dot{\hat{x}}(t)\} = [A]\ \{\hat{x}(t)\} + [B]\ \{u(t)\} + [L]\ (\{y(t)\} - \{\hat{y}(t)\}) \tag{5}$$

$$\{\hat{y}(t)\} = [C_{me}]\ \{\hat{x}(t)\} \tag{6}$$

where $[L]$ is the matrix of the observer status, according to Wang, Kuo and Hsu (1987), the matrix $[C_{me}]$ is assumed to be of full rank, i.e. the position it should be equal to the order of system.

Working with systems where parameter variations are observed, it has:

$$\{\dot{x}(t)\} = [A+\Delta A].\ \{\hat{x}(t)\} + [B]\ \{u(t)\} \tag{7}$$

$$\{y(t)\} = [C_{me}]\{x(t)\} + [D]\{u(t)\} \tag{8}$$

where $[A + \Delta A]$ is the dynamic matrix of the system with parameter variations.

Therefore, a state observer for the system of Eq (7) is given by:

$$\{\dot{\hat{x}}(t)\} = [A + \Delta A]\left\{\hat{x}(t)\right\} + [B]\{u(t)\} + [L]\left(\{y(t)\} - \left\{\hat{y}(t)\right\}\right) \tag{9}$$

$$\left\{\hat{y}(t)\right\} = [C_{me}]\left\{\hat{x}(t)\right\} \tag{10}$$

where $\{\hat{x}(t)\}$ is the state vector estimated by the observer and $\{\hat{y}(t)\}$ denotes the output vector of the observer status.

From this idea, the design of state observers is divided into two parts: the first is the assembly of the global observer and the second is the assembly of robust observers for possible parameters which can fail. In assembling the global observer is used the same array of dynamic mechanical system in question. Thus, when the system is functioning properly without signs of failure, the global observer will respond exactly like the real system. Otherwise, the viewer response will not be equal, thus being able to detect a possible failure or irregularity in the system. In the second stage, is taken a rate of loss variation of each parameter of each robust observer to the parameter subject to failure before their assembly, so the response of the robust observer that approaches the system response with evidence of failures will be the observer responsible for this location of this possible system failure. On the possibility of one or more parameters fail at the same time, the solution is to design robust state observers to all parameters subject to failures, (MELO, 1998).

In "Fig. 1" shows the scheme set up to detect and locate faults in mechanical systems with varying parameters using the technique of state observers. This observation system consists of a real system, by an excitation force vector *{u(t)},* a system response vector *{y(t)},* a bank of observers, a unit of logical precision, plotting of graphs and results.

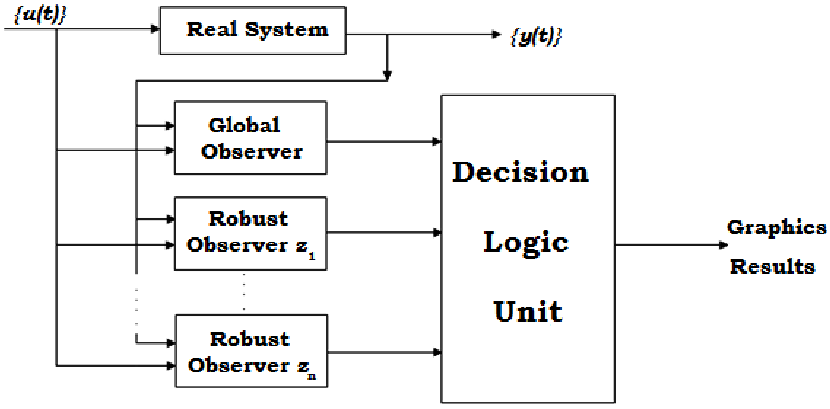

Figure 1. Observation system, Marano (2002)

3. MODEL ANALYSIS OF CRACKS

The presence of a crack in the beam, according to the principle of Saint Venant, causes a disturbance in the stress distribution around the crack. This condition is especially relevant when the crack opens and causes a local reduction of stiffness. According to Muscolino (2003) the structural properties of the stiffness matrix are more affected by the opening of the crack. Undamaged beam elements were modeled by Euler-Bernoulli using finite element with two nodes and two degrees of freedom (transverse displacement and rotation) per node. The cracked element can be modeled as an element uncracked if it is closed.

It's possible to determine the stiffness matrix of an element without crack and of the cracked element, through the Euler Bernoulli theory and the virtual work principle. Considering that a crack may reach a depth of up to 40% of the height of the beam, the expression of matrix cracked element is an explicit function of parameters involved curves obtained for various values of crack depth ($\alpha_1, \alpha_2, \alpha_3$ and α_4), being of the form:

$$k_c = \alpha_1 \begin{bmatrix} k_{11}\alpha_2 & k_{12}\alpha_2 & k_{13}\alpha_2 & k_{14}\alpha_2 \\ & k_{22}\alpha_3 & k_{23}\alpha_2 & k_{24}\alpha_4 \\ & & k_{33}\alpha_2 & k_{34}\alpha_2 \\ sim & & & k_{44}\alpha_3 \end{bmatrix} \tag{11}$$

The dynamic response of the beam in the time interval in which the crack is closed can be considered, for simplicity, as the beam without a crack because the crack interfaces interact with each other completely. Under the action of the excitation force, the opening and closing of cracks will alternate in function of time.

The equation of motion of the cracked beam discretized for N_e finite element and subjected to an external excitation vector $f(t)$ can be written by:

$$\begin{gathered} M\ddot{u}(t) + C\dot{u}(t) + (K_u - \gamma\Delta K)u(t) = f(t) \\ u(0) = u^0 \; // \; \dot{u}(0) = \dot{u}^0 \end{gathered} \tag{12}$$

where M is the mass matrix, C the damping matrix, $(K_u - \gamma\Delta K)$ the stiffness matrix $\Delta K = K_u - K_c$ and $\gamma = 1$ when the crack is open and $\gamma = 0$ when the crack is closed.

It considered $\gamma = 1$, because during the period in which the crack remains closed $(\gamma = 0)$ the stiffness matrix is composed of only the portion where the crack is not considered, so, in this moment, there is no existence of failure.

For a beam truss in three dimensions analyzing separately each member, in which each member has six degrees of freedom.

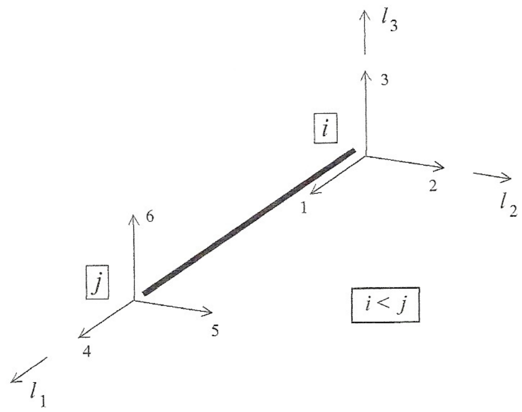

Figure 2. 3D Truss: Six degrees of freedom of bar *ij* in the reference site, (Azevedo, 2003)

So for the three-dimensional beam, together with Eq. (11) gives the expression of matrix element cracked, getting this:

$$k_c = \alpha_1 \begin{bmatrix} k_{11}\alpha_2 & k_{12}\alpha_2 & k_{13}\alpha_2 & k_{14}\alpha_2 & k_{15}\alpha_2 & k_{16}\alpha_2 \\ & k_{22}\alpha_3 & k_{23}\alpha_2 & k_{24}\alpha_4 & k_{25}\alpha_2 & k_{26}\alpha_3 \\ & & k_{33}\alpha_2 & k_{34}\alpha_2 & k_{35}\alpha_2 & k_{36}\alpha_2 \\ & & & k_{44}\alpha_3 & k_{45}\alpha_2 & k_{46}\alpha_4 \\ & & & & k_{55}\alpha_2 & k_{56}\alpha_2 \\ sim & & & & & k_{66}\alpha_3 \end{bmatrix} \tag{13}$$

4. METHODOLOGY

Methodologies were developed for detection and location of failures in frame structures using the technique of state observer for the detection of cracks. First step was the creation of the truss developed using the software *Ansys*, after, the analysis by the insertion of random cracks in structural members. Then extracted the matrices of mass and stiffness of these systems, and through them, obtained the real system and the mathematical model to calculating the state observers, through the software *Matlab for Windows*, and from the response of the system of measures matrices and the response of the state observer it was obtained, by comparing the signals, the response to the detection and location of the faults inserted.

5. NUMERICAL EXPERIMENT

The example used was a three-dimensional truss with 36 elements, with 15m in height, with the boundary condition, its nodes 1, 4, 9 and 12 supported, with restrictions on movements in the directions *x*, *y* and *z*. In this example It considered the following material properties: Young's modulus (E) of 210 GPa, cross-sectional area (A) of 2.5 x10^{-3} m^2, density (ρ) 7860kg/m^3, cracked element length (L) 7,07m, height of it h=0,0282m, with a time interval of 0 to 0.1 second.

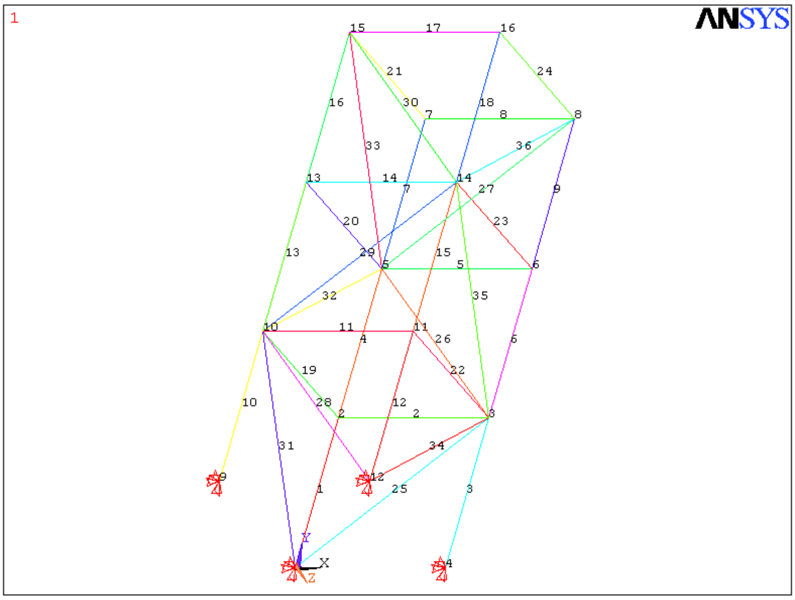

Figure 4. Example of a three-dimensional truss

Two cracks were inserted simultaneously in the truss, randomly, a crack depth of 20% of the height of the beam in element 18 and another 25% in element 29. It was given an impact speed of 5 m/s at node 16 in the negative direction of the *y*-axis. The "Figures 5, 6, 7, 8" shown the graphs of the simulation, in the ordinates are showed the displacement measured values to the second degree of freedom and their values reconstructed for the global observer, and in the abscissas the time in seconds.

Chart 1 - System Without Fail

Chart 2 - Failed System

Chart 3 - Failed System

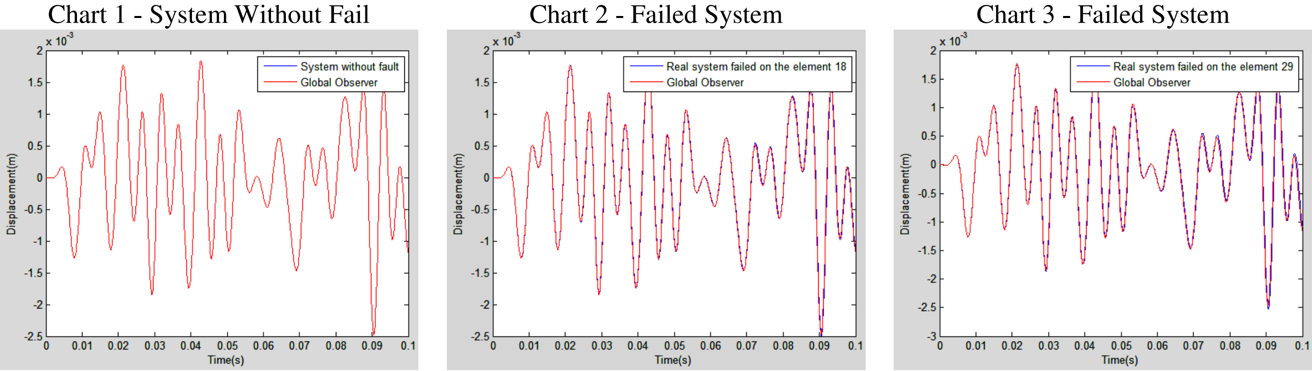

Figure 5. Results related to global observers

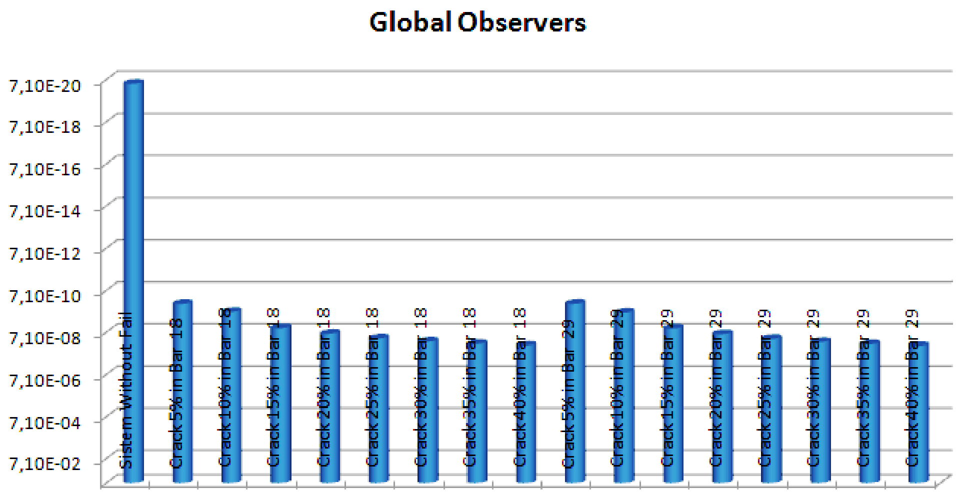

Figure 6. Difference of RMS of displacement values of the second degree of freedom with the global observer

It can be verified from "Fig. 5", which presents the results concerning the global observers, that in the first graph it first considered a system "without fault", which was the original system, since their curves were coincided, i.e., the global observer didn’t detect any irregularity in the system, since it behaves exactly like the real system without fail. Once inserted two cracks, being a 20% depth of the height of the beam in element 18 and another 25% of depth in the element 29, which have been the second and third graphic of the same figure the curves did not coincide. The second graphic refers to the global observer for the element 18, and the third graphic is related to the global observer of element 29, both the graphics don’t have their curves coincide with the real system, i.e., they detected a possible crack. Confirmed with the "Fig. 6", which presents the differences in RMS (Root Mean Square) of the values of the real system with and without fail, and it appears that the system without fail there is almost no difference in RMS, while inserted the cracks these differences increased, detecting faults in the system. The “Tab. 1” shows the results of simulations for all parameters and calculating the differences of RMS of values between the real system and global observer.

Table 1. Difference of RMS of displacement values of the second degree of freedom with the global observer with crack depth from 5% to 40% of the height

	Global Observer $(\neq RMS)$
System Without Fail	1.107e-019
System Failure	
Crack 5% in Bar 18	3.084e-009
Crack 10% in Bar 18	7.422e-009
Crack 15% in Bar 18	4.501e-008
Crack 20% in Bar 18	8.310e-008
Crack 25% in Bar 18	1.396e-007
Crack 30% in Bar 18	1.959e-007
Crack 35% in Bar 18	2.478e-007
Crack 40% in Bar 18	2.994e-007
Crack 5% in Bar 29	3.036e-009
Crack 10% in Bar 29	8.113e-009
Crack 15% in Bar 29	4.721e-008
Crack 20% in Bar 29	8.681e-008
Crack 25% in Bar 29	1.456e-007
Crack 30% in Bar 29	2.043e-007
Crack 35% in Bar 29	2.607e-007
Crack 40% in Bar 29	3.167e-007

Looking at “Tab 1”, by comparing the real system without fail with the global observers, it sees that the system without fail there is almost no difference in RMS value again, concluding that the curves are coincident, whereas when inserted the cracks, the RMS differences increase, and the curves move away, detecting the cracks.

After the cracks were detected, the next step is to locate them through robust observers, to know what sizes of their depths and in which elements was happened them. For this, it mounts a robust database of observers’ to all possible parameters of the system prone to failure, which includes robust observer for the values of crack depth and for all elements of the truss. The graph which shows the coincident curves means that the real system behaves exactly like the simulated failed system, so finding the location and size of the crack. The "Figure 7" presents examples of results for the robust observers. The observer t2018 represents the robust observer regarding the crack depth of 20% of the height of the beam in element 18, observer t4018, the observer regarding the depth of 40% in the same element 18, observer t0529, the observer regarding the depth of 5% of the element 29 and observer t2529, the observer for 25% in element 29.

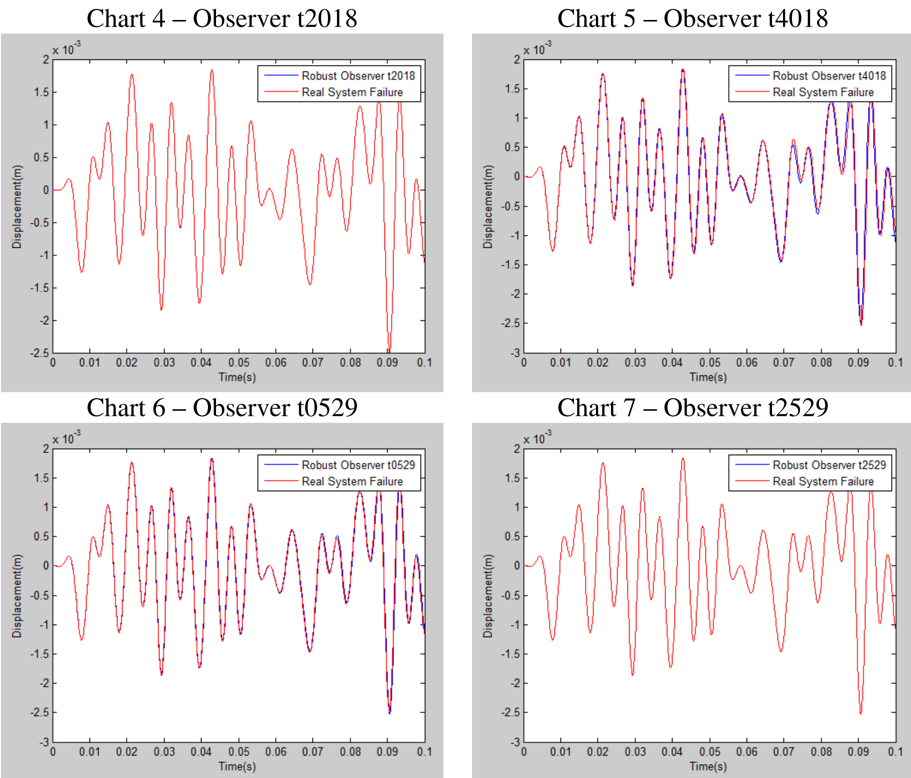

Figure 7. Examples of results obtained for robust observers

It can check that the chart 4 of "Fig. 7 " and the chart 7 on the same figure have coincident curves, meaning that the actual failed system behaved exactly as robust observers responsible for the crack depth of 20% of the height beam in element 18 and the crack depth of 25% of the height of the beam in element 29, may be concluded that those places are the location of the fault and these depths, respectively, i.e., robust observers located the faults, the system has a crack depth of 20% of the height of the beam in the truss element 18 and a crack depth of 25% in element 29. Can also be confirmed by the RMS difference, shown in "Figure 8" between the real system failure and robust observers, these differences are exactly minimum where the cracks were placed, a crack depth of 20% in element 18 and another 25% in element 29.

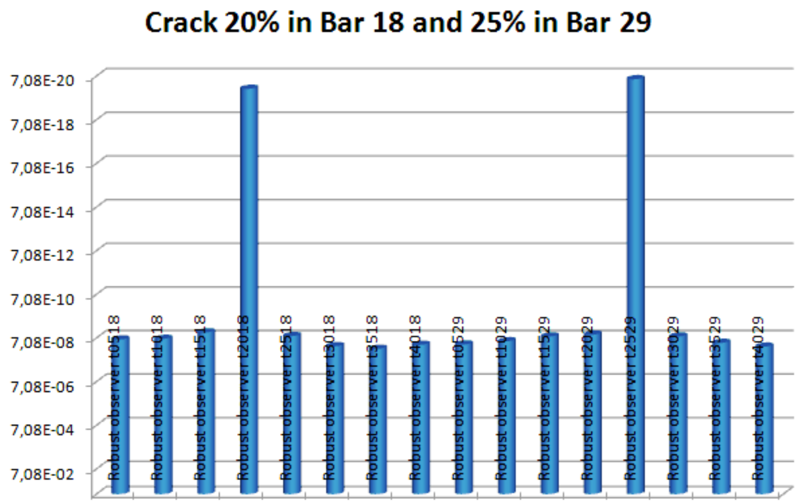

Figure 10. Difference of RMS between the real failed system and the robust observers for a crack depth of 20% of the height in the beam in the element 18 and 25% in element 29

7. CONCLUSIONS

The purpose of this study was the use of model failures and state observers in frame structures, and through this research it was noted in studies that the technique of state observers using a small number of measures with the reconstruction of the states, even in a system with many degrees of freedom this technique is very effective, detecting and locating faults perfectly.

This method was able to detect faults introduced by the sensor in the first node of the structures, which helps a lot in real systems, which can be large and difficult access points, in addition to this methodology was able to detect broken different depths and different elements simultaneously.

8. ACKNOWLEDGMENT

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