SOLUTION OF INVERSE RADIATIVE TRANSFER PROBLEMS WITH THE BEES COLONY ALGORITHM

Altamirando Colombo Ribeiro Neto, altamirando_neto@hotmail.com
Fran Sérgio Lobato, fslobato@feq.ufu.br
School of Chemical Engineering, FEQUI,
Valder Steffen Jr, vsteffen@mecanica.ufu.br
School of Mechanical Engineering, FEMEC,
Universidade Federal de Uberlândia, UFU, P.O. Box 593, 38400-902, Uberlândia-MG, Brazil.
Antônio José da Silva Neto, ajsneto@iprj.uerj.br
Department of Mechanical Engineering and Energy, Instituto Politécnico, IPRJ,
Universidade do Estado do Rio de Janeiro, UERJ, P.O. Box 97282, 28601-970, Nova Friburgo-RJ, Brazil.

Abstract. *A large increase in the use of the observation of nature, mainly the behavior of social insects or animals, in the development of new optimization algorithms has been observed in recent years. Among these algorithms, we can cite the Bees Colony Algorithm (BCA), which is based on the bees colony behavior in their search for food aiming at honey production. This approach combines global random search (through the performance of scout bees) with local search (by accentuated exploration of promising regions of the search space) to solve the optimization problem. However, due to the architecture of the canonical algorithm described in the literature, the stagnation of the population around a local optimum represents a limitation of the technique. In this context, the goal of this work is to propose a strategy to prevent the stagnation of the population and, consequently, to increase the capacity of escaping the local optima. The proposed algorithm is then applied to the estimation of radiative properties in a one-dimensional participating medium, being the radiative transfer phenomena modeled by an integro-differential equation, i.e., the Boltzmann equation. This equation describes mathematically the interaction of the radiation with the participating medium, i.e., a medium that may absorb, scatter and emit radiation. Test cases are presented for illustrating the efficiency of the proposed methodology in the treatment of inverse radiative transfer problems.*

Keywords: *Inverse Problem, Radiative Transfer, Bees Colony Algorithm, Refinement Strategy.*

1. INTRODUCTION

Nowadays, biological systems have contributed significantly to the development of new optimization techniques. These techniques are based on the use of strategies to upgrade a population of candidates so that a solution to the optimization problem is provided, which differentiates them from various techniques normally presented in the literature (Lobato *et al.*, 2010a). Among the most recent bio-inspired strategies stands the Bees Colony Algorithm (BCA) proposed by Lucic and Teodorovici (2001) for solving combinatorial optimization problems. This algorithm is based on the behavior of bees' colonies in their search of raw materials for honey production. According to Lucic and Teodorovici (2001), in each hive groups of bees (called scouts) are recruited to explore new areas in search for pollen and nectar. These bees, returning to the hive, share the acquired information so that new bees are indicated to explore the best regions visited in an amount proportional to the previously passed assessment. Thus, the most promising regions are best explored and eventually the worst end up being discarded. This cycle repeats itself, with new areas being visited by scouts at each iteration.

Among the main applications of bio-inspired methods we can cite the solution of inverse problems. In this context, the inverse analysis of radiative transfer in participating media has numerous practical applications, such as the one-dimensional plane-parallel (Silva Neto and Özişik, 1995; Alvarez Acevedo *et al.*, 2004, Lobato *et al.*, 2008; Lobato *et al.*, 2009; Lobato *et al.*, 2010b) and two-dimensional media (Carita Montero *et al.,* 2001; Carita Montero *et al.,* 2004), and radiative transfer in composite layer media (Siegel and Spuckler, 1993; Wang *et al.*, 2002), which are devoted to applications in scientific and technological areas that are related to environmental sciences (Hanan, 2001), parameter estimation (Sousa *et al.*, 2007), and tomography (Kim and Charette, 2007).

The main difficulty found in the so-called bio-inspired methods is the high number of objective function evaluations and the probability of getting stuck in a local optimum because of the architecture of these canonical algorithms. To overcome this difficulty the present work proposes the incorporation of an operator to the BCA to refine the current optimal solution so that local minima can be avoided.

In the present contribution the BCA is used for the solution of the inverse radiative transfer problem related to the simultaneous estimation of the optical thickness, single scattering albedo and the intensities of the isotropic external sources of radiation incident at τ=0 and $\tau=\tau_o$, respectively. The results obtained with this methodology are compared with those from the Differential Evolution (DE) and Simulated Annealing algorithms. This work is organized as follows. The mathematical formulations of the direct and inverse problems are presented in Sections 2 and 3, respectively. A review of the BCA and the refinement operator proposed are presented in Section 4. The results and discussion are described in Section 5. Finally, the conclusions and suggestions for future work conclude the paper.

2. MATHEMATICAL FORMULATION OF THE DIRECT PROBLEM

Consider a one-dimensional gray homogeneous participating medium of optical thickness τ_o, with transparent boundary surfaces that are subjected to external radiation. The mathematical formulation for such a problem considering no emission inside the medium and azymuthal symmetry is given by an integro-differential equation, known as Boltzmann equation (Özişik, 1973; Silva Neto and Moura Neto, 2005, de Abreu, 2005):

$$\mu \frac{\partial I}{\partial \tau}(\tau,\mu) + I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')d\mu' \tag{1}$$

with $0 < \tau < \tau_o$ and $-1 \le \mu \le -1$ and subject to the boundary conditions:

$$\begin{cases} I(0,\mu) = A_1 & \text{for } \mu>0 \\ I(\tau_o,\mu) = A_2 & \text{for } \mu<0 \end{cases} \tag{2}$$

In this equation, $I(\tau,\mu)$ is the intensity (radiance) of the radiation field, τ the optical variable, μ the cosine of the polar angle, ω the single scattering albedo, and A_1 and A_2 are the intensities of the isotropic external sources of radiation incident at τ=0 and $\tau = \tau_o$, respectively, according to Fig.(1).

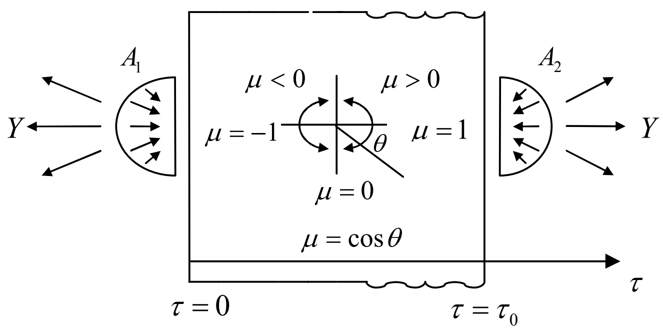

Figure 1. One-dimensional participating medium.

In the direct problem defined by Eqs. (1) and (2) the radiative properties and boundary conditions are considered as being known. Then the problem becomes the one of determining the radiation intensity $I(\tau,\mu)$. In order to solve the direct problem, the Collocation Method (Villadsen and Michelsen, 1978; Wylie and Barrett, 1985) was used. In this methodology, the general Boundary Value Problem (BVP) is described as

$$\ddot{y} = f(x, y, p), \qquad a \le x \le b \tag{3}$$

where x is the independent variable, y is a vector of dependent variables and p is a vector of unknown parameters. This BVP, subject to general nonlinear, two-point boundary conditions

$$g(y(a), y(b), p) = 0 \tag{4}$$

is approximated by a polynomial function ($S(x)$) on each subinterval $[x_n, x_{n+1}]$ of a mesh $a = x_o < x_1 < \ldots < x_N = b$. This approximation should satisfy the boundary conditions

$$g(S(a), S(b)) = 0 \tag{5}$$

and satisfies also the differential equations at both ends and at the midpoints of each subinterval

$$\dot{S}(x_n) = f(x_n, S(x_n)) \tag{6}$$

$$\dot{S}((x_n + x_{n+1})/2) = f((x_n + x_{n+1})/2, S((x_n + x_{n+1})/2)) \tag{7}$$

$$\dot{S}(x_{n+1}) = f(x_{n+1}, S(x_{n+1})) \tag{8}$$

In the context of this work, the integral terms found in the right hand side of Eq.(1) were substituted by Gauss-Legendre Quadratures (Wylie and Barrett, 1985). The Collocation Method is formally derived by evaluating the governing integro-differential equation at the collocation points, which results in a system of nonlinear ordinary differential-algebraic equations describing the evolution of the solution at the collocation points. This methodology is very attractive due to its easiness of implementation, even when the problem to be solved is highly nonlinear (Villadsen and Michelsen, 1978; Wylie and Barrett, 1985).

3. MATHEMATICAL FORMULATION OF THE INVERSE PROBLEM

In this work, the inverse problem can be stated as: utilizing the measured data $\{Y_i\}$, i=1, 2, ..., K, determine the vector of unknowns Z defined as:

$$\vec{Z} = \{\tau_o,\ \omega, A_1, A_2\}^T \tag{9}$$

Considering that the number of measured data, K, is larger than the number of parameters to be estimated (four variables), an implicit formulation is used for the inverse radiation problem at hand, in which the minimization of the least square norm is required, as given below:

$$Q(\vec{Z}) = \sum_{i=1}^{K} \left[I_i(\tau_o,\ \omega, A_1, A_2) - Y_i \right]^2 = \vec{G}^T \vec{G} \tag{10}$$

where I_i and Y_i are computed and measured exit intensities, respectively, and the elements of the vector of residues are

$$G_i = I_i(\tau_o,\ \omega, A_1, A_2) - Y_i, \quad i = 1,\ 2,\ \ldots,\ K \tag{11}$$

As real experimental data is not available, the measured exit intensities, Y_i, were obtained from simulation. For this aim, random error E (with normal distribution and standard deviation σ) was added to the exact intensities, I_{exact}, obtained from the solution of the direct problem.

$$Y_i = I_{exact_i} + \sigma E_i, \quad i = 1,\ 2,\ \ldots,\ K \tag{12}$$

4. SOLUTION OF THE INVERSE PROBLEM

4.1. Bee Colony Algorithm

As observed in biology, a colony of honey bees can extend itself over long distances (more than 10 km) and in multiple directions simultaneously to exploit a large number of food sources. In addition, the colony of honey bees presents as characteristic, the capacity of memorization, learning and transmission of information in colony, so forming the swarm intelligence (von Frisch, 1976).

In a colony the foraging process begins by scout bees being sent to search randomly for promising flower patches. When they return to the hive, those scout bees that found a patch which is rated above a certain quality threshold (measured as a combination of some constituents, such as sugar content) deposit their nectar or pollen and go to the "waggle dance".

This dance is responsible by the transmission (colony communication) of information regarding a flower patch: the direction in which it will be found, its distance from the hive and its quality rating (or fitness) (von Frisch, 1976). This dance enables the colony to evaluate the relative merit of different patches according to both the quality of the food they provide and the amount of energy needed to harvest it (Camazine *et al.*, 2003).

After waggle dancing on the dance floor, the dancer (i.e., the scout bee) goes back to the flower patch with follower bees that were waiting inside the hive. More follower bees are sent to more promising patches. This allows the colony to gather food quickly and efficiently. While harvesting from a patch, the bees monitor its food level. This is necessary to decide upon the next waggle dance when they return to the hive (Camazine *et al.*, 2003). If the patch is still good enough as a food source, then it will be advertised in the waggle dance and more bees will be recruited to that source.

In this context, Pham and co-workers (Pham *et al.*, 2006) proposed an optimization algorithm inspired by the natural foraging behavior of honey bees (Bees Colony Algorithm - BCA) and presented in Fig. 2.

Basic steps of the Bees Colony Algorithm
1. Initialise population with random solutions.
2. Evaluate fitness of the population.
3. While (stopping criterion not met)
4. Select sites for neighborhood search.
5. Recruit bees for selected sites (more bees for the best *e* sites) and evaluate fitnesses.
6. Select the fittest bee from each site.
7. Assign remaining bees to search randomly and evaluate their fitnesses.
8. End While.

Figure 2. Bees Colony Algorithm (Pham *et al.*, 2006).

The BCA requires a number of parameters to be set, namely, the number of scout bees (n), number of sites selected for neighborhood search (out of n visited sites) (m), number of top-rated (elite) sites among m selected sites (e), number of bees recruited for the best e sites (nep), number of bees recruited for the other (m-e) selected sites (ngh), and the stopping criterion.

The BCA starts with the n scout bees being placed randomly in the search space. The fitnesses of the sites visited by the scout bees are evaluated in step 2.

In step 4, bees that have the highest fitnesses are chosen as "selected bees" and sites visited by them are chosen for neighborhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to search near to the best e sites. The bees can be chosen directly according to the fitnesses associated with the sites they are visiting.

Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighborhood of the best e sites, which represent more promising solutions, are made more detailed by recruiting more bees to follow them than the other selected bees. Together with scouting, this differential recruitment is a key operation of the BCA.

However, in step 6, for each patch only the bee with the highest fitness will be selected to form the next bee population. In nature, there is no such a restriction. This restriction is introduced here to reduce the number of points to be explored. In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions.

In the literature, various applications using this bio-inspired approach can be found, such as: modeling combinatorial optimization transportation engineering problems (Lucic and Teodorovic, 2001), engineering system design (Yang, 2005; Lobato *et al.*, 2010a), transport problems (Teodorovic and Dell'Orco. 2005), mathematical function optimization (Pham *et al.*, 2006), dynamic optimization (Chang, 2006), optimal control problems (Afshar *et al.*, 2001), parameter estimation in control problems (Azeem and Saad, 2004), among other applications (http://www.bees-algorithm.com/).

4.2. Anti-Stagnation Operator

In any evolutionary approach, there is the possibility of the population to stagnate at a point that is not the global optimum. To increase the chance of the BCA to avoid this situation, an anti-stagnation operator was coupled to the original algorithm. In this operator, at the t-th generation, the average of the last k-th objective functions is calculated. If the difference between this value and the best value of the objective function in the current generation is smaller than a tolerance defined by the user, the current solution is refined through smaller perturbations generated randomly around this candidate. It should be mentioned that this procedure naturally increases the number of objective function evaluations, but also increases the probability of escaping from this stagnation point.

Next section presents an application of the methodology proposed in this work, denominated as Improved Bees Colony Algorithm - IBCA.

5. RESULTS AND DISCUSSION

In order to evaluate the performance of the IBCA to estimate both the single scattering albedo, ω, and the optical thickness, τ_o, of the layer, and also the intensities A_1 and A_2 of the external sources at $\tau = 0$ and $\tau = \tau_o$, respectively, of a given one-dimensional plane-parallel participating media, the three test cases listed in Tab.(1) have been performed.

Table 1. Parameters used to compose the illustrative examples (Lobato *et al.*, 2010c).

Parameter	Meaning	Case #		
		1	2	3
ω	Single scattering albedo	0.1	0.1	0.9
τ_o	Optical thickness of the layer	0.5	5.0	5.0
A_1	Intensity of external source at $\tau = 0$	1.0	1.0	1.0
A_2	Intensity of external source at $\tau = \tau_o$	0.0	0.0	0.0

It should be emphasized that 20 points were used for the approximation of the variable μ and 10 collocation points were taken into account to solve the direct problem. Table 2 present the parameters used in the BCA (in the IBCA algorithm; the anti-stagnation operator was activated when the current generation was larger than 20).

Table 2. Parameters used in the IBCA to solve the radiative transfer problem.

Number of scout bees (*n*)	20
Number of bees recruited for the best *e* sites (*nep*)	10
Number of bees recruited for the other (*m*-*e*) selected sites	5
number of sites selected for neighbourhood search (*m*)	10
number of top-rated (elite) sites among *m* selected sites (*e*)	5
Neighborhood search (*ngh*)	10^{-3}
Generation Number	100

In order to examine the accuracy of the inverse methodology of analysis considered, test cases incluidng noise (σ =0.02, i. e., corresponding to 5% error) or without noise (σ =0) have been studied. Also, two algorithms to solve the inverse problem, namely the Differential Evolution (DE) and the Simulated Annealing (SA) algorithms have been performed. The parameters used by DE and SA are given in Tab. (3).

Table 3. Parameters used in the two evolutionary algorithms (Lobato *et al.*, 2010c).

Parameter		SA	DE	
Generation number		100	100	
Population size		-	10	
Crossover probability		-	0.8	
Perturbation rate		-	0.8	
Strategy		-	DE/rand/1/bin	
Temperature number		50	-	
Iterations number for each temperature		10	-	
Temperature initial/final		0.5/0.01	-	
Initial Estimate	Case #1	[0.25 0.25 0.5 0.5]	Randomly generated	$0 \leq \omega \leq 1$; $0 \leq \tau_o \leq 1$; $1 \leq A_1 \leq 1.5$; $0 \leq A_2 \leq 1$
	Case #2	[0.25 0.45 0.5 0.5]		$0 \leq \omega \leq 1$; $3 \leq \tau_o \leq 5$; $1 \leq A_1 \leq 1.5$; $0 \leq A_2 \leq 1$
	Case #3	[0.75 0.45 0.5 0.5]		$0 \leq \omega \leq 1.4$; $3 \leq \tau_o \leq 5$; $1 \leq A_1 \leq 1.5$; $0 \leq A_2 \leq 1$

The present case studies are intended to observe the performance of the evolutionary algorithms for different levels of noise with standard deviation of experimental errors of 0% and 5%. For all test cases presented in this section the inverse problem algorithm was run ten times. Consequently, the worst, average and best results obtained are shown.

In Table 4 the results obtained for case #1 are presented. In this table, N_{eval} is the number of function evaluations. It can be observed that when using σ=0 (without noise) both algorithms led to good estimates for the unknown parameters. However, if noise is added, it can be seen that the estimates become poorer. The same behavior was observed for test cases #2 and #3 whose results are presented in Tabs.(5)-(6), respectively. However, the results obtained can be considered satisfactory in the context of the study conveyed.

From Tabs. (4)-(6) it should be emphasized that, in terms of the number of objective function evaluations, both BCA and IBCA lead to less evaluations as compared with the SA algorithm. Approximately the same number of objective function evaluations was obtained by the DE algorithm.

Table 4. Results obtained for case #1.

			ω	τ_o	A_1	A_2	Q (Eq.(10))
Exact	*Error in experimental data*		0.1	0.5	1.0	0.0	-
DE(1)	0.0	*Worst*	0.1003	0.5002	1.0000	0.0001	$1.5578\text{x}10^{-6}$
		Average	0.0998	0.4999	0.9999	0.0000	$5.7702\text{x}10^{-7}$
		Best	0.1000	0.4999	0.9999	0.0000	$4.4564\text{x}10^{-9}$
	5.0%	*Worst*	0.0876	0.5018	0.9992	0.0058	0.0842
		Average	0.0876	0.5018	0.9992	0.0058	0.0842
		Best	0.0870	0.5017	0.9990	0.0057	0.0842
SA(2)	0.0	*Worst*	0.0994	0.4999	1.0001	0.0000	$5.3920\text{x}10^{-7}$
		Average	0.0996	0.4998	0.9999	0.0000	$3.4741\text{x}10^{-7}$
		Best	0.0999	0.4999	0.9999	0.0000	$2.1496\text{x}10^{-7}$
	5.0%	*Worst*	0.0885	0.5012	0.9991	0.0059	0.0849
		Average	0.0880	0.5010	0.9990	0.0059	0.0844
		Best	0.0879	0.5010	0.9989	0.0056	0.0842
BCA(3)	0.0	*Worst*	0.1023	0.5087	1.0041	0.0001	$2.5900\text{x}10^{-4}$
		Average	0.1090	0.5002	1.0001	0.0000	$5.9080\text{x}10^{-5}$
		Best	0.9993	0.4999	0.9997	0.0000	$9.8978\text{x}10^{-6}$
	5.0%	*Worst*	0.0888	0.5098	0.9921	0.0078	0.0889
		Average	0.0888	0.5090	0.9922	0.0078	0.0889
		Best	0.0879	0.5080	0.9922	0.0077	0.0887
IBCA(4)	0.0	*Worst*	0.1020	0.5022	1.0014	0.0000	$7.5649\text{x}10^{-6}$
		Average	0.1000	0.5001	1.0001	0.0000	$5.9080\text{x}10^{-9}$
		Best	0.9999	0.4999	0.9999	0.0000	$3.1349\text{x}10^{-14}$
	5.0%	*Worst*	0.0877	0.5070	0.9921	0.0079	0.0839
		Average	0.0875	0.5067	0.9921	0.0079	0.0839
		Best	0.0875	0.5068	0.9920	0.0078	0.0837

(1) N_{eval}=1010, (2) N_{eval} =7015, (3) N_{eval} =852 and (4) N_{eval} =1245 (average value in all runs).

Table 5. Results obtained for case #2.

			ω	τ_o	A_1	A_2	Q (Eq.(10))
Exact	*Error in experimental data*		0.1	5.0	1.0	0.0	-
DE(1)	0.0	*Worst*	0.1024	4.9982	0.9988	0.0013	$6.3559\text{x}10^{-6}$
		Average	0.1004	4.9976	0.9992	0.0000	$2.6107\text{x}10^{-6}$
		Best	0.0998	5.0036	1.0008	0.0000	$1.1856\text{x}10^{-7}$
	5.0%	*Worst*	0.0453	4.9678	0.9683	0.0000	0.0878
		Average	0.0454	4.9675	0.9682	0.0000	0.0878
		Best	0.0455	4.9674	0.9680	0.0000	0.0878
SA(2)	0.0	*Worst*	0.0997	5.0097	1.0026	0.0004	$8.6468\text{x}10^{-7}$
		Average	0.0998	4.9981	0.9995	0.0003	$7.7231\text{x}10^{-7}$
		Best	0.0994	4.9956	0.9988	0.0005	$7.1664\text{x}10^{-7}$
	5.0%	*Worst*	0.0483	4.9578	0.9689	0.0001	0.0892
		Average	0.0484	4.9575	0.9685	0.0001	0.0890
		Best	0.0485	4.9554	0.9680	0.0001	0.0888
BCA(3)	0.0	*Worst*	0.0996	5.0077	1.0006	0.0004	$1.0948\text{x}10^{-4}$
		Average	0.0994	4.9998	0.9996	0.0001	$7.0989\text{x}10^{-5}$
		Best	0.0993	4.9996	0.9987	0.0001	$1.4222\text{x}10^{-6}$
	5.0%	*Worst*	0.0477	4.9578	0.9688	0.0002	0.0908
		Average	0.0478	4.9575	0.9686	0.0001	0.0907
		Best	0.0480	4.9554	0.9690	0.0000	0.0899
IBCA(4)	0.0	*Worst*	0.0997	5.0007	1.0002	0.0001	$8.009\text{x}10^{-7}$
		Average	0.0998	4.9999	0.9998	0.0000	$1.2223\text{x}10^{-9}$
		Best	0.0999	4.9999	0.9999	0.0000	$9.0989\text{x}10^{-13}$
	5.0%	*Worst*	0.0478	4.9580	0.9678	0.0002	0.0890
		Average	0.0480	4.9585	0.9680	0.0001	0.0880
		Best	0.0482	4.9594	0.9699	0.0001	0.0878

(1) N_{eval}=1010, (2) N_{eval} =8478, (3) N_{eval} =852 and (4) N_{eval} =1249 (average value in all runs).

Table 6. Results obtained for case #3.

			ω	τ_o	A_1	A_2	Q (Eq.(10))
Exact	*Error in experimental data*		0.9	5.0	1.0	0.0	-
DE(1)	0.0	*Worst*	0.9000	5.0002	0.9996	0.0000	2.8555×10^{-8}
		Average	0.9000	5.0001	0.9999	0.0000	2.6683×10^{-8}
		Best	0.9000	5.0000	0.9999	0.0000	2.6203×10^{-8}
	5.0%	*Worst*	0.8999	5.0599	1.0118	0.0001	0.0844
		Average	0.8992	5.0592	1.0117	0.0000	0.0824
		Best	0.8979	5.0562	1.0107	0.0000	0.0804
SA(2)	0.0	*Worst*	0.9001	5.0003	0.9998	0.0000	3.7788×10^{-8}
		Average	0.9000	5.0002	0.9999	0.0000	2.9988×10^{-8}
		Best	0.9000	5.0000	0.9999	0.0000	2.7245×10^{-8}
	5.0%	*Worst*	0.8999	5.0692	1.0090	0.0001	0.0855
		Average	0.8994	5.0691	1.0189	0.0001	0.0834
		Best	0.8981	5.0566	1.0179	0.0001	0.0811
BCA(3)	0.0	*Worst*	0.9021	5.0031	0.9987	0.0000	1.8228×10^{-4}
		Average	0.9018	5.0022	0.9987	0.0000	7.9098×10^{-5}
		Best	0.9010	5.0013	0.9999	0.0000	4.5556×10^{-6}
	5.0%	*Worst*	0.8979	5.0694	1.0091	0.0001	0.0995
		Average	0.8984	5.0684	1.0182	0.0001	0.0995
		Best	0.8981	5.0576	1.0180	0.0000	0.0994
IBCA(4)	0.0	*Worst*	0.9010	5.0011	0.9989	0.0000	9.9989×10^{-8}
		Average	0.9008	5.0008	0.9989	0.0000	9.9094×10^{-10}
		Best	0.9002	5.0005	0.9999	0.0000	6.9789×10^{-14}
	5.0%	*Worst*	0.8989	5.0698	1.0089	0.0002	0.0998
		Average	0.8985	5.0698	1.0177	0.0001	0.0993
		Best	0.8983	5.0555	1.0145	0.0000	0.0992

(1) N_{eval}=1010, (2) N_{eval} =8588, (3) N_{eval} =852 and (4) N_{eval} =1346 (average value in all runs).

6. CONCLUSIONS

In this work, the Improved Bees Colony Algorithm was applied to the simultaneous estimation of the albedo, optical thickness and the intensities A_1 and A_2 of the external sources at τ=0 and $\tau = \tau_o$, respectively, of a given one-dimensional plane-parallel participating media.

In this sense, an operator for the refinement of the current solution in the canonical Bees Colony Algorithm was proposed. The results obtained by applying the methodology showed that the incorporation of the operator designed to refine the solution was extremely important for obtaining the global optimum, even if this procedure requires more evaluations of the objective function with respect to its canonical version. The results obtained by using BCA and those from DE and SA are similar. However, in terms of the number of objective function evaluations, the BCA needs yet to be better studied, so that appropriate conclusions can be drawn, i.e., a new mechanism to explore the diversity of the design space should be proposed.

As observed in Tabs.(4)-(6), the addition of noise to the synthetic experimental points results an increase in the objective function values. Such a behavior was previously expected since noise does not permit the convergence of the optimization process to the actual experimental values. Consequently, the user should be aware of this behavior when using real experimental data, as they are always affected by noise.

As further work, we intend to extend the Bees Colony Algorithm to the multi-objective context and assess the sensitivity of the parameters with respect to the quality of the solution. The inclusion of the conduction heat transfer effect in the inverse problem of combined conduction and radiation effects in semitransparent media is also left for further studies.

7. ACKNOWLEDGEMENTS

The authors acknowledge the financial support provided by FAPEMIG. The third author acknowledges the financial support provided by FAPEMIG and CNPq (INCT-EIE). The fourth author acknowledges the financial support provided by CNPq and FAPERJ.

8. REFERENCES

Afshar, A., Haddad, O. B., Mariño, M. A. and Adams, B. J., 2001,, "Honey-Bee Mating Optimization (HBMO) Algorithm for Optimal Reservoir Operation". Journal of the Franklin Institute, 344, 452-462.

Alvarez Acevedo, N. I., Roberty, N. C. and Silva Neto, A. J., 2004, "A One-Dimensional Inverse Radiative Transfer Problem with Time-Varying Boundary Conditions". Inverse Problems in Engineering, 12, 123-140.

Azeem, M. F. and Saad, A. M., 2004. "Modified Queen Bee Evolution Based Genetic Algorithm for Tuning of Scaling Factors of Fuzzy Knowledge Base Controller". IEEE INDICON 2004 Proceedings of the India Annual Conference, 299-303.

Camazine, S., Deneubourg, J., Franks, N. R., Sneyd, J., Theraula, G. and Bonabeau, E., 2003, "Self-Organization in Biological Systems", Princeton University Press.

Carita Montero, R. F., Roberty, N. C. and Silva Neto, A. J., 2001, "Absorption Coefficient Estimation in Heterogeneous Media using a Domain Partition Consistent with Divergent Beams". Inverse Problems in Engineering, 9, 587-617.

Carita Montero, R. F. Roberty, N. C. and Silva Neto, A. J., 2004, "Reconstruction of a Combination of the Absorption and Scattering Coefficients with a Discrete Ordinates Method Consistent with the Source-Detector System". Inverse Problems in Engineering, 12 (1), 81-101.

Chang, H. S., 2006, "Converging Marriage in Honey-Bees Optimization and Application to Stochastic Dynamic Programming". Journal of Global Optimization, 35, 423-441.

de Abreu, M. P., 2005, "A Mathematical Method for Solving Mixed Problems in Multislab Radiative Transfer", J. of the Braz. Soc. of Mech. Sci. & Eng, Vol. XXVII, (4), pp. 381-393.

Hanan, N. P., 2001, "Enhanced Two-Layer Radiative Transfer Scheme for a Land Surface Model with a Discontinuous Upper Canopy". Agricultural and Forest Meteorology, 109, 265-281.

Kim, H. K. and Charette, A., 2007, "A Sensitivity Function Based Conjugate Gradient Method for Optical Tomography with the Frequency Domain Equation of Radiative Transfer". Journal of Quantitative Spectroscopy and Radiative Transfer, vol. 104, pp. 24-39.

Lobato, F. S., Steffen Jr., V. and Silva Neto, A. J. 2008, "Solution of Inverse Radiative Transfer Problems in Two-Layer Participating Media with Differential Evolution", International Conference on Engineering Optimization - EngOpt, 2008, Rio de Janeiro, Brazil.

Lobato, F. S., Steffen Jr., V. and Silva Neto, A. J. 2009, "Solution of the Coupled Inverse Conduction-Radiation Problem using Multi-objective Optimization Differential Evolution", 8th World Congress on Structural and Multidisciplinary Optimization – WCSMO-9, Lisboa, Portugal.

Lobato, F. S., Sousa, J. A., Hori, C. E. and Steffen Jr, V., 2010a, "Improved Bees Colony Algorithm Applied to Chemical Engineering System Design". International Review of Chemical Engineering (Rapid Communications), 6, 1-7.

Lobato, F. S., Steffen Jr., V. and Silva Neto, A. J., 2010b, "Estimation of Space-Dependent Single Scattering Albedo in Radiative Transfer Problems using Differential Evolution Algorithm". Inverse Problems, Design and Optimization Symposium, João Pessoa-Brazil.

Lobato, F. S., Steffen Jr., V. and Silva Neto, A. J., 2010b, "A Comparative Study of the Application of Differential Evolution and Simulated Annealing in Inverse Radiative Transfer Problems", Journal of the Brazilian Society of Mechanical Sciences and Engineering, v. XXXII, pp .518-526.

Lucic, P. and Teodorovic, D., 2001, "Bee System: Modeling Combinatorial Optimization Transportation Engineering Problems by Swarm Intelligence. Preprints of the TRISTAN IV Triennial Symposium on Transportation Analysis, 441-445.

Özişik, M. N., 1973. "Radiation Transfer and Iterations with Conduction and Convection". John Wiley.

Pham, D. T., Kog, E., Ghanbarzadeh, A., Otri, S., Rahim, S. and Zaidi, M., 2006. "The Bees Algorithm - A Novel Tool for Complex Optimisation Problems". Proceedings of 2nd International Virtual Conference on Intelligent Production Machines and Systems, Oxford, Elsevier.

Siegel, R. and Spuckler, C. M., 1993, "Refractive Index Effects on Radiation in an Absorbing, Emitting, and Scattering Laminated Layer". Journal of Heat Transfer, 115, 194-200.

Silva Neto, A. J. and Özişik, M. N., 1995, "An Inverse Problem of Simultaneous Estimation of Radiation Phase Function, Albedo and Optical Thickness". Journal of Quantitative Spectroscopy and Radiative Transfer, 53, 397-409.

Silva Neto, A. J. and Moura Neto, F. D., 2005, "Inverse Problems - Fundamental Concepts and Applications" (in portuguese). EdUERJ, Editora da Universidade do Estado do Rio de Janeiro.

Sousa, F. L., Soeiro, F. J. C. P., Silva Neto, A. J., and Ramos, F. M., 2007, "Application of the Generalized External Optimization Algorithm to an Inverse Radiative Transfer Problem", Inverse Problems in Science and Engineering, 15, 7, 699-714.

Teodorovic, D. and Dell'Orco, M., 2005. "Bee Colony Optimization - A Cooperative Learning Approach to Complex Transportation Problems". Proceedings of the 10th EWGT Meeting and 16th Mini-EURO Conference.

Villadsen, J. and Michelsen, M. L., 1978, “Solution of Differential Equation Models by Polynomial Approximation”. Prentice-Hall, Englenwood Cliffs.
von Frisch, K., 1976. “Bees: Their Vision, Chemical Senses and Language”. Revised edn, Cornell University Press, N.Y., Ithaca.
Wang, P. Y., Cheng, H. E. and Tan, H. P., 2002, “Transient Thermal Analysis of Semitrans parent Composite Layer with an Opaque Boundary”. International Journal of Heat and Mass Transfer, 45, 425-440.Wylie, C. R. and Barrett, L. C., 1985, “Advanced Engineering Mathematics”. 5th Edition, McGraw-Hill, Singapura.
Yang, X. S., 2005. “Engineering Optimizations via Nature-Inspired Virtual Bee Algorithms”. IWINAC 2005, Lecture Notes in Computer Science, 3562, Yang, J. M. and J.R. Alvarez (Eds.), Springer-Verlag, Berlin Heidelberg, 317-323.