HYBRID SOLUTION OF LAMINAR FLOW IN A CIRCULAR TUBE PARTIALLY FILLED WITH A POROUS MEDIUM

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Abstract. *This paper presents a semi-analytical solution of the velocity profile in a circular tube partially filled with a porous medium. The Darcy-Forchheimer-Brinkmam model was used to describe the flow in the porous region, coupled with the Navier Stokes equations used to describe the flow in the free region. The study also examined the effect of interfacial conditions between the porous medium and fluid layer. The solution of the problem was performed using the hybrid analytical-numerical approach of the Generalized Integral Transform Technique (GITT). The influence of the governing parameters such as the Reynolds and Darcy numbers, porosity, thickness of the porous region and interface coefficient β were studied. The present results demonstrated to be in agreement with previous experimental data in the literature.*

Keywords*: Laminar flow, Circular tube, porous medium, Integral transforms.*

1. INTRODUCTION

The Darcy's equation describes the velocity profile in porous media in which the drag due to pressure drop shows a linear behavior. Darcy's equation is linear when the seepage velocity *u* is sufficiently small, which means Reynolds number *Rep* of the flow is of order of the unity or smaller. As *u* increases, this linearity no longer exists; the Darcy's equation was modified with the addition of a nonlinear term in the constitutive equation (Dupuit, 1863; Forchheimer, 1901). Brinkman (1947) added an inertial term to Darcy's equation, which is valid only when the porosity is sufficiently large according to Bejan (2006).

Over the years, several authors studied the fluid flow at the interface between a porous medium and a clear fluid region. The work of Vafai and Kim (1990) used the Darcy-Forchheimer-Brinkman equation to describe the velocity profile in the porous medium, while for the clear fluid was used the Navier-Stokes equation, while for the interfacial conditions it was employed continuity of velocity and flux. Such model is well accepted and widely used in the literature. Xiong (2001) solved this problem in transient regime using the finite differences method. Vafai and Alazmi (2000) studied the velocity profile of a fluid layer sandwiched between a porous medium from above and a solid boundary from below considering different interfacial conditions. In same way, Ochoa-Tapia and Whitaker (2001) have proposed a hybrid interface condition, in which a jump in the shear stress at the interface region was assumed and compared with experimental data.

In this context, the present work aims at solving the problem of a porous medium and clear fluid region using the Generalized Integral Transform Technique (GITT). The Generalized Integral Transform Technique (GITT) (Cotta, 1990; Cotta, 1993) is an eigenfunction expansion methodology for solving linear or nonlinear convection-diffusion problems, especially those not a priori transformable by the classical approach. Also, we used the method of lines [discretization by finite difference in space and solution of the ODE in time by using the subroutine DIVPAG from the IMSL Library (1991)] in order to verify the agreement of the results obtained via GITT.

2. GENERAL FORMULATION

The geometrical configuration considered in this research is shown in Fig. 1. The problem consists of an incompressible Newtonian fluid flowing in a cylinder partially filled with a porous medium saturated in transient state, with fully developed flow in the axial direction and variable in the radial direction, and constant physical properties. This system has wide applications in engineering such as electronic cooling, transpiration cooling, drying processes, thermal insulation, porous bearing, solar collectors, heat pipes, nuclear reactors, crude oil extraction, fuel cell and geothermal engineering. Therefore, the mathematical formulation of the problem is defined below as:

- For the fluid region:

$$\rho_f \frac{\partial u}{\partial t} = -\nabla P + \mu_f \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \quad 0 < r < R_i \tag{1}$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{at} \quad r = 0 \tag{2}$$

- For the porous region:

$$\rho_f \frac{\partial u}{\partial t} = -\varepsilon\nabla P + \mu_f \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\varepsilon\mu_f}{k}u - \frac{\varepsilon^2\rho_f c_F}{k^{1/2}}u^2, \quad R_i < r < R_e \tag{3}$$

$$u = 0 \quad \text{at} \quad r = R_e \tag{4}$$

with initial and interface conditions

$$u = 0 \quad \text{for} \quad t = 0 \tag{5}$$

$$u\big|_{r=R_i^-} = u\big|_{r=R_i^+}; \qquad \frac{\mu_f}{\varepsilon}\frac{\partial u}{\partial r}\bigg|_{r=R_i^+} - \mu_f\frac{\partial u}{\partial r}\bigg|_{r=R_i^-} = \frac{\beta\mu_f}{k^{1/2}}u\bigg|_{r=R_i} \tag{6,7}$$

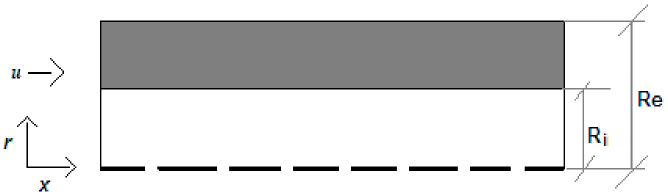

Figure 1. Schematic diagram of the problem.

where, c_F is the Forchheimer coefficient, k is the permeability of the porous medium, R_i is the inner radius, R_e is the outer radius, μ_f is the fluid viscosity, ε is the porosity, μ_{ef} is the effective viscosity of porous medium, d_p is particle diameter, ∇P is the pressure gradient, ρ_f is the fluid density, t is the time variable and r is the radial coordinate. The model for the permeability and for the Forchheimer coefficient are given, respectively, as:

$$k = \frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2}; \quad c_F = \frac{1.75}{\sqrt{150}\varepsilon^{5/2}} \tag{8,9}$$

The following dimensionless parameters and physical quantities are defined as:

$$R = \frac{r}{R_e};\ \tau = \frac{\nu t}{R_e^2};\ U = \frac{u}{u_c};\ a = \frac{R_i}{R_e};\ u_c = \frac{GR_e^2}{\mu_f};\ \mathbb{R}e_p = \frac{u_c d_p}{\nu_f};\ Da = \frac{k}{R_e^2} \tag{10.a-g}$$

$$\gamma = \frac{R_e}{d_p};\ \mathbb{C} = \frac{\beta}{\sqrt{Da}};\ \Omega = \frac{\mu_{ef}}{\mu_f};\ G = -\frac{\partial P}{\partial z};\ \nu_f = \frac{\mu_f}{\rho_f};\ \mu_{ef} = \frac{\mu_f}{\varepsilon} \tag{10.h-m}$$

where, $\mathbb{R}e_p$ is the Reynolds number based on the particle diameter and Da is the Darcy number.

Introducing Eqs. (10) into Eqs. (1) to (7), the following dimensionless governing equations can now be written in the form:

- For the fluid region:

$$\frac{\partial U}{\partial \tau} = 1 + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right), \quad 0 < R < a \tag{11}$$

$$\frac{\partial U}{\partial R} = 0 \quad \text{at} \quad R = 0 \tag{12}$$

- For the porous region:

$$\frac{\partial U}{\partial \tau} = \varepsilon + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right) - \frac{\varepsilon}{Da}U - \mathbb{R}e_p\gamma\frac{\varepsilon^2 c_F}{Da^{1/2}}U^2, \qquad a \le r \le 1 \tag{13}$$

$$U = 0 \quad \text{at} \quad R = 1 \tag{14}$$

- initial and interface conditions:

$$U = 0 \quad \text{for} \quad \tau = 0 \tag{15}$$

$$U\big|_{R=a^-} = U_{R=a^+}; \quad \Omega\frac{\partial U}{\partial R}\bigg|_{R=a^+} - \frac{\partial U}{\partial R}\bigg|_{R=a^-} = \mathbb{C}U\big|_{R=a} \tag{16,17}$$

3. ANALYSIS

In order to improve the solution methodology, it is better to split the problem into two potential velocities:

$$\frac{\partial U_1}{\partial \tau} = 1 + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U_1}{\partial R}\right), \qquad 0 \le R < a \tag{18}$$

$$\frac{\partial U_2}{\partial \tau} = \varepsilon + \frac{1}{R}\frac{\partial}{\partial (R)}\left(R\frac{\partial U_2}{\partial R}\right) - \frac{\varepsilon}{Da}U_2 - \mathbb{R}e_p\gamma\frac{\varepsilon^2 c_F}{Da^{1/2}}U_2{}^2, \qquad a \le R \le 1 \tag{19}$$

$$U_1 = U_2 = 0 \quad \text{for} \quad \tau = 0 \tag{20}$$

$$\frac{\partial U_1}{\partial R} = 0 \quad \text{at} \quad R = 0 \tag{21}$$

$$U_1 = U_2 \ \text{e}\ \Omega\frac{\partial U_2}{\partial R} - \frac{\partial U_1}{\partial R} = \mathbb{C}U_2 \quad \text{at} \quad R = a \tag{22}$$

$$U_2 = 0 \quad \text{at} \quad R = 1 \tag{23}$$

To solve the problem, it is necessary to use a filter to reduce the weight of the non-homogeneities in the PDEs and in the boundary conditions. The solution can be split in two parts U_p (particular solution) and U_H (filtered solution) as follows:

$$U_1 = U_{p1}(R) + U_{H1}(R,\tau); \quad U_2 = U_{p2}(R) + U_{H2}(R,\tau) \tag{24,25}$$

3.1. Formulation for the particular problem (U_p)

The particular problems for the potentials U_{p1} and U_{p2} are:

$$1 + \frac{1}{R}\frac{d}{dR}\left(R\frac{dU_{p1}}{dR}\right) = 0; \qquad \varepsilon_0 + \frac{1}{R}\frac{d}{dR}\left(R\frac{dU_{p2}}{dR}\right) = 0 \tag{26,27}$$

$$\frac{dU_{p1}}{dR} = 0 \quad \text{at} \quad R = 0 \tag{28}$$

$$U_{p1} = U_{p2}; \quad \Omega\frac{dU_{p2}}{dR} - \frac{dU_{p1}}{dR} = \mathbb{C}U_{p2} \quad \text{at} \quad R = a \tag{29,30}$$

$$U_{p2} = 0 \quad \text{at} \quad R = 1 \tag{31}$$

Solving Eqs. (26) to (30), it obtains

$$U_{p1} = \frac{(a-R)(a+R)\Omega + a\left(-a(2+a\mathbb{C}) + \mathbb{C}R^2\right)\ln a + \Omega\left(1 - a^2 + 2a^2\ln a\right)\varepsilon_0}{4(\Omega - a.\mathbb{C}.\ln a)} \tag{32}$$

$$U_{p2} = \frac{2a^2\ln R + \left(-\left(-1+R^2\right)(\Omega - a\mathbb{C}\ln a) + a\left(\mathbb{C} - a^2\mathbb{C} + 2a\Omega\right)\ln R\right)\varepsilon_0}{4(\Omega - a\mathbb{C}\ln a)} \tag{33}$$

3.2. Formulation for the filtered problem (U_H)

The formulation for the filtered problems U_{H1} and U_{H2} are:

$$\frac{\partial U_{H1}}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U_{H1}}{\partial R}\right), \qquad 0 < R < a \tag{34}$$

$$\frac{\partial U_{H2}}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial (R)}\left(R\frac{\partial U_{H2}}{\partial R}\right) + F(\tau, R), \qquad a < R < 1 \tag{35}$$

$$U_{H1} = -U_{p1}(R); \quad U_{H2} = -U_{p2}(R) \quad \text{for} \quad \tau = 0 \tag{36,37}$$

$$\frac{\partial U_{H1}}{\partial R} = 0 \quad \text{at} \quad R = 0 \tag{38}$$

$$U_{H1} = U_{H2}; \quad \Omega\frac{\partial U_{H2}}{\partial R} - \frac{\partial U_{H1}}{\partial R} = \mathbb{C}U_{H2} \quad \text{at} \quad R = a \tag{39,40}$$

$$U_{H2} = 0 \quad \text{at} \quad R = 1 \tag{41}$$

where,

$$F(\tau, R) = \Delta\varepsilon - \frac{\varepsilon}{Da}\left(U_{H2} + U_{p2}\right) - q\left(U_{H2} + U_{p2}\right)^2; \quad q = \mathbb{R}e_p \gamma \frac{\varepsilon^2 c_F}{Da^{1/2}} \tag{42,43}$$

4. SOLUTION METHODOLOGY

A hybrid analytical-numerical solution of the problem, given by Eqs. (34) to (43), is then here developed by extending the ideas of the Generalized Integral Transform Technique (GITT) as described in the references of Cotta (1990), Cotta (1993), Cotta and Mikhailov (1997) Cotta (1998) and Cotta and Mikhailov (2006). First, an appropriate auxiliary eigenvalue problem is selected to offer a basis for the eigenfunction expansion. Therefore, the following eigenvalue problem is proposed for the transformation in the R direction (Frankel and Vick, 1987):

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\psi_{1i}}{dR}\right) + \mu_i^2\psi_{1i} = 0, \quad 0 < R < a; \quad \frac{1}{R}\frac{d}{dR}\left(R\frac{d\psi_{2i}}{dR}\right) + \mu_i^2\psi_{2i} = 0, \quad a < R < 1 \tag{44,45}$$

$$\frac{d\psi_{1i}}{dR} = 0 \quad \text{at} \quad R = 0 \tag{46}$$

$$\psi_{1i} = \psi_{2i}; \qquad \Omega\frac{d\psi_{2i}}{dR} = \frac{d\psi_{1i}}{dR} \quad \text{at} \quad R = a \tag{47,48}$$

$$\psi_{2i} = 0 \quad \text{at} \quad R = 1 \tag{49}$$

The solution of the auxiliary eigenvalue problem is:

$$\psi_{1i} = J_0(\mu_i R); \quad \frac{d\psi_{1i}}{dR} = -\mu_i J_1(\mu_i R), \quad 0 < R < a \tag{50,51}$$

$$\psi_{2i} = A_{2i}J_0(\mu_i R) + B_{2i}Y_0(\mu_i R); \quad \frac{d\psi_{2i}}{dR} = -A_{2i}\mu_i J_1(\mu_i R) - B_{2i}\mu_i Y_1(\mu_i R), \quad a < R < 1 \tag{52,53}$$

$$A_{2i} = \frac{J_0(\mu_i a)Y_1(\mu_i a) - \frac{1}{\Omega}J_1(\mu_i R)Y_0(\mu_i a)}{J_0(\mu_i a)Y_1(\mu_i a) - J_1(\mu_i a)Y_0(\mu_i a)}; \quad B_{2i} = \frac{J_0(\mu_i a)\frac{1}{\Omega}J_1(\mu_i R) - J_1(\mu_i a)J_0(\mu_i a)}{J_0(\mu_i a)Y_1(\mu_i a) - J_1(\mu_i a)Y_0(\mu_i a)} \tag{54,55}$$

The eigenvalues μ_i are given from the following transcendental equation:

$$\det\begin{bmatrix} J_0(\mu_i a) & -J_0(\mu_i a) & -Y_0(\mu_i a) \\ \frac{1}{\Omega}J_1(\mu_i R) & -J_1(\mu_i a) & -Y_1(\mu_i a) \\ 0 & J_0(\mu_i) & Y_0(\mu_i) \end{bmatrix} = 0 \tag{56}$$

The eigenfunctions enjoy the following orthogonality property:

$$\int_0^a R\psi_{1i}\psi_{1j}dR + \int_a^1 R\psi_{2i}\psi_{2j}dR \begin{cases} 0, & i \neq j \\ N_i, & i = j \end{cases} \tag{57}$$

$$N_i = \int_0^a R\psi_{1i}^2 dR + \int_a^1 R\psi_{2i}^2 dR \tag{58}$$

where N_i is the norm or normalization integral.

Eigenvalue problem allows the definition of the following integral transform pair:

$$\bar{U}_{Hi}(\tau) = \int_0^a R\psi_{1i}(R)U_{H1}(\tau,R)dR + \int_a^1 R\psi_{2i}(R)U_{H2}(\tau,R)dR, \qquad \text{transform} \tag{59}$$

$$U_{Hm}(\tau,R) = \sum_{i=1}^{\infty} \frac{\psi_{mi}(R)}{N_i}\bar{U}_{Hi}(\tau), \quad m = 1,2, \qquad \text{inverse} \tag{60}$$

The next step is thus to accomplish the integral transformation of the original partial differential system given by Eqs. (34) to (43). For this purpose, Eq. (34) and the initial condition (36) are multiplied by $R\psi_{1i}$, integrated over the domain $[0,a]$ in R, and the inverse formula given by Eq. (60) is employed. Similarly, Eq. (35) and the initial condition (37) are multiplied by $R\psi_{2i}$, integrated over the domain $[a,1]$ in R, and the inverse formula given by Eq. (60) is employed. After the appropriate manipulations, the following coupled ordinary differential system results, for the calculation of the transformed potentials $\bar{U}_{Hi}(\tau)$:

$$\frac{d\bar{U}_{Hi}}{d\tau} = -\mu_i^2\bar{U}_{Hi} + A_i(\tau) + B_i(\tau) + \tilde{F}_i(\tau) \tag{61}$$

$$\bar{U}_{Hi}(0) = -\mathbb{C}_i \tag{62}$$

where the integral coefficients are calculated from:

$$A_i(\tau) = a\psi_{1i}(a)\frac{\partial U_{H1}(\tau,a)}{\partial R} - a\frac{\partial \psi_{1i}(a)}{\partial R}U_{H1}(\tau,a) \tag{63}$$

$$B_i(\tau) = a\frac{\partial \psi_{2i}(a)}{\partial R}U_{H2}(\tau,a) - a\frac{\partial U_{H2}(\tau,a)}{\partial R}\psi_{2i}(a) \tag{64}$$

$$\tilde{F}_i(\tau) = \int_a^1 R\psi_{2i}(R)F(\tau,R)dR \tag{65}$$

$$\mathbb{C}_i = \left[\int_0^a R\psi_{1i}(R)U_{p1}(R)dR + \int_a^1 R\psi_{2i}(R)U_{p2}(R)dR\right] \tag{66}$$

The calculation of $\mathbb{C}_i$ is numerically performed from the subroutine DQDAGS from the IMSL Library (1991), and the calculation of $\tilde{F}_i(\tau)$ is performed by a semi–analytical integration with the linear approximation of $F(\tau,R)$, as follows:

$$\tilde{F}_i(\tau) = \sum_{k=1}^{Nk}\int_{x_{k-1}}^{x_k} R\psi_{2i}(R)F(\tau,R)dR; \quad F(\tau,R) = \alpha_k R + \varphi_k \tag{67,68}$$

$$\tilde{F}_i(\tau) = \sum_{k=1}^{Nk}\left\{\alpha_k\int_{x_{k-1}}^{x_k} R^2\psi_{2i}(R)dR + \varphi_k\int_{x_{k-1}}^{x_k} R\psi_{2i}(R)dR\right\}; \quad \alpha_k = \frac{F_k - F_{k-1}}{x_k - x_{k-1}}; \quad \varphi_k = F_k - \alpha_k x_k \tag{69-71}$$

5. RESULTS AND DISCUSSION

A computer code was developed in FORTRAN 95/2003 programming language and executed on a personal computer Intel Core 2 Duo 2.0 GHz. The system of coupled ordinary differential equations for the transformed potentials was solved employing the subroutine DIVPAG from the IMSL Library (1991) with a relative error target of 10^{-8} prescribe by the user. This subroutine solves initial value problems with stiff behavior, and provides the important feature of automatically controlling the relative error in the solution of the ordinary differential equations system, allowing the user to establish error targets for the transformed potentials. The dimensionless times of interest were selected in the range of 10^{-5} to 10.

The convergence behavior of the present solution is illustrated in tabular form, according to Table 1 for the parameters $\varepsilon_0=0.37$; $Re = 10^3$;$\beta = 0.0$; $a = 0.8$; $Da = 0.01$. Within the wide range of dimensionless times were required a truncation order around 60 terms, while in the lower value of the time the convergence was reached with around 40 terms to four significant digits.

Table 1. Convergence behavior of the dimensionless potential $U(R,\tau)$ at various radial positions for times 0.01, 0.1 and 1, with $\varepsilon_0=0.37$; $Re = 10^3$;$\beta = 0.0$; $a = 0.8$; $Da = 0.01$.

τ	*R*	20	40	60	80	100	120	140	150
0.01	0.08	0.01001	0.01000	0.01000	0.01000	0.01000	0.01000	0.01000	0.01000
	0.4	0.00999	0.00999	0.01000	0.01000	0.01000	0.01000	0.01000	0.01000
	0.8	0.00433	0.00433	0.00433	0.00433	0.00433	0.00433	0.00433	0.00433
	0.9	0.00170	0.00169	0.00169	0.00169	0.00169	0.00169	0.00168	0.00168
0.1	0.08	0.09024	0.09024	0.09023	0.09023	0.09023	0.09023	0.09023	0.09023
	0.4	0.07618	0.07625	0.07626	0.07627	0.07627	0.07627	0.07627	0.07627
	0.8	0.01029	0.01028	0.01028	0.01028	0.01028	0.01028	0.01028	0.01028
	0.9	0.00229	0.00225	0.00224	0.00224	0.00224	0.00224	0.00224	0.00224
1	0.08	0.17145	0.17164	0.17165	0.17166	0.17166	0.17166	0.17167	0.17166
	0.4	0.13295	0.13322	0.13326	0.13327	0.13327	0.13327	0.13328	0.13328
	0.8	0.01330	0.01330	0.01330	0.01330	0.01330	0.01330	0.01330	0.01330
	0.9	0.00249	0.00242	0.00241	0.00241	0.00241	0.00241	0.00241	0.00241

Figure 2 shows a comparison of the present GITT results against those of the method of lines and of Xiong (2001), for the fully developed velocity distribution as a function of the radial position. This figure shows an excellent agreement among the three sets of results obtained. Also, it is observed that the fluid region presents a parabolic profile characteristic of fluid flow in duct free, while for the porous region a more flattened profile is found, since in this region the resistance to the flow is higher.

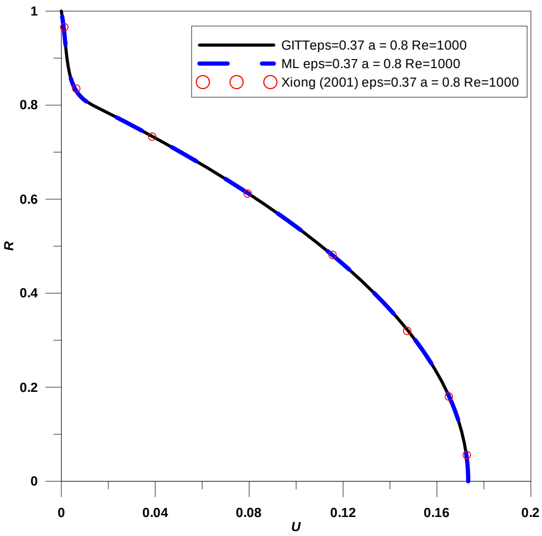

Figure 2. Comparison of the present GITT results with those of the Method of Lines (ML) and of Xiong (2001) for the fully developed velocity profile for $\varepsilon_0=0.37$; $Re = 10^3$;$\beta = 0.0$; $a = 0.8$; $Da = 0.01$.

Figure 3 shows the effect of thickness of porous medium in the velocity profile. As would be expected, as *a* increases from 0.4 to 0.8, the thickness of the porous layer decreases and the velocity shows an increase. When *a* tends to zero, it has a duct completely filled by the porous medium, on the other hand, when *a* tends to unity it has a totally clear duct.

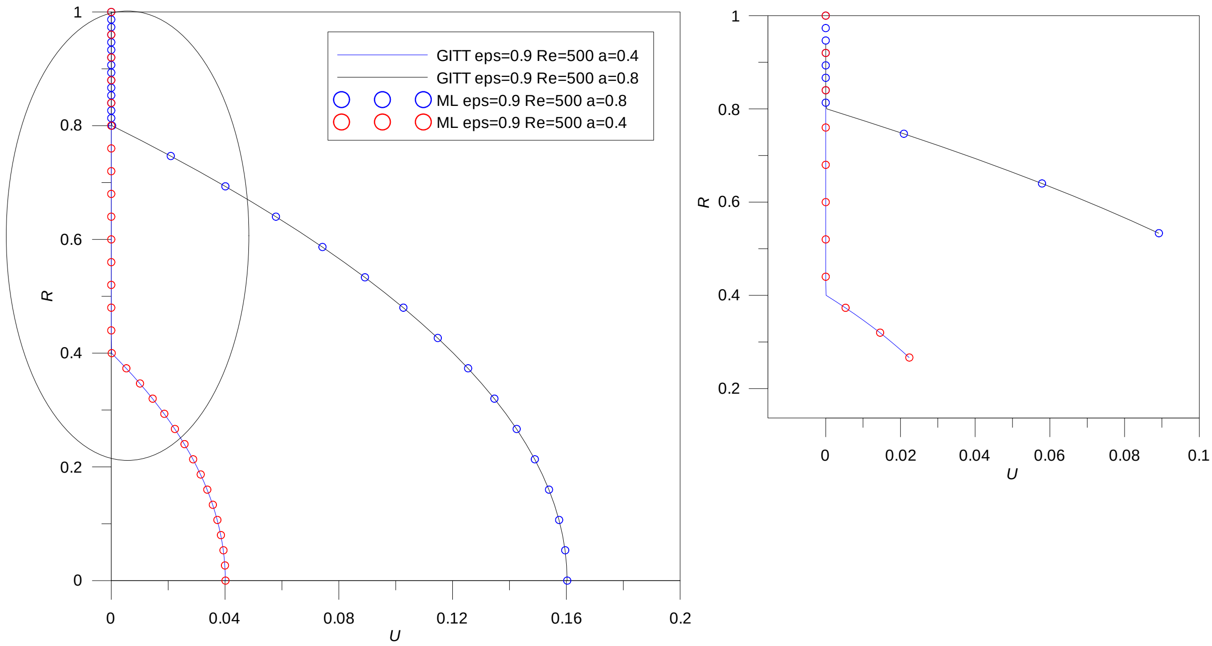

Figure 3. Effect of the porous medium thickness in the velocity profile for ε_0=0.9; Re = 500; β=0.0.

Figure 4 shows the effect of the porosity on the velocity profile for Re = 500; a=0.4; β=0.0, and ε=0.4 and 0.9. It is shown that for increasing values of the porosity, it has an increase in the permeability coefficient, and as a consequence there is an increase in the velocity throughout the domain. At the interface, for β=0.0 and ε=0.9, the curve is less sharp than for ε=0.4, because μ_{ef} is closer to μ_f warranting the continuity of flows between the two regions.

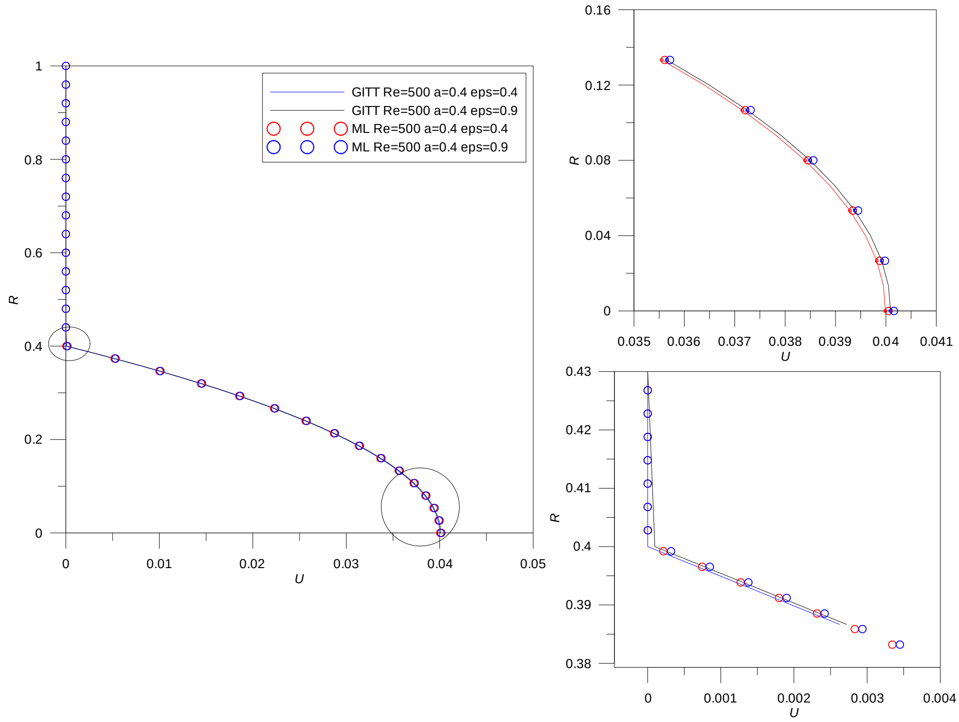

Figure 4. Effect of the porosity on the velocity profile for Re = 500; a=0.4; β=0.0.

Figure 5 shows the effect of Reynolds number on the velocity profile for ε=0.9; a=0.4; β=0.0, and Re = 500 and 1000. According to dimensionless equations, Eqs. (11) and (13), the Reynolds number has influence only on the coefficient of U^2 that represents the Forchheimer term. For the porous region, this term only has influence on neighborhood of the interface, where the velocity is close to zero, therefore, there is no change with the Reynolds number variation.

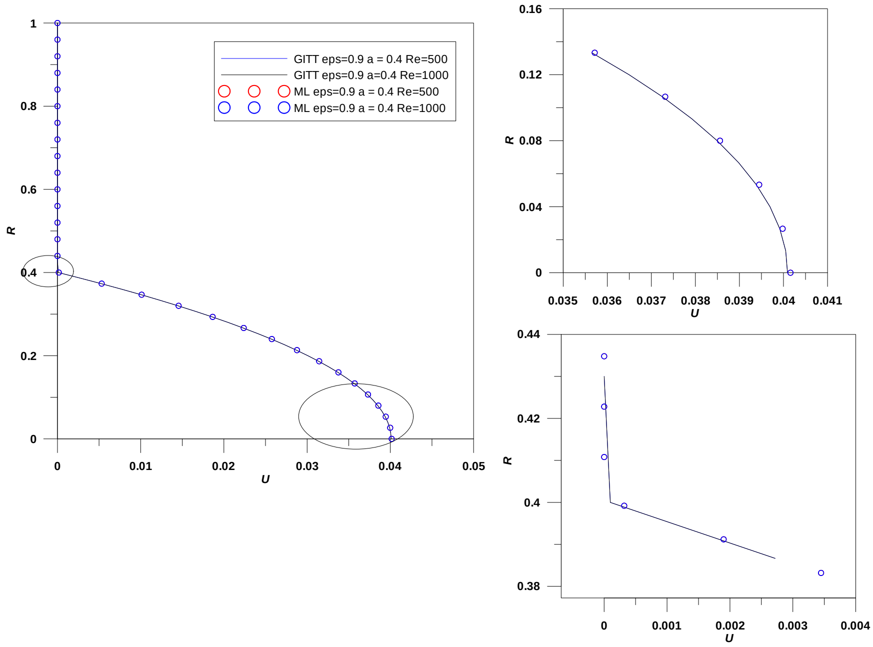

Figure 5. Effect of the Reynolds number on the velocity profile for ε=0.9; a=0.4; β=0.0.

6. CONCLUSIONS

The Integral Transform Method was further extended towards the hybrid numerical-analytical solution of the velocity profile in a circular tube partially filled with a porous medium. An eigenvalue problem that brings information of the two layers (fluid and porous medium) has allowed an integral transformation of the problem similar to that for a single region, this way permitting the GITT application. Numerical results for the velocity field were thus produced. The excellent agreement of the present results with those of method of lines and with previously reported ones demonstrates the consistency of this approach and adequacy for benchmarking such class of problems. The approach presented, in addition, opens up new perspectives in the hybrid numerical-analytical solution of nonlinear heat and mass problems in non-Cartesian coordinates systems.

7. ACKNOWLEDGEMENTS

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