

FAULT-TOLERANT NONLINEAR FLIGHT CONTROL

Filipe Alves Pereira da Silva, filipe.alves@gmail.com

Pedro Paglione, paglione@ita.br

ITA – Instituto Tecnológico da Aeronáutica, Praça Marechal Eduardo Gomes, 50 - Vila das Acácias, São José dos Campos – SP – Brasil CEP: 12.228-900

Abstract. *This work demonstrates the use of the backstepping method to develop a fault tolerant control method for flight control applications. The desire to create a control system that makes the aircraft have similar response to pilots commands with or without a flight control system failure, demands the knowledge of the possible failures and how to deal with them. The proposed method herein integrates the airframe, the flight control law, the control allocation and the fault detection and identification. Control allocation technique enables the control law to be designed regardless of the failure detection method or the designed control.*

Keywords: *backstepping, nonlinear control, control allocation, fault-tolerant flight control*

1. INTRODUCTION

The objective of this work is to demonstrate the study of nonlinear control techniques – mainly backstepping – applied to aircraft flight control system, considering control surfaces actuators failures. Besides the nonlinear control techniques, control allocation will be used in order to determine the distribution of control for each control surface movement to achieve the aircraft desired response.

At the end of this paper results comparing the response of a failure in one of the flight control actuators of a six degree of freedom aircraft model normal response, with an actuator failure not considering the fault tolerant control and with an actuator fault with the fault tolerant control integrated will be demonstrated.

2. AIRCRAFT AERODYNAMIC MODEL

The aircraft model used for this study is a model of an airliner for 250 passengers based on the model used in Givisiez(2009), considering the coupled longitudinal and lateral-directional movements. Its nonlinear model is modeled in Eq. (1) and Eq. (2), where X is the aircraft state vector and U is the command vector applied by the control surfaces and the throttle lever position.

$$\dot{X} = f(X) + g(X)U \quad (1)$$

$$Y = h(X) \quad (2)$$

For linear analysis the model is defined in Eq. (3) and Eq. (4).

$$\dot{x} = A.\dot{x} + B.\Delta\dot{u} \quad (3)$$

$$\dot{y} = H.\dot{x} \quad (4)$$

The inputs to the aircraft model are the control surfaces control (left and right elevators, left and right ailerons and rudder) and the throttle lever position. Usually it is considered that the elevator and the throttle lever are responsible only for the longitudinal movement and the ailerons and the rudder are responsible for the lateral-directional movement. This assumption cannot be done when considering actuators failure scenarios, because an elevator asymmetry caused by a fault can generate a rolling moment and one aileron fault can generate a pitching moment. For a model considering these faults, the input vector must be in the form of the Eq. (5), where all the five control surfaces are treated independently, where δe is the left elevator position, δre is the right elevator position, δa is the left aileron position, δra is the right aileron position, δr is the rudder position and δpi is the throttle lever position.

$$U = [\delta e \quad \delta re \quad \delta a \quad \delta ra \quad \delta r \quad \delta pi]^T \quad (5)$$

The control surfaces actuators and the throttle lever response can be modeled as first order systems combined with position and rate limiters as in Fig. 1, where a is the time constant (0.1 second for the control surfaces and 5 seconds for the throttle lever).

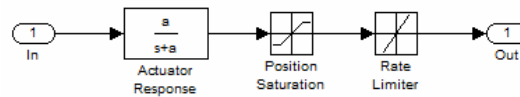


Figure 1. Actuators Model

Considering the aerodynamics and propulsion external forces and moments a usual six degree of freedom rigid body aircraft can be modeled by the Eq. (6) to Eq. (19). These equations represent the responses of twelve aircraft states. The aircraft velocities (u , v and w) in the body axis coordinates, Euler angles (Ψ , θ and ϕ), body rotation rates (p , q and r) and the aircraft position relative to the earth (x , y and z), where Xa , Ya and Za are aerodynamics coefficients, Xf , Yf and Zf are coefficients related to the propulsion system, L , M and N are the roll, pitch and yaw moments respectively and I_x , I_y , I_z , I_{xz} are inertia components of the aircraft.

State Vector

$$X = [u \quad v \quad w \quad p \quad q \quad r \quad \Psi \quad \theta \quad \phi \quad x \quad y \quad z]^T \quad (6)$$

Velocities

$$\dot{u} = \frac{(Xa + Xf)}{m} - g \cdot \sin(\theta) + r \cdot v - q \cdot w \quad (7)$$

$$\dot{v} = \frac{(Ya - Yf)}{m} - g \cdot \sin(\phi) \cdot \cos(\theta) + p \cdot w - r \cdot u \quad (8)$$

$$\dot{w} = \frac{(Za + Zf)}{m} - g \cdot \cos(\phi) \cdot \cos(\theta) + q \cdot u - p \cdot v \quad (9)$$

Kinematics Equations

$$\dot{\phi} = p + \tan(\theta) \cdot (q \cdot \sin(\phi) + r \cdot \cos(\phi)) \quad (10)$$

$$\dot{\theta} = q \cdot \cos(\phi) - r \cdot \sin(\phi) \quad (11)$$

$$\dot{\Psi} = \frac{(q \cdot \sin(\phi) + r \cdot \cos(\phi))}{\cos(\theta)} \quad (12)$$

Moment Equations

$$\Gamma \dot{p} = q \{ I_{xz} [I_x - I_y + I_z] p - [I_z (I_z - I_y) + I_{xy}^2] r \} + I_z L - I_{xz} N \quad (13)$$

$$\Gamma \dot{q} = (I_z - I_x) p r - I_{xz} (p^2 - r^2) + M \quad (14)$$

$$\Gamma \dot{r} = q ((I_x - I_y) I_x + I_{xz}^2) p - I_{xz} (I_x - I_y + I_z) r + I_{xz} L + I_x N \quad (15)$$

$$\Gamma = I_x \cdot I_z - I_{xz}^2 \quad (16)$$

Navigation Equations

$$x = u \cdot \cos(\theta) \cdot \cos(\Psi) + v \cdot (\sin(\phi) \cdot \sin(\theta) \cdot \cos(\Psi) - \cos(\phi) \cdot \sin(\Psi)) + w \cdot (\cos(\phi) \cdot \sin(\theta) \cdot \cos(\Psi) + \sin(\phi) \cdot \sin(\Psi)) \quad (17)$$

$$\dot{y} = u \cdot \cos(\theta) \cdot \sin(\Psi) + v \cdot (\sin(\phi) \cdot \sin(\theta) \cdot \sin(\Psi) + \cos(\phi) \cdot \cos(\Psi)) + w \cdot (\cos(\phi) \cdot \sin(\theta) \cdot \sin(\Psi) + \sin(\phi) \cdot \cos(\Psi)) \quad (18)$$

$$\dot{z} = -u \cdot \sin(\theta) + v \cdot \sin(\phi) \cdot \sin(\Psi) + w \cdot (\cos(\phi) \cdot \cos(\Psi)) \quad (19)$$

The nonlinear model time response analysis are executed considering variables such as total speed (VT), angle of attack (α) and sideslip angle (β). To achieve these variables, the above equation must be transformed from the body axis to the wind axis as in Eq. (20) to Eq. (22).

$$\dot{VT} = \frac{u.\dot{u} + v.\dot{v} + w.\dot{w}}{VT} \quad (20)$$

$$\dot{\beta} = \frac{\dot{v}.VT - v.\dot{VT}}{VT.(u^2 + w^2)^{1/2}} \quad (21)$$

$$\dot{\alpha} = \frac{u.\dot{w} - w.\dot{u}}{u^2 + w^2} \quad (22)$$

The longitudinal and lateral-directional aircraft models can be isolated, creating two sets of equations. Equation (23) to Eq. (29) are used for the longitudinal (vertical) mode and Eq. (30) to Eq. (33) are used for lateral-directional modes. This approach is commonly used in aircraft modeling literature (STEVENS and LEWIS, 2003). Where L the aircraft lift, D is the aircraft drag, F is thrust generate by the engines, α_f is the angle of attack relative to the engine installation, m is the aircraft mass, g is the gravity acceleration, γ is the flight path and h is altitude..

$$\dot{VT} = \frac{1}{m} (-D + F.\cos(\alpha + \alpha_f) - m.g.\sin(\gamma)) \quad (23)$$

$$\dot{\gamma} = \frac{1}{m.VT} (L + Ft.\sin(\alpha + \alpha_f) - m.g.\cos(\gamma)) \quad (24)$$

$$\dot{\alpha} = \dot{\theta} - \dot{\gamma} = q - \dot{\gamma} \quad (25)$$

$$\dot{\alpha} = q + \frac{1}{m.VT} (-L - Ft.\sin(\alpha + \alpha_f) + m.g.\cos(\gamma)) \quad (26)$$

$$\dot{\theta} = q \quad (27)$$

$$\dot{q} = \frac{1}{I_y} (M + Ft.Z_f) \quad (28)$$

$$\dot{h} = VT.\sin(\gamma) \quad (29)$$

$$\dot{p} = \frac{I_{zz}.L + I_{xz}.N - \{I_{xz} - (I_{yy} - I_{xx} - I_{zz}).p + [I_{xz}^2 + I_{zz}(I_{zz} - I_{yy})]r\}q}{(I_{xx}.I_{zz} - I_{xz}^2)} \quad (30)$$

$$\dot{r} = \frac{I_{xz}.L + I_{xx}.N - \{I_{xz}(I_{yy} - I_{xx} - I_{zz}).p + [I_{xy}^2 + I_{xx}(I_{xx} - I_{yy})]r\}q}{(I_{xx}.I_{zz} - I_{xz}^2)} \quad (31)$$

$$\dot{\phi} = p + (q.\sin(\phi) + r.\cos(\phi)).\tan(\theta) \quad (32)$$

$$\dot{\beta} = -r + \frac{1}{m.VT} (Y - Ft.\sin(\alpha + \alpha_f) + m.g) \quad (33)$$

3. FAULT-TOLERANT CONTROL

There are two most common fault tolerant control techniques; the passive fault-tolerant controllers (i.e. fixed robust feedback controllers) and the active fault-tolerant controllers. Here it will be discussed the active fault-tolerant technique that will lead to a system which can be adjusted according to the problem necessity. The control system (Fig. 2) has two functions, the first is to identify changes in parameters by fault detection and isolation (FDI), and the second is the actual controller that will be adapted according to the faults identified by the FDI.

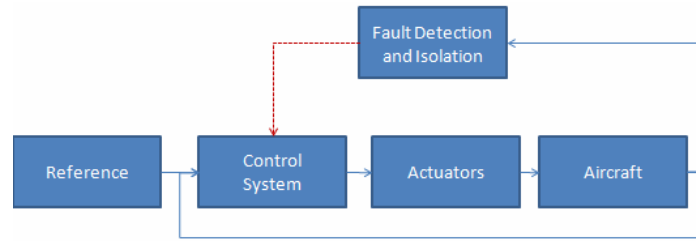


Figure 2. Fault Tolerant Control Diagram

This paper is not covering the first function of the system, but is focusing in the control system synthesis that will be based on nonlinear control.

3.1. Backstepping

The chosen nonlinear control technique is the backstepping, a technique based on the Lyapunov theory of stability. The objective of the development of a backstepping control is to find a Lyapunov function, that once found, means that the system is able to become stable. The backstepping works as a recursive design methodology to achieve the feedback control law and the Lyapunov function systematically. Its advantage is its nonlinearities treatment, in a way to take advantage of some helpful linearities that help to stabilize the system. First, a small system is considered and a virtual controller is designed for it, then recursively the design is conducted to have control laws for the whole system. Together with the control law a Lyapunov function is created. Some control Lyapunov functions are used in the process.

Backstepping can be applied to flight controls, in this application the controller is designed to determine what are the desired fast rotational velocities ($\dot{\omega}$) in Eq. (34), \dot{p}_{ref} to the rolling angular velocity, \dot{q}_{ref} to the pitching angular velocity and \dot{r}_{ref} to the yawing angular velocity in order to control the aircraft to move according the desired control. The allocation technique will be responsible to distribute these desired values to the correct control surfaces depending on the existence of a fault.

$$\dot{\omega} = \begin{bmatrix} \dot{p}_{ref} \\ \dot{q}_{ref} \\ \dot{r}_{ref} \end{bmatrix} \quad (34)$$

Using this technique, it is possible to control the rolling velocity p , bank angle ϕ , pitching velocity q , angle of attack α , flight path angle γ and sideslip angle β . For the purpose of demonstrating the technique, here it will be demonstrated only the rolling velocity and the bank angle control.

3.1.1 Backstepping applied for second order systems

The bank angle model can be considered as a second order system, so it will be demonstrated the second order system approach of backstepping. We consider a second order system as Eq. (35), where w is a vector of states, $f(w_1, y)$ represents the nonlinearities and u is the control.

$$\begin{aligned} \dot{w}_1 &= w_2 + f(w_1, y) \\ \dot{w}_2 &= u \end{aligned} \quad (35)$$

The desired value of w_1 is w_1^{des} and defines a new system based on the error between the current and the desired equilibrium state, the result is Eq. (36).

$$\begin{aligned} x_1 &= w_1 + w_1^{des} \\ x_2 &= w_2 + f(w_1^{des}, y) \\ \Phi(w_1) &= f(x_1 + w_1^{des}, y) - f(w_1^{des}, y) = f(w_1, y) - f(w_1^{des}, y) \end{aligned} \quad (36)$$

This latest system results in $\Phi(0) = 0$ in equilibrium. Working on Eq. (35) it is possible to reach to Eq. (37).

$$\begin{aligned}\dot{x}_1 &= \Phi(w_1) + x_2 \\ \dot{x}_2 &= u\end{aligned}\quad (37)$$

Now we can deal with this system using the backstepping. The first step is to create a virtual function for the system Eq. (36). The most general form of control law used is:

$$x_2^{des} = -\Psi(x_1) \quad (38)$$

Then a control Lyapunov function Eq. (39), is proposed.

$$W(x_1) = \frac{1}{2} x_1^2 \quad (39)$$

Lyapunov stability theory proves that, if the derivative of this function is less than zero the system will be stable. Deriving Eq. (39) the result is Eq. (40).

$$\dot{W} = x_1(\Phi(x_1) + x_2) = x_1(\Phi(x_1) - \Psi(x_1)), x_1 \neq 0 \quad (40)$$

In order to this function results in $\dot{W} < 0$, it is needed to choose a good value of $\Psi(x_1)$.

The second step is to consider the residual of the errors caused by the difference between the current state x_2 and the desired state x_2^{des} , as in Eq. (41).

$$\tilde{x}_2 = x_2 - x_2^{des} = x_2 + \Psi(x_1) \quad (41)$$

Rewriting Eq. (37), considering the errors of Eq. (41) the result is the system Eq. (42).

$$\begin{aligned}\dot{x}_1 &= \Phi(x_1) - \Psi(x_1) + \tilde{x}_2 \\ \dot{x}_2 &= u + \Psi'(x_1)(\Phi(x_1) - \Psi(x_1) + \tilde{x}_2)\end{aligned}\quad (42)$$

Also, for this system, recursively, there is a CLF of the form of Eq. (43).

$$V(x_1, x_2) = F(x_1) + \frac{1}{2} \tilde{x}_2^2 \quad (43)$$

This CLF uses a non quadratic function in order to avoid the control u of canceling the dependencies of x_1 on \tilde{x}_2 . $F(x_1)$ is any valid CLF for the system x_1 , resulting in Eq. (44)

$$\dot{F}(x_1) \Big|_{x_2=x_2^{des}} = F'(x_1)(\Phi(x_1) - \Psi(x_1)) = -U(x_1) \quad (44)$$

Differentiating Eq. (43) we get to:

$$\dot{V} = -U(x_1) + \tilde{x}_2 \cdot [F'(x_1) + u + \Psi'(x_1)(\Phi(x_1) - \Psi(x_1)) + \Psi'(x_1) \cdot \tilde{x}_2] \quad (45)$$

It is possible to simplify this equation, choosing a $F'(x_1)$ that cancels some terms of x_1 :

$$F'(x_1) = -\Psi(x_1)(\Phi(x_1) - \Psi(x_1)), F(0) = 0 \quad (46)$$

$$U(x_1) = \Psi'(x_1)(\Phi(x_1) - \Psi(x_1))^2 \quad (47)$$

Now Eq. (45) becomes Eq. (48)

$$\dot{V} = -U(x_1) + \tilde{x}_2 \cdot [u + \Psi'(x_1) \cdot \tilde{x}_2] \quad (48)$$

Here also, \dot{V} needs to be negative, so u needs to dominate the term $\Psi'(x_1) \cdot \tilde{x}_2^2$. If $\Psi'(x_1)$ is limited, a possibility is a linear control like Eq. (49).

$$u = -k \cdot \tilde{x}_2 = -k(x_2 + \Psi(x_1)) \quad (49)$$

One requirement of Eq. (47) is that $k \geq \max_{x_1} \Psi'(x_1)$ to achieve the negative CLF:

$$\dot{V} = -U(x_1) - (k - \Psi'(x_1)) \cdot \tilde{x}_2^2 \quad (50)$$

Then, there is a control law based on Eq. (51), in which:

$$(\Phi(x_1) - \Psi(x_1)) \cdot x_1 < 0, x_1 \neq 0 \quad (51)$$

From this point, it is still missing a real control law to be applied. Considering the system in Eq. (37) we have that a globally asymptotically stable (GAS) form that partially linearizes the dynamic is given by Eq. (52).

$$u = -k_2(x_2 + k_1 \cdot x_1 + \Phi(x_1)) \quad (52)$$

Where,

$$\begin{aligned} k_1 &> \max\{0, -\min \Phi'(x_1)\} \\ k_2 &> k_1 + \max_{x_1} \Phi'(x_1) \end{aligned} \quad (53)$$

Admitting that the maximum and minimum values of $\Phi'(x_1)$ exist.

3.1.3. Backstepping applied for bank angle control

The backstepping control developed for second order system can be applied to the bank angle control of the aircraft, modeled by a simplified version of Eq. (32), where r , q are neglected and Eq. (34). These equations must be rearranged to achieve the following system, the desired command u is a roll velocity applied to the control allocation.

$$\dot{\phi} = p + (q \cdot \sin(\phi) + r \cdot \cos(\phi)) \cdot \tan(\theta) \quad (54)$$

$$\dot{p} = u_1 \quad (55)$$

$$u_1 = \frac{I_{zz} \cdot L}{(I_{xx} \cdot I_{zz} - I_{xz}^2)} \quad (56)$$

We must search for the nonlinearities on this equations and considering Eq. (54) it is possible to consider that the nonlinearities are developed as Eq. (57).

$$f(\phi, y) = (q \cdot \sin(\phi) + r \cdot \cos(\phi)) \cdot \tan(\theta) \quad (57)$$

The states of this system can be defined as:

$$\begin{aligned} w_1 &= \phi \\ w_2 &= p \\ u &= u_1 \\ x_1 &= w_1 - w_1^{des} = \phi - \phi^{des} \\ x_2 &= p - f(\phi, y) \end{aligned} \quad (58)$$

Now, it is proposed the use of the control defined on Eq. (52) that will result in Eq. (59). The executed simulations demonstrate that this control, although stable, results in a non null error on steady state. This way, it is necessary to include a integral command on the controller, achieving the control of Eq. (60).

$$\begin{aligned} u &= -k_2(x_2 + k_1 \cdot x_1 + \Phi(x_1)) \\ u &= -k_p \left(p + k_\phi (\phi - \phi_{ref}) + f(\phi, y) \right) \end{aligned} \quad (59)$$

$$u = -k_p \left(p + k_\phi \left(1 + \frac{1}{s} \cdot k_{\phi_int} \right) (\phi - \phi_{ref}) + f(\phi, y) \right) \quad (60)$$

Until now, we have the modeled system for bank angle and the GAS controller to achieve stability. The control designed in Eq. (60) generates a demand of \dot{p} , but this are not yet the needed inputs for the control allocation scheme. What is necessary is the desired quantity of movement (L) that needs to be created by the control surfaces. The rolling moment of an aircraft is defined as Eq. (61). Where Cl is the aerodynamic coefficient for the rolling moment, ρ is the air density at the current altitude, VT is true airspeed, S is the wing surface and l is the aircraft mean aerodynamic chord. For the calculus of Cl , there is Cl_0 that is the fixed rolling moment (usually zero for symmetric aircraft), Cl_β the rolling coefficient related to the sideslip angle β , Cl_p the rolling coefficient related to the rolling rate, Cl_r the rolling coefficient related to the yaw rate, Cl_{la} , Cl_{ra} , Cl_{le} and Cl_{re} the rolling coefficients related to the deflection of the left aileron, right aileron, left elevator and right elevator respectively. Only aerodynamic moment is considered here.

$$L = \frac{1}{2} \cdot \rho \cdot VT^2 \cdot S \cdot l \cdot Cl \quad (61)$$

$$Cl = Cl_0 + Cl_\beta \cdot \beta + Cl_p \cdot \frac{p \cdot l}{VT} + Cl_r \cdot \frac{r \cdot l}{VT} + Cl_{la} \cdot \delta a + Cl_{ra} \cdot \delta ra + Cl_{le} \cdot \delta le + Cl_{re} \cdot \delta re \quad (62)$$

Equation (62) is one of the most important differences in this fault tolerant control, because it considers all the control surfaces independently. Usually for Cl calculus it is considered a global value for both ailerons deflection and a global value for both elevators deflection, here they are considered separate so the system will be able to control them one by one.

3.2. Control allocation

It will be used control allocation methods to calculate the necessary aerodynamic control surfaces deflections to generate the desired rolling moment. The advantage of the control allocation can be seen when used in system with a large quantity of actuators. Using control allocation, it is possible to release the controller from determining which control surface will be used. In this design the control allocation is used to determine which surface will be used to compensate the detected failure.

The method to be demonstrated is a control allocation based on Ducard (2009), capable of compensate a generated failure. Considering that there is a fault identification and isolation system that identifies correctly the failure, the control allocation chooses a determined algorithm to each situation, without the need to redesign the controller for each case.

The controller shall generate a virtual command $Cv=[Cl \ Cm \ Cn]^T$ for roll, pitch and yaw torque coefficient. The virtual command serves as input to the control allocation that will determine the needed surfaces deflection necessary in the current condition to achieve that torque.

In order to translate the virtual command on surface deflection, it is needed to solve the system on Eq.(63) .

$$\begin{bmatrix} Cl \\ Cm \\ Cn \end{bmatrix} = \begin{bmatrix} Cl_{la} & Cl_{ra} & Cl_{le} & Cl_{re} & Cl_r \\ Cm_{la} & Cm_{ra} & Cm_{le} & Cm_{re} & Cm_r \\ Cn_{la} & Cn_{ra} & Cn_{le} & Cn_{re} & Cn_r \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta a \\ \Delta \delta ra \\ \Delta \delta le \\ \Delta \delta re \\ \Delta \delta r \end{bmatrix} \quad (63)$$

In this system we have three inputs from the backstepping controller Cl , Cm and Cn , and five unknowns to be solved (the surfaces deflection variation), the others are aircraft parameters. So, it is a system that cannot be solved directly, to solve this we need to consider the failure cases.

3.2.1 Control allocation for a failure free scenario

Considering a failure free condition (all actuators working properly), it is possible to consider that the elevator deflection will not produce roll moment, neither ailerons deflection will produce pitch moment and we can go back to the usual notation using one global value for elevator deflection and one global value for aileron deflection according to Eq. (64) and Eq. (65).

$$\begin{aligned}\Delta\delta a &= \Delta\delta r a \\ \Delta\delta a &= \frac{\Delta\delta a + \Delta\delta r a}{2}\end{aligned}\quad (64)$$

$$\begin{aligned}\Delta\delta e &= \Delta\delta r e \\ \Delta\delta e &= \frac{\Delta\delta e + \Delta\delta r e}{2}\end{aligned}\quad (65)$$

With these considerations it is possible to transform Eq. (63) in Eq. (66). That can be easily solved.

$$\begin{bmatrix} \Delta Cl_{ref} \\ \Delta Cm_{ref} \\ \Delta Cn_{ref} \end{bmatrix} = \begin{bmatrix} Cl_a & 0 & Cl_r \\ 0 & Cm_e & 0 \\ Cn_a & 0 & Cn_r \end{bmatrix} \begin{bmatrix} \Delta\delta a \\ \Delta\delta e \\ \Delta\delta r \end{bmatrix}\quad (66)$$

3.2.2 Control allocation for an aileron failure scenario

In the event of failure of an aileron – here considered an aileron stuck on neutral position – the asymmetry between the ailerons positions causes the generation of an undesired pitch moment, inexistent on normal conditions. Since the pitch moment generated by one aileron is compensated by the pitch moment generated by the other aileron. In this failure situation the elevators will be used to compensate this undesired pitch moment.

For the control allocation, first the non faulty aileron deflection must be defined to compensate the lack of effectiveness of the other aileron, considering a left aileron failure, this is done by Eq. (67).

$$\Delta\delta r a = \frac{Cl}{Cl_{ra}}\quad (67)$$

As only the right aileron will be deflected, due to the left aileron failure, a pitch moment $Cm_{ar} \cdot \Delta\delta ar$ will be generated; this will be compensated by the elevators like in Eq. (68) and Eq. (69).

$$\Delta\delta e = \frac{Cm_{ra} \cdot \Delta\delta r a}{Cm_e}\quad (68)$$

$$\delta r e = \delta e = \frac{Cm_{ref}}{Cm_e} + \Delta\delta e\quad (69)$$

Besides the pitch moment compensation, there is the necessity to deflect the rudder, since the yaw moment will be also affected. The new rudder deflection value is on Eq. (71).

$$\Delta \delta_r = \frac{Cn_{ra} \cdot \Delta \delta_{ra}}{Cn_r} \tag{70}$$

$$\delta_{re} = \delta e = \frac{Cn_{ref}}{Cn_r} + \Delta \delta_r \tag{71}$$

The Matlab/Simulink® model used for this calculus is on Fig. 3a and if we consider the failure of the right aileron the calculus is done by the model of Fig. 3b.

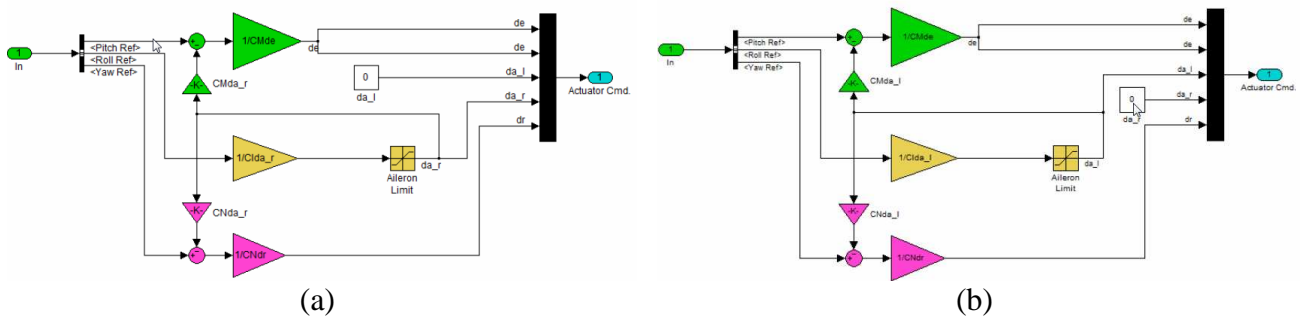


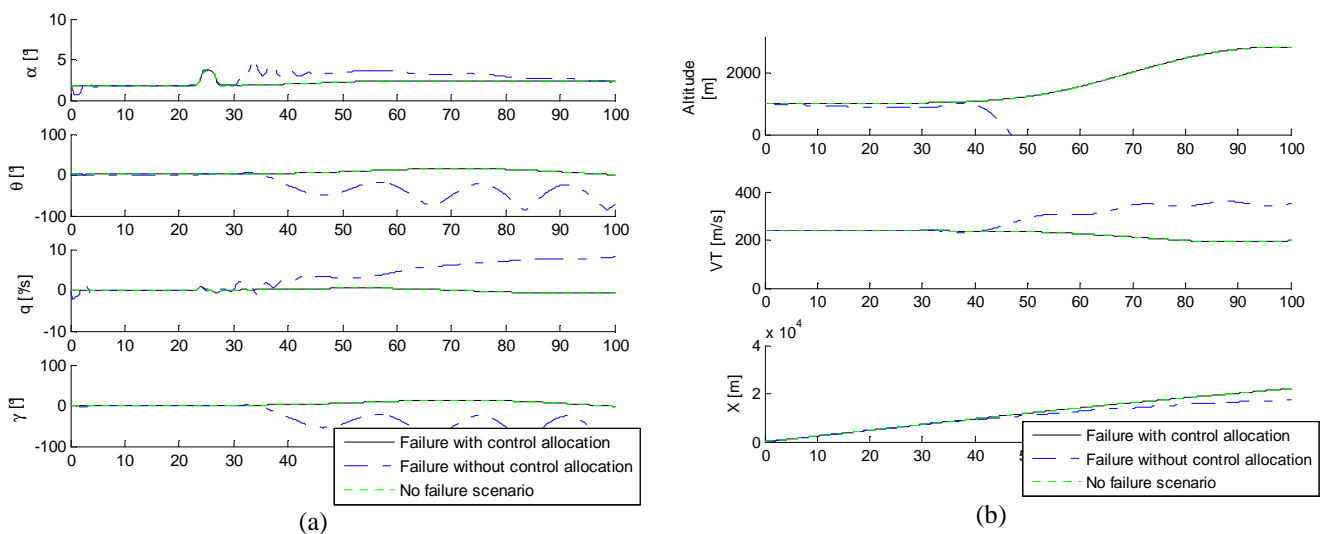
Figure 3. Control Allocation: (a) Left Aileron Failure, (b) Right Aileron Failure

4. RESULTS

The presented aircraft model, backstepping control and control allocation were simulated in the following initial trimmed conditions: altitude: 1000 ft, airspeed: 240 m/s, angle of attack and pitch angle: 1.81°, elevators deflection: -2.1°, throttle lever position: 69,78%. The result graphics demonstrate the non failure scenario, a failure scenario with the fault-tolerant control active and a failure scenario without the fault-tolerant control. A wind gust is applied close to 25 seconds of simulation.

The objective of the control demonstrated is to achieve and maintain 5 degrees of bank angle (ϕ) for 30 seconds and then return to bank angle equal to zero considering a left aileron failure.

In this simulation is possible to see in Fig. 4c that the correct bank angle was achieved and maintained during the 30 seconds in the fault free and in the fault-tolerant control scenarios. In the lines that demonstrates the failure without the fault-tolerant control an undesired pitch acceleration was generated, the desired bank angle was not achieved and the system became unstable in the lateral-directional mode (ψ and ϕ) after the wind gust and the return to zero bank angle. The aircraft loses altitude (Fig. 4b) very fast and attain zero altitude. This simulation demonstrates that the fault-tolerant control based on backstepping and control allocation would cause the aircraft to have a similar behavior with or without this failure, avoiding critical situations that could happen without the fault-tolerant control.



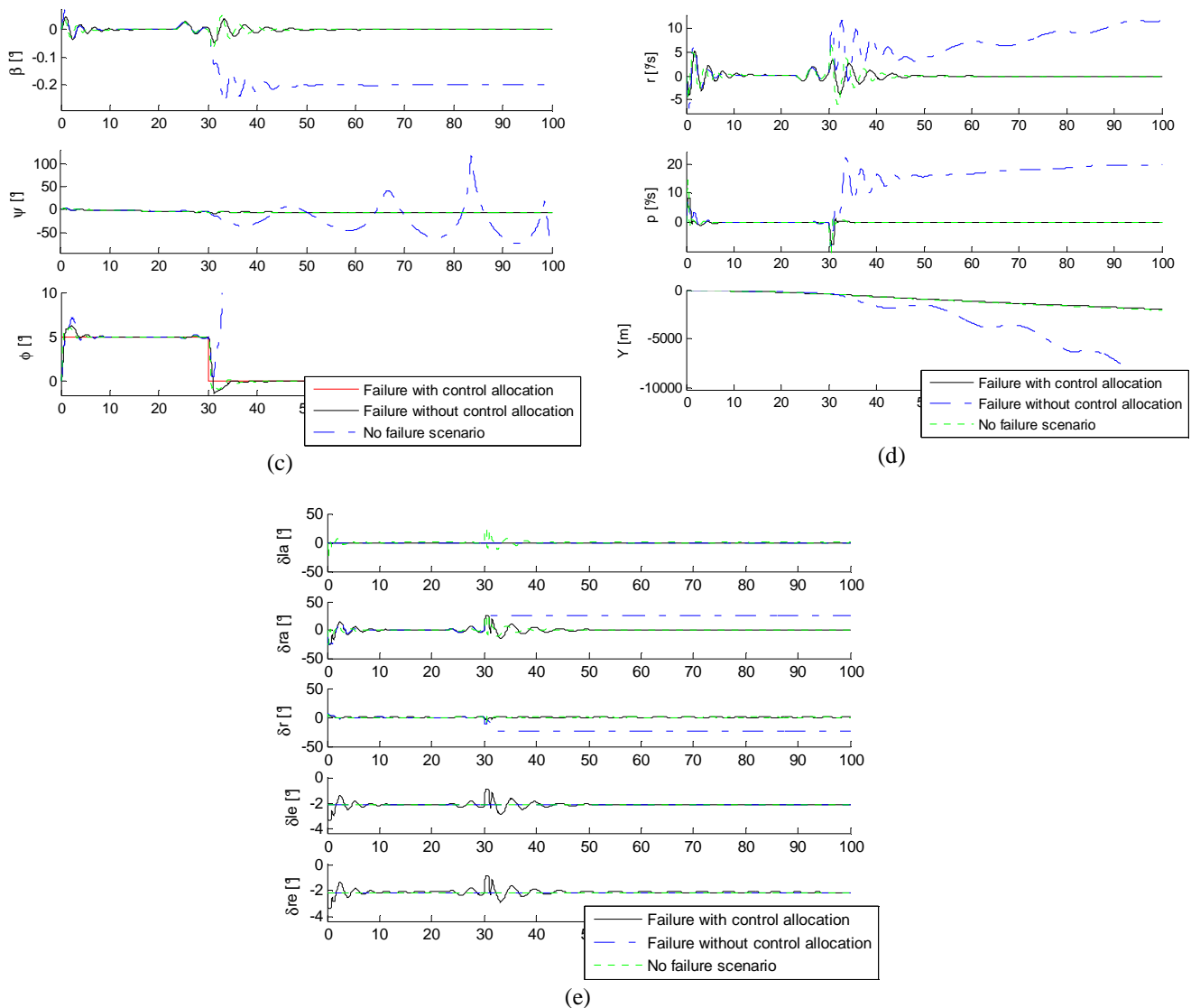


Figure 4. Roll Control (5 degrees) simulations with left aileron failure

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