# MODELING AND SIMULATION OF COOLING OF A FOOD PACKAGED IN GLASS CYLINDRICAL GEOMETRY

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***Abstract.*** *Food processing sometimes requires that the packaging and sealing is performed at high temperature. The commonly used container is glass. For production reasons it is necessary to bring the product packaging to the environmental temperature as quickly as possible so they can perform other processing steps. The glass container can suffer abrupt cooling thermal shock fracture, which requires to perform the cooling in stages. For companies with this type of process is important to have mathematical models that allow for analysis with a view to optimizing the process. The mathematical model of the cooling process of the food in glass is given by a system of partial differential equations (PDE) coupled at the interface food-glass and glass-environment. The solution is obtained by the numerical method of lines (MELIN) which has proven to be efficient for the numerical solution for these systems.*

***Keywords:*** *Density, thermal conductivity, specific heat, convection, stability, decoupled solution, Method of lines, cooling systems, thermal shock.*

## 1. INTRODUCTION

Even when the food packaging in glass at high temperature has long industrial methods, mathematical models describing how cooling occurs are really recent. In 1996, the multinational food company Kraft raised during the week of modeling in Melbourne Australia, the need for a model to describe the cooling process cheddar cheese in cylindrical glass containers, in terms of variables such as diameter container[1], in such a way that allowed them to adjust the process to produce a cooling. As discussed in Jenkins (1998) the proposed model required adjustments which were raised in it. In the week of modeling from the University of Kaiserslautern in 2002, we worked again on the issue raised by Kraft and obtained a linear model, assuming constant properties of food in time, as with temperature. In the present work proposes a cooling model that takes into account the properties of food which are temperature dependent, Section 2.it is possible to obtain a more complete model. By the type of model proposed is necessary to find the solution in numerical form, which is performed by the method of lines.

## 2. FOOD

Foods that humans and animals eat are made of protein ($p$), fat ($f$), carbohydrate ($c$), fiber ($fi$), ash ($a$) and water ($waf$) (frozen or not) and in principle difference between the different food groups to the percentage content of these constituents.

The physical properties of foods such as density ($\rho$), thermal conductivity ($k$) and specific heat ($c_p$) depend on the percentage of the constituents of food, and temperature, which are referenced in Toledo (1991) and specific studies in Dincer (1995), Yang *et al.* (2002), Fontana and et al (2001), Zhong *et al.* (2004) among others.

It is also possible to obtain expressions to determine the specific heat of food components as a function of temperature Toledo (1991).

The specific heat of the mixture at temperatures above the freezing point is given by Toledo (1991):

$$C_{avg} = P * C_{pp} + F * C_{pf} + C * C_{pc} + Fi * C_{pfi} + A * C_{pa} + M * C_{pwaf} \quad (1)$$

where $P$, $F$, $C$, $Fi$, $A$ and $M$ represent the mass fraction of protein, fat, carbohydrates, fiber, flour and water, respectively Toledo (1991). These equations for the specific heat were obtained by Choi & Okos (1987) by correlation procedures as indicated in Toledo (1991).

The thermal conductivity varies with the composition of food. Choi et al (1987) (Toledo (1991)) reported that the

[1]Kraft cheese packaged in such presentations and thus in various pack sizes, namely.

variation of thermal conductivity with composition Toledo (1991) is described by

$$k = \sum_i k_i X_{vi} \tag{2}$$

where $k_i$ is the conductivity of pure component, $X_{vi}$ is the volume fraction of component $k$ is the thermal conductivity of food. The units of conductivity are $W/m * K$.

Given the dependence of $k_i$ with temperature, the thermal conductivity of food depends on the temperature at which the food is Toledo (1991).

The volume fraction, $X_{vi}$ of each component indicates how much of the total volume for the component, it is determined from the mass fraction $X_i$, $\rho_i$ component density and composite density $\rho$ as Toledo (1991):

$$X_{vi} = \frac{X_i}{\rho_i}\rho \qquad \rho = \frac{1}{\sum_i (X_i/\rho_i)} \tag{3}$$

The densities of each component in $kg/m^3$, temperature dependent and are obtained from the following equations Toledo (1991), Water ($\rho_{waf}$), ice ($\rho_{ic}$), proteins ($\rho_p$), fat ($\rho_f$), carbohydrates ($\rho_c$), fiber ($\rho_{fi}$) and ash ($\rho_a$).

$\rho_{waf} = 997.18 + 0.0031439T - 0.0037574T^2$ $\quad \rho_{ic} = 916.89 - 0.13071T$ $\quad \rho_p = 1329.9 - 0.51814T$
$\rho_f = 925.59 - 0.41757T$ $\quad \rho_c = 1599.1 - 0.31046T$ $\quad \rho_{fi} = 1311.5 - 0.36589T$
$\rho_a = 2423.8 - 0.28063T$

In the case of cheese, with the following mass composition: $70\%$ water, $4\%$ protein, $14\%$ fat, $10\%$ carbohydrate and $2\%$ fiber. For our purposes it is interesting to consider how it changes the specific heat, thermal conductivity and density for the temperature range between $10°$C and $120°$C. As shown in Figures 1 and 2 show the behavior of these properties in the temperature range of interest. In simulations, the behavior of these thermal properties depend on the content of food components, fat, protein, etc. as mentioned in Marschoun *et al.* (2001).

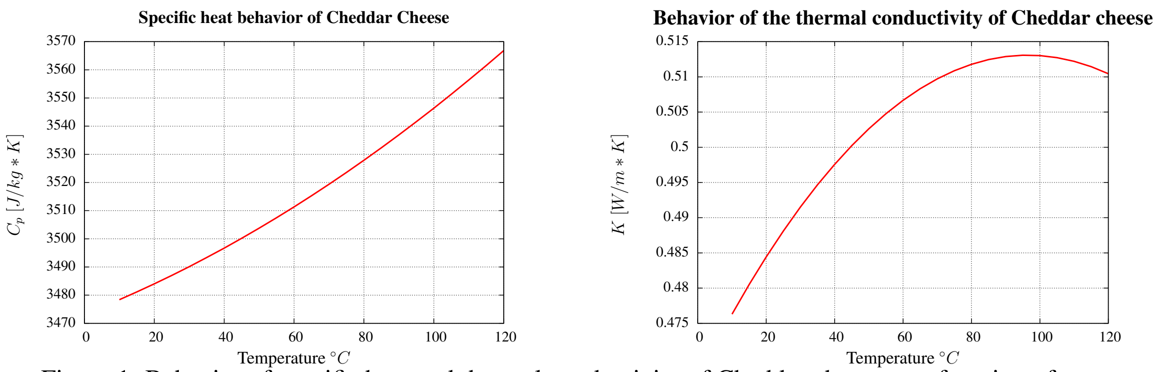


Figure 1. Behavior of specific heat and thermal conductivity of Cheddar cheese as a function of temperature.

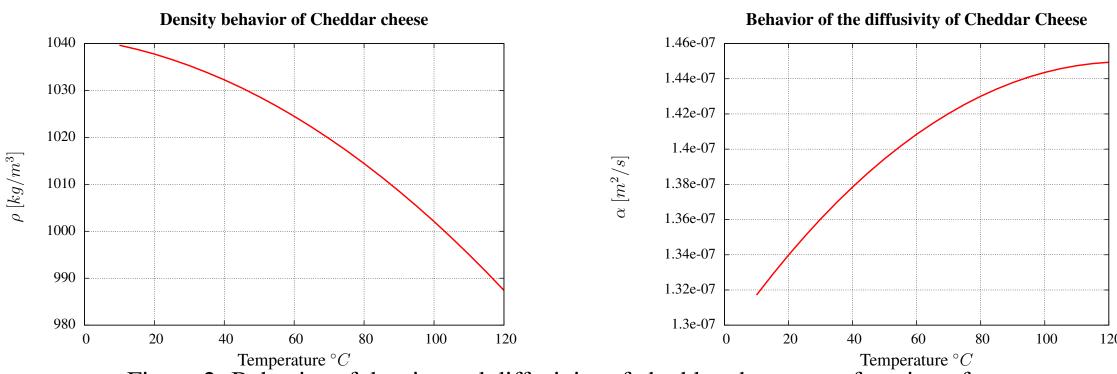


Figure 2. Behavior of density and diffusivity of cheddar cheese as a function of temperature.

## 3. MODEL

Consider a cooling process of food packaging which moves on a conveyor belt at a speed $\nu$, so do the packaged product is subjected to a temperature $T_0$ to a shower with a fluid, usually water, at room temperature $T_{amb}$, ($T_0 \geq T_{amb}$). Figure 3 illustrates a container during the cooling process.

The cooling process is described by heat transfer between the packaged food and fluid, and is not considered mass transfer of momentum and because the container is completely closed and there is no movement of food within the container. The equations describing the process for food (subscript a) and container (subscript e) are:

$$\frac{\partial(\rho_a c_a T_a)}{\partial t} = \nabla \cdot (k_a \nabla T_a) \qquad \frac{\partial(\rho_e c_e T_e)}{\partial t} = \nabla \cdot (k_e \nabla T_e) \qquad \rho = \rho(\vec{r}, t, T) \quad c = c(\vec{r}, t, T) \quad k = k(\vec{r}, t, T) \tag{4}$$

The properties, in general, may depend on the position, time and temperature.

The cooling occurs by conduction and convection heat: conduction between the food and container in the radial direction and convection between the food and the container lid and between the container and the environment. Figure 4 shows the different ways to transfer heat to the cooling process.

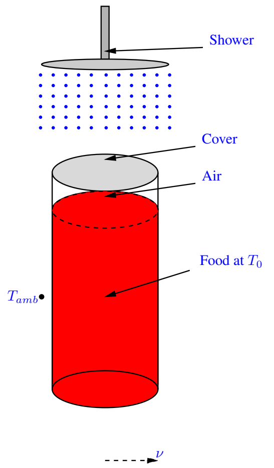


Figure 3. Cooling scheme of a food poured into a glass jar.

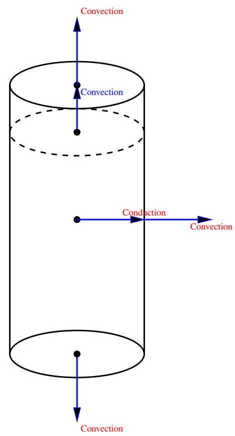


Figure 4. Forms of heat transfer in the food packaging during cooling.

Companies must ensure, quality control, once a food product manufactured, their properties must be maintained for a period of time which is specified as the expiration date. This allows us to consider in our models that the thermal properties of food does not depend on time. If one also considers that the food is isotropic then the cooling process can be described by equations

$$\rho_a c_a \frac{\partial T_a}{\partial t} = \nabla \cdot (k_a \nabla T_a) \qquad \rho_e c_e \frac{\partial T_e}{\partial t} = \nabla \cdot (k_e \nabla T_e) \tag{5}$$

When considering the geometry of the vessel and the temperature range does not generate a significant change in the properties of food, taking into account the mentioned above equations take the form

$$\rho_a c_a \frac{\partial T_a}{\partial t} = k_a \left( \frac{\partial^2 T_a}{\partial r^2} + \frac{1}{r} \frac{\partial T_a}{\partial r} \right) \qquad \rho_e c_e \frac{\partial T_e}{\partial t} = k_e \left( \frac{\partial^2 T_e}{\partial r^2} + \frac{1}{r} \frac{\partial T_e}{\partial r} \right) \tag{6}$$

To adequately characterize the process we add the boundary conditions and initial condition:

1. We have finite temperature at the origin ($r = 0$), this means that:

$$\left. \frac{\partial T_a}{\partial r} \right|_{r=0} = 0 \tag{7}$$

   which guarantees that no singularities are present in the term $\frac{1}{r}\frac{\partial T}{\partial r}$.

2. On the border between the container and the food ($r = R_1$) as there is continuity of the thermodynamic temperature and heat flux at the interface.

$$T_a|_{r=R_1} = T_e|_{r=R_1} \qquad \left. \frac{\partial T_a}{\partial r} \right|_{r=R_1} = \left. \frac{\partial T_e}{\partial r} \right|_{r=R_1} \tag{8}$$

3. On the border between the container and the medium ($r = R_2$), to account for cooling, has a heat exchange with the external environment according to Newton's law

$$-k_e \left. \frac{\partial T_e}{\partial r} \right|_{r=R_2} = h(T_{fluid} - T_e)|_{r=R_2} \tag{9}$$

4. In addition the system when starting the process (t = 0) has the following temperature values:

$$T_a(r,0) = T_0 \qquad T_e(r,0) = T_0 \tag{10}$$

It is noted that the mathematical description of a food cooling poured into a glass container and a shower is modeled by two partial differential equations coupled by the border, with the initial boundary condition mentioned.

The ratio between the thermal conductivity and density product with the specific heat diffusivity of the material known as $\alpha$ ($\alpha = \frac{k}{\rho c}$). For the food mentioned, we have: $k = 0.48W/m°\text{C}$, $\rho_a = 1037kg/m^3$ and $c_a = 3485kJ/kg°\text{C}$, in the case of a glass container used for this purpose $k_e = 0,78W/m°\text{C}$, $\rho_e = 2700kg/m^3$ and $c_e = 0.84kJ/kg°\text{C}$. With the above values have $\alpha_a = 1.33 * 10^{-7}m^2/s$ and $\alpha_e = 3.44 * 10^{-4}m^2/s$, respectively.

In order to observe the cooling in the container and the food at the same time scale as the time constants are very different, it is necessary to make a adimensionalización of equations, this will define the variables

$$\rho = \frac{r}{R_1} \quad 0 \le \rho \le 1; \qquad T' = \frac{T - T_{fluid}}{T_0 - T_{fluid}} \quad 1 \ge T' \ge 0; \qquad t_1 = \frac{t}{\tau} \quad \tau := \text{(material characteristic time)}$$

Replacing these variables in the original equations is reached

$$\frac{1}{\tau}\rho_i c_i \frac{\partial T_i'}{\partial t_1} = k_i \frac{1}{R_1^2}\left(\frac{\partial^2 T_i'}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T_i'}{\partial \rho}\right) \qquad \dots \qquad \frac{\partial T_i'}{\partial t_1} = \frac{\partial^2 T_i'}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T_i'}{\partial \rho}$$

where we find that the characteristic time of material is given by $\tau = \frac{R_1^2}{\alpha}$ being $\alpha$ the diffusivity of material.

$$\begin{aligned} \frac{\partial T_a}{\partial t} &= \frac{\partial^2 T_a}{\partial r^2} + \frac{1}{r}\frac{\partial T_a}{\partial r} \qquad 0 \le r \le 1; \quad 0 \le T_a \le 1; \quad t \ge 0 \\ \frac{\partial T_e}{\partial t} &= \frac{\partial^2 T_e}{\partial r^2} + \frac{1}{r}\frac{\partial T_e}{\partial r} \qquad 1 \le r \le \frac{R_2}{R_1}; \quad 0 \le T_e \le 1; \quad t \ge 0 \end{aligned} \tag{11}$$

where $R_2$ is the outer radius of the container and the inner radius $R_1$, $(R_2 > R_1)$.

With the new variables, the initial and boundary conditions take the form

$$\begin{aligned} & T_a(r,t)|_{r=1} = T_e(r,t)|_{r=1} \qquad -k_a \left.\frac{\partial T_a}{\partial r}\right|_{r=1} = -k_e \left.\frac{\partial T_e}{\partial r}\right|_{r=1} \qquad -k_e \left.\frac{\partial T_e}{\partial r}\right|_{r=\frac{R_2}{R_1}} = h\, T_e(r,t)|_{r=\frac{R_2}{R_1}} \\ & \left.\frac{\partial T_a}{\partial r}\right|_{r=0} = 0 \qquad T_a(r,t)|_{t=0} = T_e(r,t)|_{t=0} = 1 \end{aligned} \tag{12}$$

Using data for the glass used in most packaging and if found with an outer radius $28mm$ and a thickness of $1mm$ (a container with these dimensions is frequently used by food manufacturers), it must be time characteristic of the package is $\tau_e = \frac{R^2}{\alpha_e} = 1.39\ s$ while for the food is $\tau_a = \frac{R_1}{\alpha_a} = 5488.68s = 1.52h$.

Characteristic time ratio is $\tau_a/\tau_e = 3938$, indicating that the food takes much longer to reach the desired temperature than it takes the package, which means that the speed with which the container is cooled much larger than the corresponding food. Taking into account this situation the cooling process of food spillage can be modeled as

$$\rho_a c_a \frac{\partial T_a}{\partial t} = \nabla(k_a \nabla T_a) \qquad 0 = k_e \nabla^2 T_e \tag{13}$$

The initial and boundary conditions are given by equation 12.

## 4. DECOUPLED SOLUTION

As the equation describing the cooling in the container does not depend on time and is linear, $\nabla^2 T = 0$ the explicit analytical solution:

$$T_e(r) = E\ln(r) + F \qquad E, F \in \Re; \quad 1 \le r \le \frac{R_2}{R_1}; \qquad E = \frac{k_a}{k_e}\frac{\partial T_a(1,t)}{\partial r} \qquad F = T_a(1,t) \tag{14}$$

By applying the boundary conditions between the container and the food and after a few steps is the border

$$\left.\frac{\partial T_a}{\partial r}\right|_{r=1} = -\beta T_a(1,t) \qquad \beta = \frac{k_e}{k_a}\frac{h}{\frac{k_e}{(R_2/R_1)} + h\ln(\frac{R_2}{R_1})} \tag{15}$$

 

only depends on $T_a$ and coupling with the container is described by $\beta$ which is a constant that depends on the thermal properties of the materials involved in the cooling process, packaging, food and fluid, as well as geometric dimensions container.

It has been simplified dramatically since $T_a$ is sufficient to determine the problem decoupled.

$$\frac{\partial T_a(r,t)}{\partial t} = \frac{\partial^2 T_a(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T_a(r,t)}{\partial r} \quad 0 \le r \le 1 \tag{16}$$

with the boundary conditions and initial

$$\frac{\partial T_a}{\partial r} = -\beta T_a(r,t) \quad r = 1 \qquad \left.\frac{\partial T_a}{\partial r}\right|_{r=0} = 0 \qquad T_a(r,0) = 1 \tag{17}$$

This is a great simplification of the problem, as there are only reducing the number of equations.

## 5. NUMERICAL SOLUTION

To determine the solution using the method of lines, which has proven to be adequate to solve these equations Schiesser (1991).

Before performing the discretization process is necessary to consider the uncertainty that occurs in the equation for food at $r = 0$, and consider for the term L'hopital.

$$\frac{1}{r}\frac{\partial T_a}{\partial r} = \frac{\frac{\partial T_a}{\partial r}}{r} \tag{18}$$

In applying that rule holds:

$$\lim_{r\to 0}\frac{\frac{\partial T_a}{\partial r}}{r} = \frac{\lim_{r\to 0}\frac{\partial^2 T_a}{\partial r^2}}{\lim_{r\to 0} 1} = \frac{\partial^2 T_a}{\partial r^2} \tag{19}$$

With this, the equation for the food is

$$\frac{dT_a}{dt} = \begin{cases} 2\frac{\partial^2 T_a}{\partial r^2} & r = 0 \quad 0 \le T_a \le 1 \\ \frac{\partial^T_a}{\partial r^2} + \frac{1}{r}\frac{\partial T_a}{\partial r} & 0 < r \le 1 \quad 0 \le T_a \le 1 \end{cases} \tag{20}$$

Discretizing the spatial variable $r$ in units of $\Delta r$, the temperature at point $i$, is given by

$$T_i(t) = T(i\Delta r, t) \quad i = 1(1)N; \qquad \Delta r = \frac{1}{N+1} \tag{21}$$

thus obtains the discretized form of the equation for the first derivative

$$\frac{\partial T_a}{\partial r} = \begin{cases} \frac{-3T(1)+4T(2)-T(3)}{2\Delta r} & i = 1 \\ \frac{-T(i-1)+T(i+1)}{2\Delta r} & i = 2(1)N-1 \\ \frac{T(N-2)-4T(N-1)+3T(N)}{2\Delta r} & i = N \end{cases}$$

and the second derivative

$$\frac{\partial^2 T_a}{\partial r^2} = \begin{cases} \frac{-T(3)+4T(2)-5T(1)+2T(1)}{(\Delta r)^2} & i = 1 \\ \frac{T(i-1)-2T(i)+T(i+1)}{(\Delta r)^2} & i = 2(1)N-1 \\ \frac{-T(N-3)+4T(N-2)-5T(n-1)+2T(N)}{(\Delta r)^2} & i = N \end{cases}$$

Below are the simulation results obtained from the numerical solution for the system of equations resulting first order with the help of the Runge-Kutta-4 method. The number of discretization points used in ensuring the stability of the method, as indicated by Ames (1992). Figure 5 shows the temperature behavior by varying $r$ and $t$, considering a single cooling zone.

Figure 5 shows that at $r = 1$ the temperature changes quite quickly, whereas the variation in $r = 0$ is lower. This is because the speed with which cools the container is much greater than the rate of cooling of the food, we must remember the difference in characteristic times for container and for food. However, after approximately $0.6$ time constants ($0.912$ $h$), food is already at the temperature of the fluid.

## 6. FRACTURE IN THE GLASS JARS

Due to the impossibility of having a single cooling zone by the rupture of the container by the thermal stress is the need for several stages to complete the process.

The equations describing the heat exchange for different zones consist of the same expressions, seen before, how different are the fluid temperature and the initial temperature of the food and container for the relevant area.

Several cooling schemes can be raised, namely:

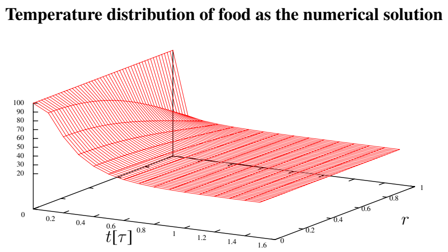


Figure 5. Temperature behavior in the food as a function of characteristic time and position.

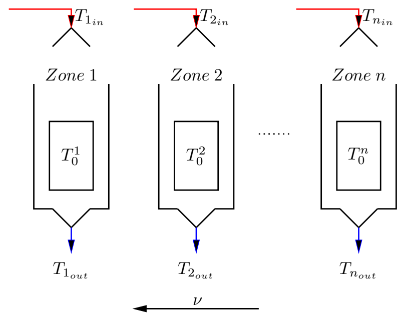


Figure 6. Independent cooling System Diagram.

### 6.1 Independent cooling systems

In this way, cooling fluid enters each of the zones at temperature $T_{in_{fluid_i}}$, which is suitable for the area. Withdrawal of food energy is removed by subjecting the fluid output to a cooling process, in order to be able to reuse the same zone. Figure 6 shows an outline of how this form of cooling.

This way of making the cooling has the advantage that the equations describing the process are independent cooling zones, which facilitates its solution. However, the problem is that energy is extracted and removed by cooling can be expensive.

### 6.2 Independent cooling systems heating

In this situation, the fluid coming out of an area $i$ at temperature $T_{out_{fluid_i}}$ is heated to the temperature $T_{in_{fluid_{i+1}}}$ before it is used for cooling in the area $i+1$ which requires a higher temperature to make the process cooling. Figure 7 shows an outline of how this form of cooling. This way of making the cooling has the advantage that the equations that describe all the zones are independent, as in the previous case which facilitates its solution, however the problem is, if it is a problem, is that Adder Required power so it can be reused which can be costly.

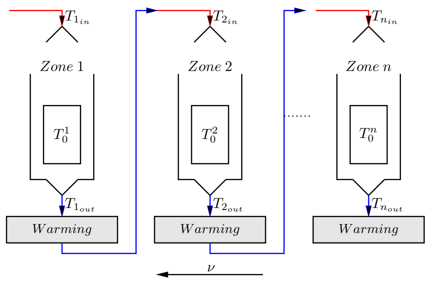


Figure 7. Cooling System Diagram independent heating.

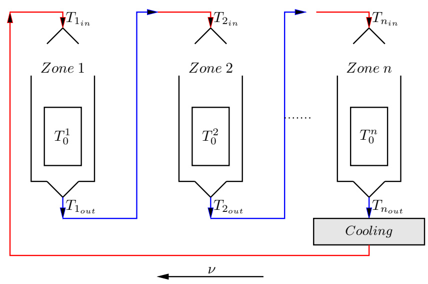


Figure 8. Cooling System Diagram interdependent without heating.

### 6.3 Interdependent cooling systems

In this case the fluid leaving each of the zones, $T_{out_{fluid_i}}$, circulated to the next higher temperature zone without heating, as in the previous case. Figure 8 shows a diagram of the operation. This way of making the cooling has the disadvantage that the equations describing the cooling process in all areas are not independent, as in previous cases, which hinders their solution. However, the advantage is that no energy is required to add or to heat or to cool the fluid out of different areas, with the exception of fluid coming out of the area $n$ which is cooled to the temperature required for the first zone, so that the whole process is carried out by recirculation of the cooling fluid, which allows you to design the system as a unit.

For each stage, if you leave aside the transfer of heat in the container, the transfer between the food and fluid can be expressed by:

$$c_a V_a \Delta T_a = c_f V_f \Delta T_f \tag{22}$$

$c_a$ and $c_f$ being the specific heat per unit volume for food and fluid respectively, $V_a$ and $V_f$ are the volumes and $\Delta T$ is the temperature variation in the food and fluid. The equation expresses the equality that must exist for heat transfer to effectively between food and fluid specific heat incorporates the two elements as well as the volume of these.

The temperature variation for the two materials in the $i-th$ region, are:

$$\Delta T_a = T^i_{a_b} - T^i_{a_e} \qquad \Delta T_f = T^i_{f_e} - T^i_{f_b} \tag{23}$$

$b$ and $e$ indicate where rates start and end respectively. Combining the last two equations gives:

$$T^i_{a_b} - T^i_{a_e} = \frac{c_f}{c_a}\frac{V_f}{V_a}(T^i_{f_e} - T^i_{f_b}) \tag{24}$$

Taking into account all zones have the system of equations

$$T^1_{a_b} - T^1_{a_e} = \frac{c_f}{c_a}\frac{V_f}{V_a}(T^1_{f_e} - T^1_{f_b}) \quad T^2_{a_b} - T^2_{a_e} = \frac{c_f}{c_a}\frac{V_f}{V_a}(T^2_{f_e} - T^2_{f_b}) \quad \ldots \quad T^n_{a_b} - T^n_{a_e} = \frac{c_f}{c_a}\frac{V_f}{V_a}(T^n_{f_e} - T^n_{f_b}) \tag{25}$$

This is a system of $n$ equations with $2n$ unknowns, inlet and outlet temperatures for the $n$ zones.

Taking into account the recirculation of the fluid, $T^i_{f_e} = T^{i+1}_{f_b}$ the previous system is reduced to a system of $n$ equations with $n+1$ unknowns. The unknowns are the fluid outlet temperature of each zone fluid temperature input to the whole system, which is called the $T^0_{f_b}$, thus

$$T^1_{a_b} - T^1_{a_e} = \frac{c_f}{c_a}\frac{V_f}{V_a}(T^1_{f_e} - T^0_{f_b}) \quad T^2_{a_b} - T^2_{a_e} = \frac{c_f}{c_a}\frac{V_f}{V_a}(T^2_{f_e} - T^1_{f_e}) \quad \ldots \quad T^n_{a_b} - T^n_{a_e} = \frac{c_f}{c_a}\frac{V_f}{V_a}(T^n_{f_e} - T^{n-1}_{f_e}) \tag{26}$$

based on the above, in this way to make cooling the solution of the problem requires, in addition to the solution of temperature for the food and packaging, the solution of the previous system of simultaneous equations which greatly complicates the problem.

In the cooling scheme one and two solutions for different areas are obtained similarly, the only thing different for the solution of each zone is the initial temperature of food and fluid. This cooling scheme is used to obtain the results shown below.

Figure 9 shows the temperature behavior in the food when you have two cooling zones. For this program was modified to admit the change in fluid temperature ($t = 0.8\tau$). In the first zone is intended that the food cools to a temperature that is $40°C$ lower than the initial temperature, holding for a time of $0.8\tau$ subject to the fluid temperature, then switch to a lower temperature fluid of $40°C$ near room temperature and let the food cool for a time of $0.7\tau$. Behavior is perceived as two cooling zones exhibits the same characteristics observed in the case of a cooling zone, Figure 9, ie the boundary between food and the container is cooled faster than the center of the food.

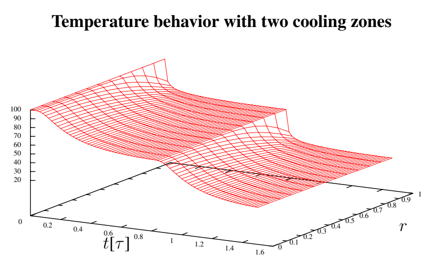


Figure 9. Temperature behavior when performing with two cooling zones.

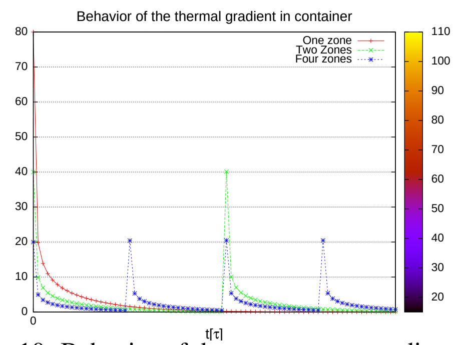


Figure 10. Behavior of the temperature gradient in the package as a function of the number of areas.

Since it is necessary to observe the behavior of the thermal gradient in the container, Figure 10, shown for different cooling zones.

Graph 10 shows that if you set only one cooling zone shows the rupture of the container by thermal stress, however, having two zones on the thermal gradient that occurs is $40°C$ which is close to the maximum value according to the manufacturer's packaging. When the cooling system uses four zones, the thermal gradient in the container does not exceed the critical gradient. Were placed equidistant in time zones, however, can be placed at different distances and the results are similar gradient.

The graph 11 can observe the behavior of the temperature for food when there are four cooling zones. You can see here that these four areas of cooling the food takes longer to come to room temperature, which is clearly shown in the diagram 10, this is mainly due to having increased the number of zones and the distance which has been placed in different areas. To reduce this effect should determine the appropriate distance to place the cooling zones. This is based, as shown in Figure 10, which can put food on the second cooling zone in a time less than $0.4\tau$ without this breaking of the container.

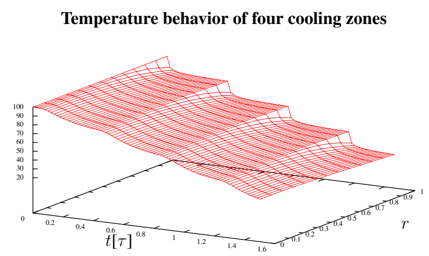


Figure 11. Temperature behavior of the food to be cooled with four cooling zones.

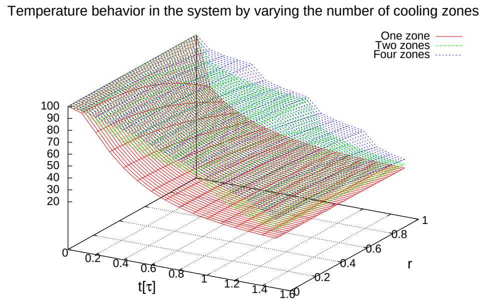


Figure 12. Temperature behavior of the food to be cooled with one, two or four cooling zones.

Deciding how far to long and at what temperature should drop the different cooling zones, and the number of zones, is something that depends on the food with which it is working because other factors come play, as is the speed which is required of the process without loss of sensory properties in food. Of course this is an issue that can be addressed as part of a master's degree or higher still.

Figure 11 can also be noted that the center of the food is cooled at a rate constant. This may or may not have consequences on the properties of food as well as on the potential for bacterial growth, as mentioned earlier, this depends on the food with which you are working.

To observe the effect of the number of zones in cooling to room temperature has been done in Figure 12 where it is represented the behavior of temperature for one, two and four cooling zones. Shown in the figure that increasing the number of time to wound areas at room temperature is increased, for the case shown.

It would certainly be interesting to perform a cooling with multiple zones, to prevent breakage of the container, but the temperature has a behavior as due to a single area.

## 7. CONCLUSIONS

Based on the foregoing it can be concluded before:

- The cooling process of a food expressed in this type of packaging depends on the characteristics of the food and container. What is presented here is valid only for the food in question, for other food is necessary to verify the assumptions, in particular the time scale of the equations.
- The cooling model is more complete with respect to that proposed by Jenkins (1998), in considering the variation of the thermal properties of food with temperature.
- PDE describing cooling are Laplace type for the container and quasilinear for food, coupled by the border. In this situation there is no analytical solution so that the use of numerical methods, in particular the method of lines, is essential.
- The ability to decouple the problem of two PDEs, coupled by the border, to a EDP for food appreciably simplify the problem, from the viewpoint of numerical computing power and computer equipment needed.
- Since the simplified problem contains all the information complete problem through the boundary condition for the food, it is possible to analyze the effect of changes in the properties and characteristics of food, container and fluid used for cooling.
- You can also study the amount of necessary cooling zones and temperatures that make the process time is optimal.

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