IMPLEMENTATION OF AN ANALYTICAL FORMULATION FOR ENERGY RELEASE RATE IN ADHESIVE JOINTS WITH A COHESIVE CRACK

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Abstract. *This paper describes a computer implementation of an analytical formulation developed by Krenk (1992)[1] for cracked bonded joints. This formulation was based on elasticity theory to derive analytical expressions for the energy release rate in mode I, mode II, and mixed mode. This formulation can be used for fatigue failure prediction in adhesive joints under cyclic loading. The computational implementation was built using MATLAB® language. The software presents a friendly graphical user interface to facilitate their use in engineering analysis.*

Keywords*: adhesive joints, energy release rate, analytical solution, computer implementation.*

1. INTRODUCTION

The bonded, riveted, and bolted joints to assemble components or structural parts is often used in aeronautic industries. With recent development of new materials and new manufacturing techniques, bonded joints have been increasingly used due to some distinct advantages over traditional riveted and bolted ones, namely: more efficient load transfer, better sealing, better finishing and, most important for aeronautical applications, less weight. The design of bonded joints is based upon analyses to estimate peeling and shear stresses in the adhesive and the displacement field along the bonded region. Over the last 50 years simple models based on the representation of the adherends as beams have been developed [4], [5]. In the first of this models, [2], only extension of the adherends is accounted for, and the adhesive is assumed to be in a state of pure shear. In practice most joint geometries lead to some measure of normal stress, the peel stress, at the extremities of the adhesive layer. Beam models for the single lap joint with bending and normal stress were developed by Goland&Reissner [3].

With the success of fracture mechanics, and the use of the associated concept of the stress intensity factors defining a local stress singularity, the relevance of beam-type finite stress concentrations has been questioned. Krenk [1] showed that the expression for the energy release rate in terms of the finite stress concentrations is similar in form to the expression in terms of stress intensity factors, known from two-dimensional elasticity theory. If a crack propagates in the adhesive, this equivalence permits the derivation of simple explicit formulas for the stress intensity factors K_I and K_{II} in terms of the finite stress concentrations from beam theory. The implication is that if the beam approximations are good enough for keeping track of overall energy balance of the joint, then the stress intensity factors associated with the local crack tip behavior can be calculated accurately from the beam theory results. This suggests that, even for some mixed mode adhesive joint configurations, extensive finite element calculations are not necessary for the determination of local stress field characteristics. Naturally the possibility of interface cracks needs to be considered, but the unique split in mode I and mode II contributions with respect to the line of symmetry suggests that the ratio between the finite peel and shear stress can be used to describe the ratio between mode I and mode II in the near-field.

2. MATHEMATICAL DEVELOPMENT

The energy release rate associated with the growth of a crack is the energy that becomes available by a unit increase of the crack area A when the load is kept constant. The connection between the energy release rate and the compliance of the cracked body is used by Krenk [1] to develop its formulation. The principle is illustrated in Fig.1.

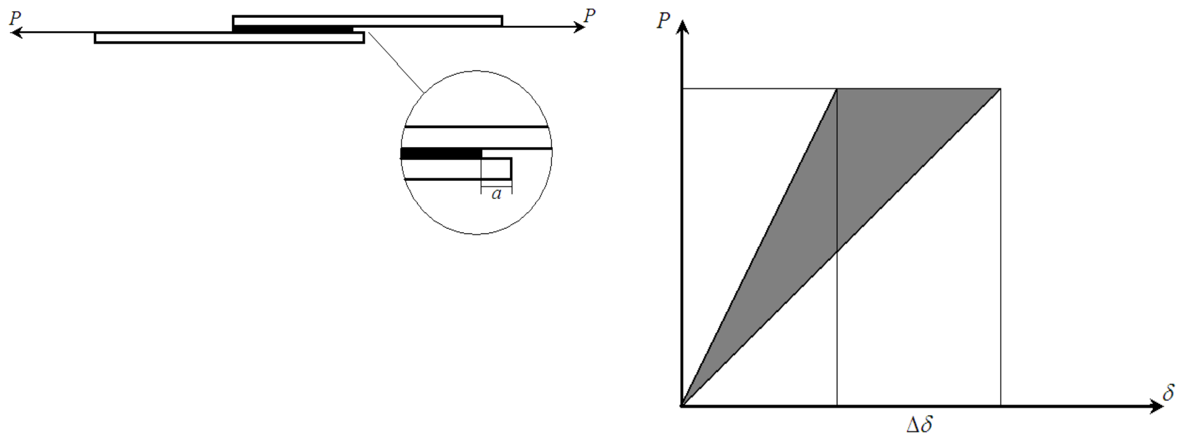

Fig.1. Energy release rate and compliance

For the sake of argument, the load consists of two opposing forces P. With the original crack area A, the mutual displacement of two forces is:

$$\delta = C(A)P \tag{1}$$

where C is the compliance. If the crack area is extended by ΔA, the additional displacement is:

$$\Delta\delta = (C(A+\Delta A) - C(A))P = \frac{\partial C}{\partial A}\Delta A P \tag{2}$$

The associated external work is $P\Delta\delta$, while the associated change in the internal energy, shown as the hatched area in Fig.1, is $\frac{1}{2}P\Delta\delta$. Thus, the energy release rate is:

$$G = \frac{\partial}{\partial A}(V-U) = \frac{1}{2}\frac{\partial C}{\partial A}P^2 \tag{3}$$

It is easily shown that the same formula holds for crack growth under fixed relative displacement δ.

2.1. Formulation development for Mode I

Krenk [1] has used a double cantilever beam (DCB) geometry to develop the formulation for mode I. The classic DCB test specimen is shown in Fig.2. Two identical beams of rectangular cross-section with height h and width b are joined by a thin adhesive layer of thickness t. The adhesive bond length is l, leaving two cantilever beams of length a. Fig. 2 shows symmetric loading (mode I) by two opposing forces of magnitude P.

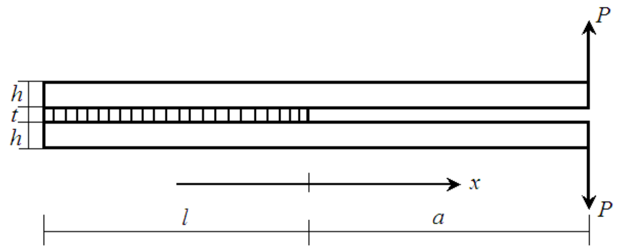

Fig.2. Double cantilever beam, mode I

The adhesive layer is approximated by springs with stiffness $E_a^{'}/t$ per unit area, where the plane strain value $E_a^{'} = E_a/(1-\nu^2)$ of the adhesive elastic modulus is used on account of the constraints. Due to the symmetry of the problem, each beam can be treated separately as a beam on elastic foundation. The bending stiffness of the individual beam is $EI = \frac{1}{12}Ebh^2$ and the associated spring stiffness is $k = 2E_a^{'}b/t$. The corresponding differential equation is:

$$\frac{d^4w}{dx^4} + 4\lambda_\sigma^4 w = 0 \tag{4}$$

where the parameter λ_σ is defined by:

$$\lambda_\sigma^4 = \frac{k}{4EI} = \frac{6}{h^3 t}\frac{E_a^{'}}{E} \tag{5}$$

λ_σ^{-1} stands fo as a length scale of the mode I problem. Along the bonded length of the beam, $x \le 0$, the displacement is:

$$w(x) = \frac{2P\lambda_\sigma}{k} e^{(\lambda_\sigma x)}.((1+\lambda_\sigma a)\cos(\lambda_\sigma x) + \lambda_\sigma a.sen(\lambda_\sigma x)), \quad x \le 0 \tag{6}$$

This solution provides the kinematic boundary conditions for the cantilever beam:

$$w(0) = \frac{2P\lambda_\sigma}{k}(1+\lambda_\sigma a) \tag{7}$$

$$\frac{dw(0)}{dx} = \frac{2P\lambda_\sigma^2}{k}(1+2\lambda_\sigma a) \tag{8}$$

The maximum peel stress occurring at the end of the bond line also follows from (6):

$$\sigma_{\max} = \frac{k}{b}w(0) = \frac{2P\lambda_\sigma}{b}(1+\lambda_\sigma a) \tag{9}$$

Applying concepts of differential and integral calculus, it is possible to obtain an expression for the energy release rate in terms of the local finite stress concentration $\sigma_{\max}$:

$$G_I = \frac{1}{2}\frac{t}{E_a^{'}}\sigma_{\max}^2 \tag{10}$$

Usually, the energy release rate of the DCB is expressed directly in terms of the moment $P.a$ and the bending stiffness of the cantilevers EI . The equation (10) can be written in the form:

$$G_I = \frac{(Pa)^2}{bEI}\left(1+\frac{1}{\lambda_\sigma a}\right)^2 \tag{11}$$

In the case of a stiff adhesive layer $\lambda_\sigma a \gg 1$, the last factor is close to unity. The important point here is that the local representation (10) remains valid in this limit, although $\sigma_{\max}$ may not provide a very accurate representation of the actual stress concentration in the adhesive.

2.2. Formulation development for Mode II

Fig.3 shows the DCB loaded in mode II. Thickness t of the adhesive layer is assumed to be small in relation to the thickness h of the adherends. Due to antisymmetry of the load the peel stress vanish identically.

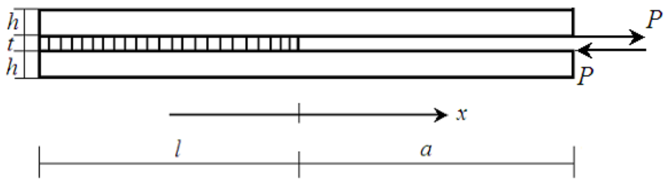

Fig.3. Double cantilever beam, mode II

Thus, the adherends are loaded only by a shear force $b\tau(x)$ at the bond line. With normal forces N_1 and N_2 in the two adherends, the equilibrium conditions are:

$$b\tau = \frac{dN_1}{dx} = -\frac{dN_2}{dx} \tag{12}$$

When $M_1 = N_1.h/2$ is the bending moment in adherend no.1 around its center line, the axial strain in adherend no.1 at the bond line is:

$$\varepsilon_1 = \frac{dN_1}{Ebh} + \frac{M_1}{EI}\frac{h}{2} = \frac{4N_1}{Ebh} \tag{13}$$

The compatibility over the adhesive layer requires:

$$u_1 - u_2 = \frac{t}{G_a}\tau \tag{14}$$

where G_a is the shear modulus of the adhesive. Differentiation and use of (12) and (13) gives:

$$\frac{d^2\tau}{dx^2} - \lambda_\tau^2 \tau = 0 \tag{15}$$

where the parameter λ_τ is defined by:

$$\lambda_\tau^2 = \frac{8}{th}\frac{G_a}{E} \tag{16}$$

The solution is given by:

$$N_1(x) = -N_2(x) = P\frac{senh\lambda_\tau(l+x)}{senh(\lambda_\tau l)}, \quad -l < x < 0 \tag{17}$$

from which:

$$\tau(x) = \frac{P\lambda_\tau}{b}\frac{\cosh\lambda_\tau(1+x)}{senh(\lambda_\tau l)}, \quad -l < x < 0 \tag{18}$$

The maximum shear stress occur at the end of the bond line:

$$\tau_{\max} = \frac{P\lambda_\tau}{b}\coth(\lambda_\tau l) \tag{19}$$

Finally, applying concepts of differential and integral calculus and fracture mechanics,it is possible to derive the expression of the mode II energy release rate in terms of the local shear stress concentration $\tau_{\max}$:

$$G_{II} = \frac{1}{2}\frac{t}{G_a}\tau_{\max}^2 \tag{20}$$

Also in this case the energy release rate can be expressed with the main emphasis on the properties of the cantilever beams:

$$G_{II} = \frac{4h}{E}\left(\frac{P}{bh}\right)^2 \coth^2(\lambda_\tau l) \tag{21}$$

2.3. Single lap joint, mixed mode

In practice the geometry and loading of most adhesive joints lead to a mixture of mode I and mode II. In these cases, it is important to obtain an estimate of the energy release rate as well as of the ratio between the mode I and mode II contributions. The Fig.4 shows a centrally loaded symmetric lap joint. This is a simple but typical example of a mixed mode adhesive joint.

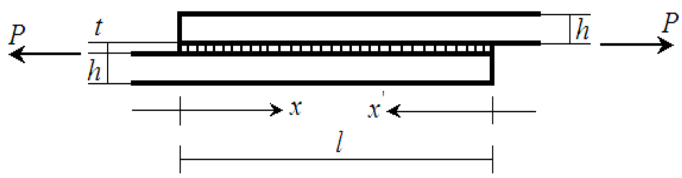

Fig.4. Centrally loaded symmetric lap joint

It is convenient to split the load into a symmetric and an antisymmetric part corresponding to mode I and mode II, respectively, as shown in Fig.5.

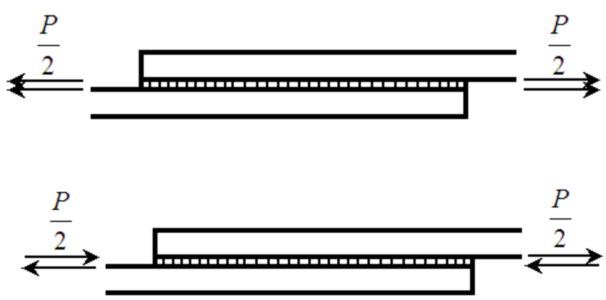

Fig.5. Separation of load into modes I and mode II

So, each part of the problem can now be solved independently by proceeding in the same way as in previous sections. Next, the mixed mode energy release rate for the centrally loaded lap joint is given by:

$$G_{mix} = \frac{1}{2}\frac{t}{E_a'}\sigma_{\max}^2 + \frac{1}{2}\frac{t}{G_a}\tau_{\max}^2 \tag{22}$$

This formula is a natural generalization of (10) and (20) for mode I and mode II of the double cantilever beam. The energy release rate is equal to the strain energy in the adhesive per unit area ahead of the crack tip.

3. COMPUTATIONAL IMPLEMENTATION AND RESULTS

The main objective of this paper was to develop a user-friendly software, which calculates the energy release rate in mode I, mode II and mixed mode. The software allows interaction with a data base. This will be useful when there exist a considerable amount of data that needs to be processed, Ex.: When normalized tests are carried on. The analysis data is input manually by using the MATLAB® interface or by reading the internal data base. The fowchart of the software is shown in Fig.6. The software interface is shown in Fig.7 and results obtained with the software are shown in Fig.8.

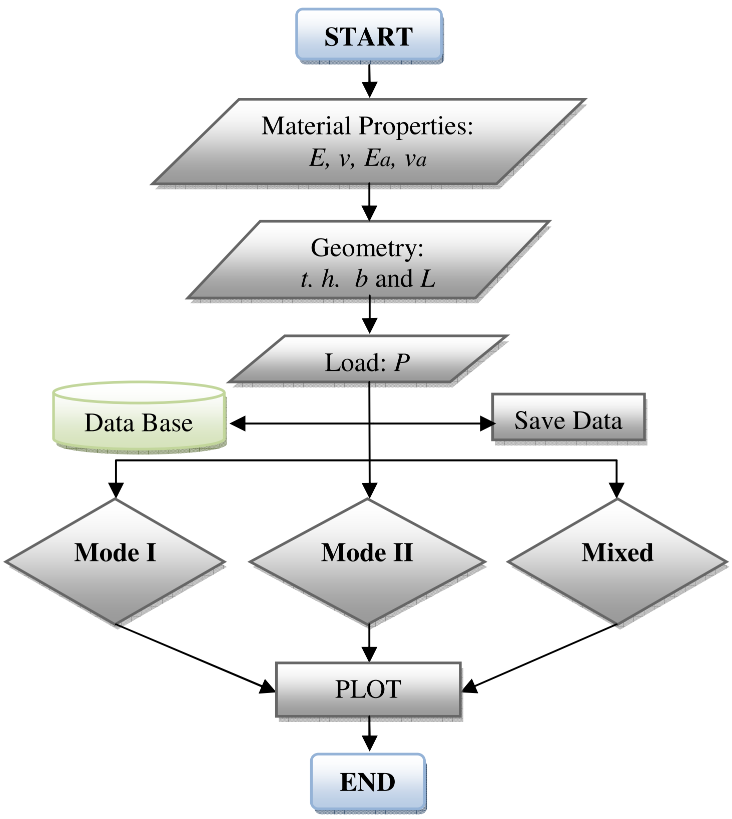

Fig.6. Flux diagram of the software

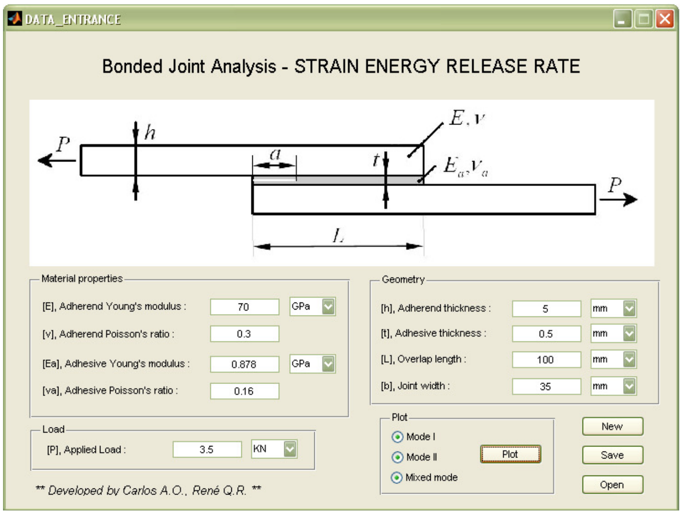

Fig.7. Program developed by the GUI (Graphical User Interface) of MATLAB®

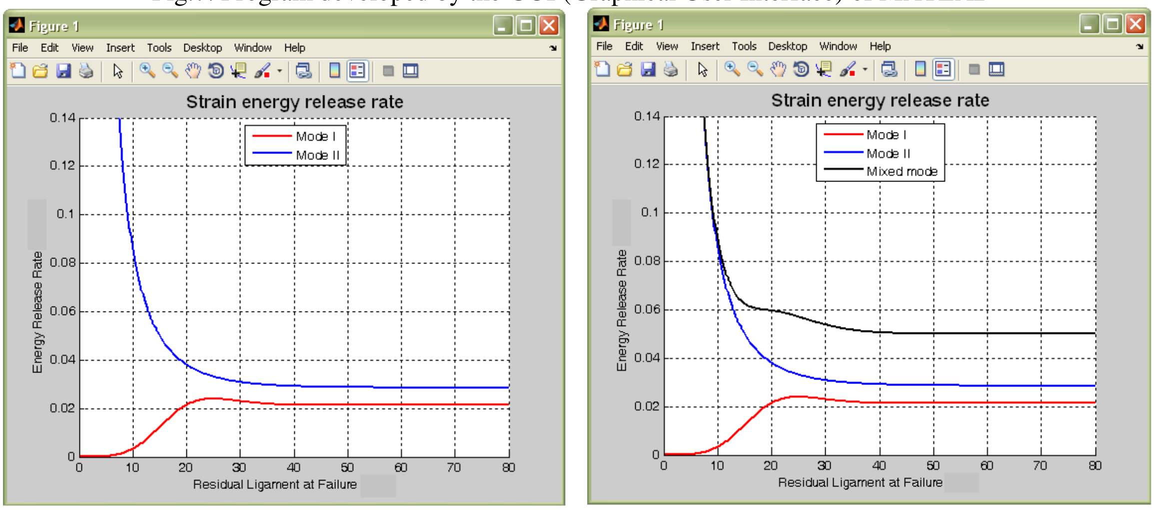

Fig.8. Figures created by the program, residual ligament vs. energy release rate in the three modes

3.1. Sensitive study

For a better understanding of the behavior of the energy release rate was performed a sensitive study by increasing the adhesive thickness t from 0.1 to 0.5 mm. It was considered a single lap joint geometry, as shown in Fig.9. The adherend used was aluminium 6082-T4, which have an elasticity modulus E of 70 GPa and a Poisson ratio ν of 0.32. The adhesive has elastic modulus E_a of 878MPa and a Poisson ratio ν_a of 0.16. Adherend thickness h is the same for both adherends and is equal to 5 mm. The applied load P considered was 3.5 kN, the joint length L was 80 mm and the joint width b was 35mm. Results are shown in Figures 10, 11 and 12 for the three modes, respectively.

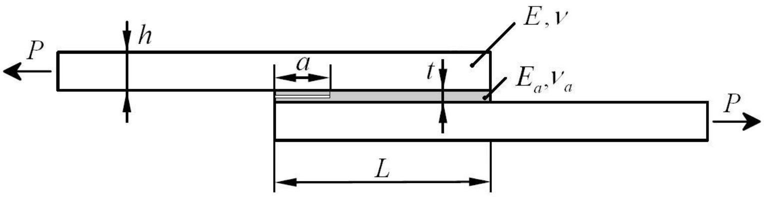

Fig.9. Geometry considered in the example

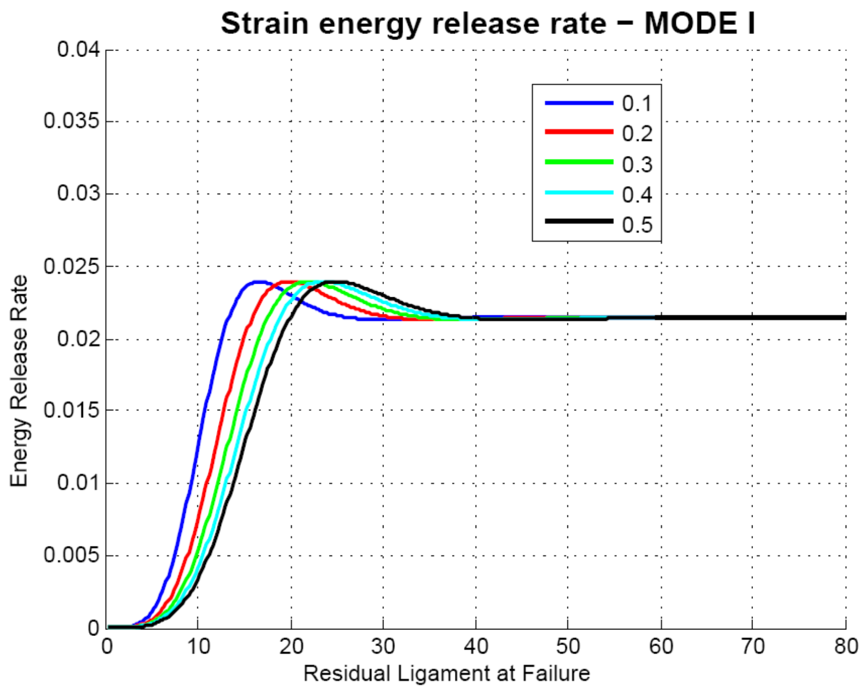

Fig.10. Energy release rate for mode I

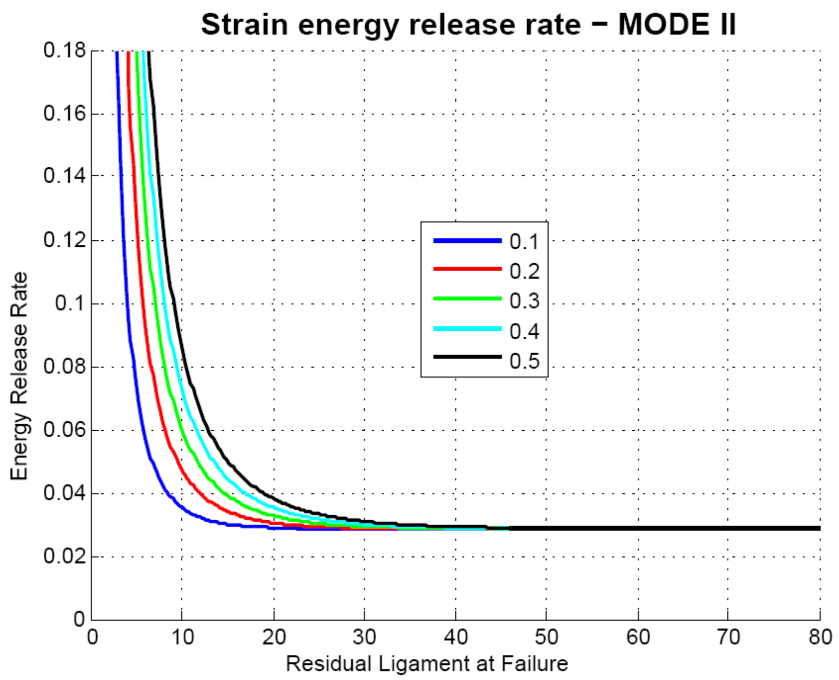

Fig.11. Energy release rate for mode II

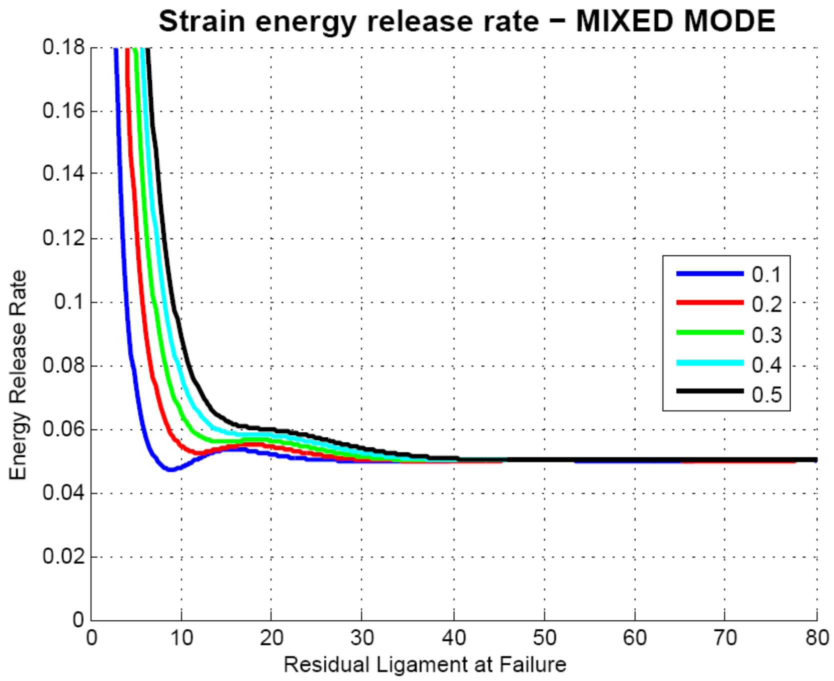

Fig.12. Energy release rate for mixed mode

4. CONCLUSIONS

This paper showed a computer implementation of an analytical formulation developed by Krenk [1]. As can be seen, the MATLAB® software presented as an appropriate tool for the implementation of the analytical formulations presented in this work. The facilities offered can be summarized in the presence of pre-programmed functions, which facilitate matrix operations and graphical display of results, besides allowing the construction of a graphical user interface to facilitate the engineering analysis. It was performed a sensitive study by changing the adhesive thickness. It can be noted that for mode I, when increasing the adhesive thickness, the energy release rate for a given crack length decreases. However, for modes II and mixed, when increasing the adhesive thickness, the energy release rate increases too.

5. ACKNOWLEDGEMENTS

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