ALGORITHM FOR COMPUTING OPTIMIZED FACTORS FOR ABSORBING BOUNDARY TECHNIQUES USING THE FINITE DIFERENCE METHOD APPLIED FOR WAVE ACOUSTIC PROBLEMS.

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Abstract. *This paper presents an algorithm for computing the best factors for the absorbing boundary methods applied for numerical simulations of acoustic waves via 10-4 10th order in space and 4nd order in time staggered-grid finite difference scheme. First, the absorbing boundary methods – Damping Zone (DZ) and Perfect Matched Layer (PML) – are showed and optimized in order to minimize the spurious reflections associated with, improving the quality of the numerical results and reducing its computational effort. Then, an algorithm based on Golden Section Search Scheme is implemented and tested for both PML and DZ optimized in order to computes the best coefficient absorbing factors in 2D problems. With a few iterations the algorithm computes those coefficients reducing energy reflection. Finally, trough examples, the algorithm applicability is shown and the results of each absorbing technique are compared. The results also show that this algorithm can reduces considerably the analysis of 3D problems.*

Keywords*: Geophysics, Acoustic Waves, Finite Difference Time Domain Method, Absorbing Boundary Techniques*

1. INTRODUCTION

One of the major problems in numerical simulations of wave diffusion problems involving infinite domains consist on how to deal with the artificial limits introduced by the use of finite mesh refinement. These undesired reflections waves mask the true solution and override the seismic signals.

To avoid these side effects, researchers used to enlarge the computational domain, delaying the backward reflections, though increasing the numerical mesh and its computational demand. An alternative is to use an absorbing technique that reduces the reflection at its boundary, simulating an infinite domain.

In the late 70's, nonreflecting those techniques were introduced aiming to treat such problems. Clayton and Engquist (1977) proposed the Absorbing Boundary Condition (ABC) technique by applying a one-way wave equation in the boundary region, which proved to be efficient for events not at shallow angles on the contour. In the early 80's, Cerjan et al. (1985) introduced the Damping Zone (DZ) concept in which a gradual reduction of the wave amplitude is imposed along an absorption layer, without any loss of effectiveness due to shallow angles of wave incidence (see figure 1). More recently, Berenger (1994) proposed the Perfect Matched Layer (PML) method for solving electromagnetic and elastic wave equations. A new matched medium is designed to absorb without reflection the incident waves at any frequency and at any incidence angle. Those methods are very sensitive to the number of grid points used in the absorbing layer, with better results found for larger discretization points. Another problem is that the coefficients factors can vary from one problem to another.

This work aims to develop an algorithm that searches for optimized factors for DZ and PML coefficients to reach a greater absorption at the artificial boundary in acoustic wave's propagation problems using the Finite Differences Method in the 2D domain of the time, with a 10ª order approximation for the space and a 2ª order for the time. The algorithm applies the Golden Section Search Scheme for accelerating the convergence. The number of grid points at the absorbing boundary layer for the least reflected waves inside the medium used is 20. Our goal is to enhance and optimize the existing absorbing boundary methods in order to minimize the errors associated with, computing the best factors and improving the quality of the numerical results and reducing its computational effort.

First, the traditional DZ and PML methods are presented and optimized aiming to reduce wave reflection at the borders, with results shown in terms of the total energy for "infinite" and nonreflecting models for varying absorbing layers. In addition, the algorithm for computing the best factors is presented and various tests are performed in order to shown in terms of the time sum of squared energy difference between infinite and nonreflecting models for varying absorbing layers.

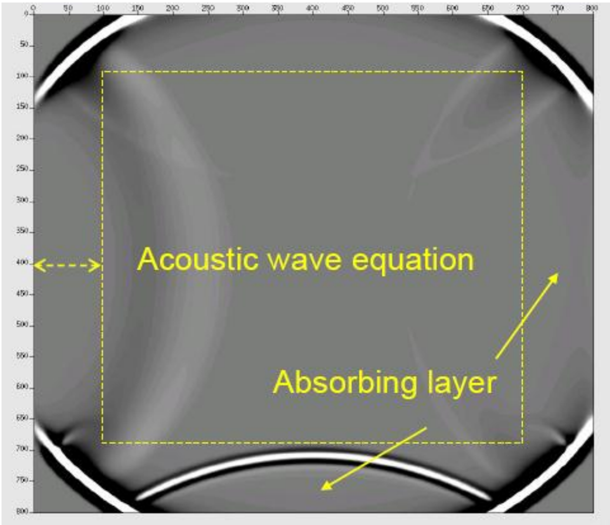


Figure 1. Wave propagation domain with absorbing boundary layers.

2. DZ OPTIMIZED TECHNIQUE

The original Cerjan's (1986) method introduces a damping zone around the domain consisting of Na points where the wave amplitude is absorbed by the relation,

$$Fac = e^{-(factor*(Na-i))^2} \tag{1}$$

The coefficient *factor* is 0,015 and it is constant for *Na* Boundary Layer points in the damping layer and *i* varies from *1* to *Na*.

In order to improve the original Cerjan's Method, others coefficients factors were calculated varying the number of points from 20 to 100 on the boundary layer and computing the energy reflection by the square amplitude difference at each time step, between the model with the infinite domain and the order with the artificial boundary.

Figure 2 shows a comparison of the amount of energy reflection, between the original Cerjan's method and its optimization. It can be seen that the original Cerjan curve is constant after 25 grid points on the Boundary Layer, while on the optimized one, the error decrease with the number of grid points on the boundary layer.

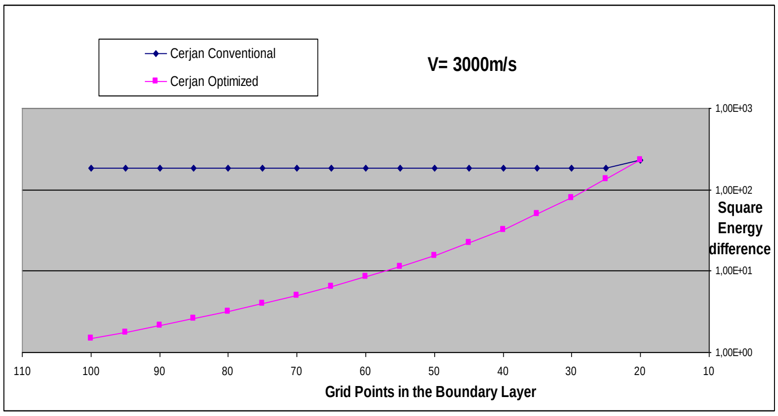


Figure 2. Comparison between Standard Cerjan and Optimized Cerjan Method.

Another optimization can be done by coupling a factor that reduces the propagation velocity. It was verified that Cerjan scheme works better in low velocities. The equation becomes:

$$\frac{\partial^2 U}{\partial t^2} = (FRv.C)^2 * \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) \tag{2}$$

Where the velocity reduction factor *(FRv)* follows a quadratic form:

$$FRv = \frac{(1-Fv)}{Na^2}x^2 - 2\frac{(1-Fv)}{Na}x + 1 \tag{3}$$

FRv varies from 1.0 (when x=Na) to *Fv* (when x=1). This factor enables a reduction of the wave energy reflection in almost 60% compared to original Cerjan for boundary layers until 50 points, that is shown in Figure 3.

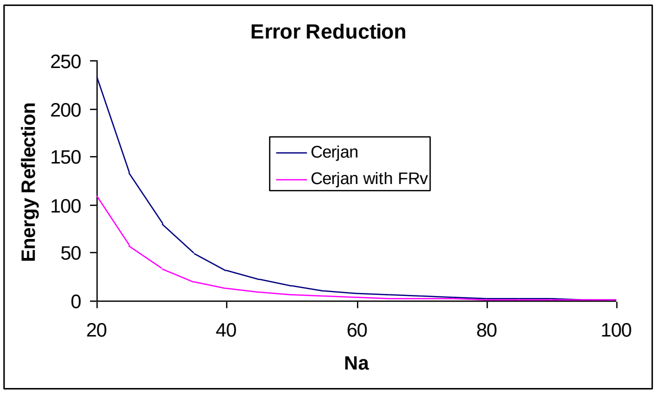


Figure 3. Wave energy reflection using the velocity reduction factor

3. PML OPTIMIZED TECHMIQUE

Berenger (1994) developed the PML technique, in which a new region that surrounds the FDTD domain is defined, where a set of non-physical equations are applied giving a high attenuation of the incident waves. For acoustics, the 2D linearized continuity and Euler equations take the following form at the PML absorbing layer,

$$p_t + B\alpha p = -B\nabla \cdot \vec{u}\,, \tag{4}$$

$$\vec{u}_t + B\alpha\vec{u} = -\frac{1}{\rho}\nabla p\,, \tag{5}$$

where ρ, p and $\vec{u}$ are, respectively, the medium density and the acoustics pressure and vector velocity, while α is the attenuation coefficient and B ($=\rho c^2$) the medium bulk modulus. c is the medium wave speed.

Differentiating in time and space equations (4) and (5) and subtracting the resulting expressions gives the PML acoustic equation,

$$p_{tt} + \alpha(1+B)p_t + \alpha^2 Bp = c^2\nabla^2 p \tag{6}$$

The attenuation coefficient α varies accordingly to,

$$\alpha(i) = \frac{1}{B\delta\delta}\ln\left(\frac{1}{r_{PML}}\right)\left[\frac{x(i)}{x(n_{PML})}\right]^k \tag{7}$$

in which the maximum applied absorption rate rPML is equal to 1/10 and the exponent k=2. Therefore α oscillates from 0 (when x is at the border of the absorbing layer, thus satisfying the acoustic wave equation) to ln(10)/(Bδt), where δt is the time step and nPML the number of PML grid elements. The integer i represents the grid element such that

$$1 \le i \le n_{PML} \tag{8}$$

An optimization can be done varying those coefficients. In a general form, α can be rewritten as,

$$\alpha(i) = c_{PML}\, f\left[x(i)\right] \tag{9}$$

Changing the values of cPML and the function $f\left[x(i)\right]$, for a fixed nPML, improves the effectiveness of the absorption, reducing its side effects by increasing the absorption rate and using smoother polynomials at the absorbing layer.

4. THE MINIMUM SEARCH METHOD

The minimization method of functions used was based in the Golden Section Search (Press (2007)), which, from three initials points (a,b,c), searches the minimum value of a function at the interval (a,c). Given this interval, a new point x is chosen, either between a and b or between b and c . Suppose that the new point is between b and c. Then, f(x) is evaluated. If f(b) < f(x) then the new triplet of points is (a,b,x), contrariwise, if f(b) > f(x), then the new triplet is (b,x,c). This is done in a manner that the middle point of the triplet is always the one with the best minimum achieved so far. This process is repeated until the distance between the two outer points is tolerably small. An example is shown at fig. 4. The initial points are 1, 3, and 2. The function is evaluated at 4, which replaces 2; then at 5, which replaces 1; then at 6, which replaces 4.

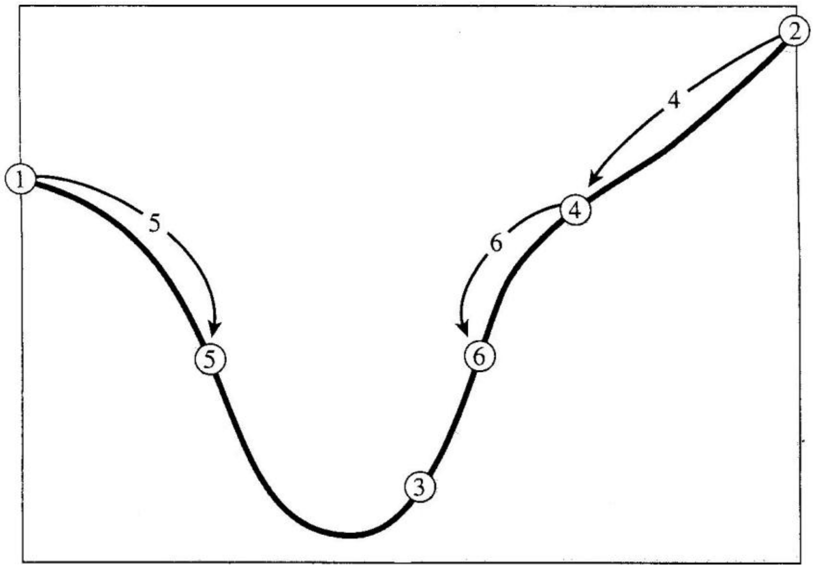


Figure 4 – Convergence process for a one dimension function

The new point x to be tried is that which is a fraction 0.38197, measuring from the middle point of the triplet, into the larger of the two intervals. So, then new point x is calculated by:

$$x = b + 0.38197*(c - b) \tag{10}$$

Or

$$x = b - 0.38197*(b - a) \tag{11}$$

This fraction is called the golden mean, which guarantees the fastest way to achieve the minimum.

However, the Golden method is applicable only to one dimension functions, while the total absorbed energy varies with a pair of factors. In other words, it's a two dimension function. The method developed works as follows: given an initial pair of factors (x_0, y_0), x remains constant while is searched the value of y which gives the minimum of the function at this conditions, at the point (x_0, y_1). Later, the new value of y, y_1, remains constant while is searched the value of x which gives the minimum of the function at this condition, at the point (x_1, y_1). After that, x remains constant at $x=x_1$ and the value of y varies in a way that the function value decreases, at the point (x_1, y_2). This procedure is repeated until the difference between two successive interactions is smaller than a given tolerance.

5. RESULTS

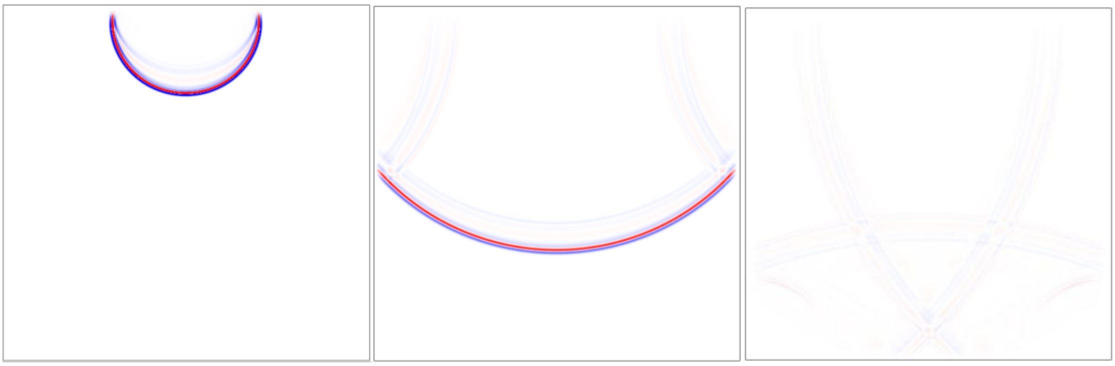
The effectiveness of the algorithm was tested with a boundary Layer of 20 points. At time t=0, a Ricker type source with 30Hz was generated at the surface of the model. As an energy measure, the square of the amplitude over the whole domain was taken to evaluate the effectiveness of each absorption boundary,

$$E_{Effectiveness} = \sum (U)^2 \tag{12}$$

5.1. Example 1 – Uniform Medium

As a first example, a 2D square domain with 3000x3000m with constant medium velocity of 3000 m/s was used, as shown in Figure 1. The region was discretized with 600x600 points with 5 meters of spacing and was used a time step of 0,0002s, to avoid instability and divergence problems with the numerical method. The source was positioned at the middle top of the domain and the frequency was 30Hz. Around this region, a boundary layer was created with 20 grid points. The finite differences operator used is a second order in time and 10th order in space operator. The algorithm was applied on these boundary layers to compute the best factors. The total energy of the wave generated was 7480 $m^{2.}$

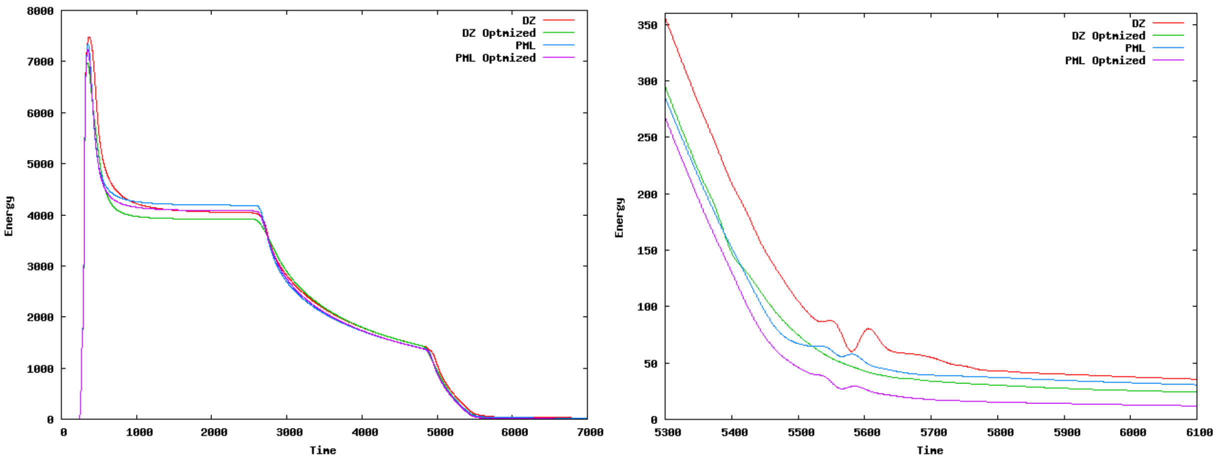
Figure 5 shows snapshots of the progression of the wave at an uniform medium



(a) (b) (c)

Figure 5. Amplitudes at progressive times with nonreflecting boundaries at an uniform medium

On figure 6 (a) one can see a plot graph time-Energy. In the beginning, the wave generated propagates throw the domain without the boundary limits. On the second range, the wave reaches the two sides, and on the third the bottom. It was solved with PML and Standard Cerjan Methods, and also the optimized ones. The good performing of boundary absorbing techniques can be clearly seen.



(a) (b)

Figure 6. (a) Total energy of each method versus time steps in uniform medium and (b) a detail comparing the performing of the PML and Dz Methods, with and without the optimization.

On figure 6(b) a detail is shown and it can be seen the increase of the performance of the optimized methods. A detail of the absorption is shown on table 1.

Table 1. Factors used in uniform medium and final energy (Time Step = 7000)

Technique	Factor 1	Factor 2	Final Energy	Absorption
DZ	0.013	1.0	27.468	99.632 %
DZ Optimized	0.0101	0.5	16.025	99.786 %
PML	3.55e-8	2.0	20.384	99.727 %
PML Optimized	1.134e-07	1.056	7.539	99.899 %

5.2. Example 2 – Non-uniform Medium with a salt region

On example 2, a simulation of a pre-salt region with an heterogeneous medium was done. The same source and mesh of example 1 was adopted and a boundary layer with 20 grid points. Also, to avoid instability and divergence problems with the numerical method, grid spacing used was 5 meters and the time step 0,0002 s. The algorithm was applied on these boundary layers to compute the best factors. The total energy of the wave was 17180 m^2. The domain with the 3 medium types and the pre-salt region is shown on figure 7.

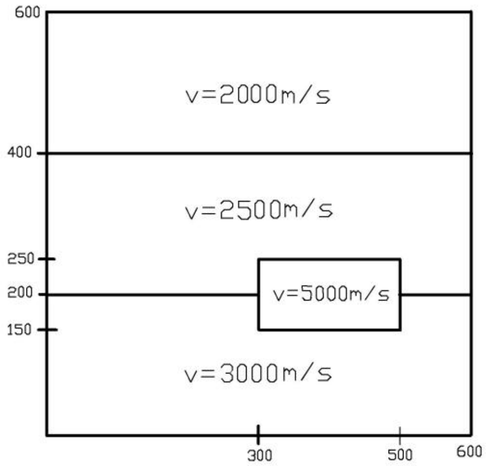
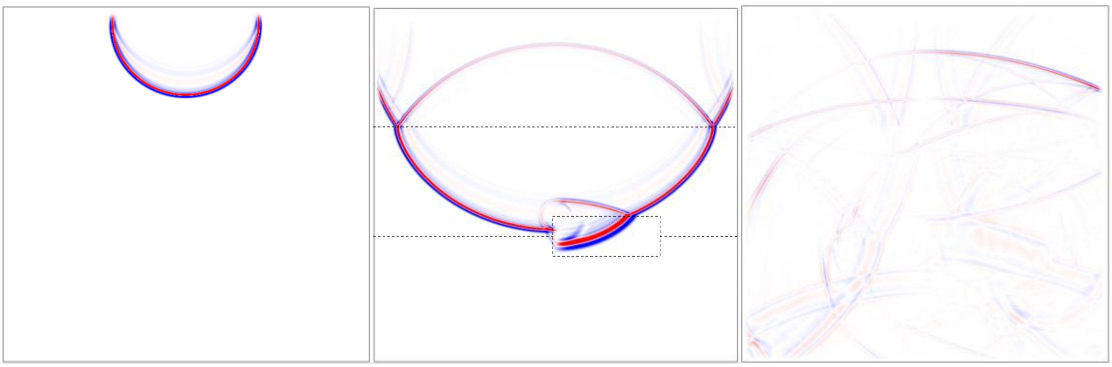


Figure 7. Heterogeneous domain with the three different mediums types and a salt region.

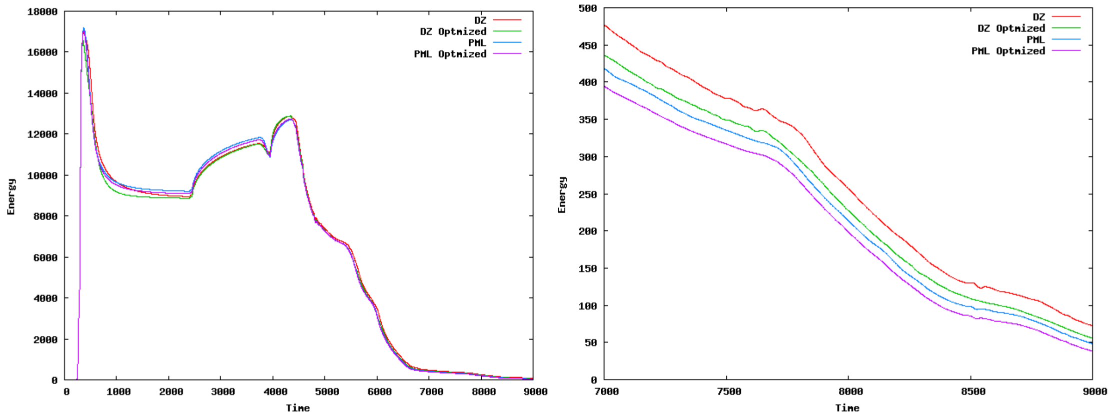
The figure 8 shows snapshots of the progression of the wave at an heterogeneous medium. Figure 8a shows the beginning of the wave propagation at the 2000m/s velocity medium. Figure 8b shows the waves being absorbed at the domain both sides and reflected by the change of mediums, and fig. 8c illustrates the results after the absorption at the bottom of the domain.



(b) (b) (c)

Figure 8. Amplitudes at progressive times with nonreflecting boundaries at an heterogeneous medium

Figure 9a shows how the energy varies throw the wave propagation. In the beginning, the wave generated propagates throw the first domain without reaching the boundary limits. The next phases of the graph, show the waves reaching the two others domains, the pre-salt and the bottom. Even in an heterogeneous media one can see the good performance of the Boundary absorbing techniques. On figure 9b a detail is shown and it can be seen the increase of the performance of the optimized methods. A detail of the absorption is shown on table 1.



(b) (b)

Figure 9. (a) Total energy of each method versus time steps in non-uniform medium, and
(b) a detail showing a comparison between the energy of the absorbing Methods at the final steps.

Table 2. Factors used in non-uniform medium and final energy (Time Step = 8000)

Technique	Factor 1	Factor 2	Final Energy	Absorption
DZ	0.013	1.0	256.622	98.506 %
DZ Optimized	0.0105	0.618	227.234	98.677 %
PML	3.55e-8	2.0	213.741	98.756 %
PML Optimized	1.738e-07	1.292	198.493	98.845 %

Comparing example 1 and 2, one can see that the best coefficients factors vary from one example to another, and that the best method was PML.

6. CONCLUSIONS

An algorithm to compute the best absorbing factors for two classical nonreflecting boundary methods – PML, DZ – in the FDTD 2D computational domain was presented. It has been used 20 points. The problems shown that the best coefficient varies from one problem to another.

This algorithm can be applied to compute the best coefficient factors for larger 3D problems in heterogeneous media. The steps for solving those problems can be summarized as:

- first, create a reduced 2D version of the 3D problem representing the medium, source and mesh refinement;
- second, apply the algorithm and find the best coefficients absorbing factors for PML and DZ Methods, and even which of those two methods have better results.
- take the best method and coefficient factor from step 2 and apply to your original 3D problem, and solve it in a Finite Difference Scheme. Then the algorithm computes the best absorbing factors.

7. ACKNOWLEDGMENTS

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