

TREATMENT OF MULTI-RESPONSE SURFACE APPLIED TO MACHINABILITY OF STAINLESS STEEL USING MULTI-OBJECTIVE OPTIMIZATION DIFFERENTIAL EVOLUTION

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Abstract. Nowadays, due to the growing needs of market, the simultaneous optimization of various responses is configured as a necessary strategy in real process. Traditionally, the treatment of this problem is done through the application of the Desirability Function, that consists in transforming the original multi-response problem in a similar with one objective. In spite of various applications involving this methodology, the quality of the solution obtained is dependent on the choice of the inferior and superior limits and on goals for each one of the responses. To overcome this disadvantage, the present work proposes a methodology to solve the original multi-objective problem by using the Differential Evolution Algorithm (DE). This algorithm consists in the extension of the DE to problems with multiple objectives, through the incorporation of two operators into the original algorithm: i) the rank ordering, and ii) the neighborhood exploration of potential candidates. The proposed algorithm is applied to the machinability of stainless steel ABNT420 using a model that considers the tool life and cutting forces responses in terms of cutting speed, feed per tooth and depth of cut, in slot milling process. The effects of these variables in the responses were investigated crossing information contained in response surfaces of material removal rate and cutting forces. The results obtained showed that the methodology used represents an interesting approach to the treatment of the optimization problem formulated.

Keywords: Multi-objective Optimization, Differential Evolution Algorithm, Multi-response, Machinability of stainless steel.

1. INTRODUCTION

Naturally, real-world problems involve the simultaneous optimization of two or more (often conflicting) objectives, called multi-objective optimization problem (MOOP). The solution of such problems is different from that of a single-objective optimization problem. The main difference is that multi-objective optimization problems normally have not one but a set of solutions which are all equally good (Stadler, 1984).

Traditionally, the treatment of such problems is done transforming the original MOOP into one-objective problem. However, the development of specific methodologies allows the formulation of the optimization problem in a way that various objectives can be taken into account simultaneously. In addition, as a number of points, that constitutes the optimal solution, are found, it is possible to explore these solutions according to the practical application studied (Deb, 2001). In the literature, several methods for solving MOOP can be found (Deb, 2001). These methods follow a preference-based approach, in which a relative preference vector is used to scalarize multiple objectives. Since classical searching and optimization methods use a point-by-point approach, at which the solution is successively modified, the outcome of this classical optimization method is a single optimized solution. However, Evolutionary Algorithms (EA) can find multiple optimal solutions in one single simulation run due to their population-based search approach. Thus, EA are ideally suited for multi-objective optimization problems. A detailed account of multi-objective optimization using EA and some of the applications using genetic algorithms can be widely found in the literature (Deb, 2001; Lobato, 2008;).

In many engineering applications, it is necessary to find the conditions under which a certain process attains the optimal results. That is, they want to determine the levels of the design parameters at which the response reaches its optimum. The optimum could be either a maximum or a minimum of a function of the design parameters. One of methodologies for obtaining the optimum is Response Surface technique (RS). This approach is a collection of statistical and mathematical methods that are useful for the modeling and analyzing engineering problems. The main objective is to optimize the response surface that is influenced by various process parameters.

Response Surface also quantifies the relationship between the controllable input parameters and the obtained response surfaces. The design procedure of response surface methodology is as follows (Myers and Montgomery, 1995): i) Designing of a series of experiments for adequate and reliable measurement of the response of interest; ii) Developing a mathematical model of the second order response surface with the best fittings; iii) Finding the optimal set of experimental parameters that produce a maximum or minimum value of response.

Associate with the MOOP, the multi-response surface technique is configured as good strategy to treatment of real-world problems. In this sense, the desirability function approach, originally developed by Harrington (1965) and later modified by Derringer and Suich (1980), to multi-response optimization is a useful technique for the analysis of experiments in which several responses have to be optimized simultaneously. The basic idea of the desirability function approach is to transform a multi-response problem into a single response problem by means of mathematical transformations.

Machinability must be understood as a system of properties which depend on complex interactions among workpiece, tool material, cutting fluid and cut conditions. Trent (1989) suggests that machinability is not only a property, but the “way” material behaves during machining. Therefore, machinability is much more than a test function, and it’s improvement is characterized by, at least, one of the following factors: - increase of tool life, - higher rate of material removal, - improvement of surface finishing, - better control of the chip, - reduction of cutting forces and power consumption, reduction on the cutting temperature, etc. According to its duration, the tests of machinability are classified in to short and long duration. And the best example of long lasting test is the tool life test and their results, generally presented using Taylor’s equation.

Stainless steel is one of the main materials employed in critical parts for installation of power plants and modern chemical industries due to combination of appropriate mechanical properties and high corrosion resistance. However, the composition required allowing such properties results in poor machinability of this steel, right below to that for the carbon steel. High rate of strain hardening, high toughness and low thermal conductivity are the main factors that cooperate for this. As a consequence, the machinability of stainless steel tends to present short tool life, especially in intermittent cut operations like milling, where thermal and mechanical shocks are observed (Bhattacharya et al., 1988).

In this work, the machinability of stainless steel ABNT420 in slot milling operation is analyzed using a model that foresees the responses of tool life and cutting forces in terms of cutting speed, feed per tooth and depth of cut. The effects of these variables in responses were investigated crossing information contained in bound surfaces of material removal.

In this context, the main contribution of this paper is to introduce a systematic methodology for the solution of multi-objective optimization problems by using the Differential Evolution Algorithm.

2. DESIRABILITY FUNCTION

The desirability function approach is one of the most widely used methods in industry for the optimization of multiple response processes. It is based on the idea that the “quality” of a product or process that has multiple quality characteristics, with one of them outside of some “desired” limits, is completely unacceptable. The method finds the operating conditions x that provide the “most desirable” response values.

For each response $Y_i(x)$, a individual desirability function $d_i(Y_i)$ assigns numbers between 0 and 1 to the possible values of $Y_i(x)$, with $d_i(Y_i)=0$ representing a completely undesirable value of $Y_i(x)$ and $d_i(Y_i)=1$ representing a completely desirable or ideal response value. The individual desirabilities are then combined using the geometric mean, which gives the overall desirability D :

$$D = (d_1(Y_1) \times d_2(Y_2) \times \dots \times d_k(Y_k))^{1/k} \quad (1)$$

with k denoting the number of responses.

Depending on whether a particular response $Y_i(x)$ is to be maximized, minimized, or assigned a target value, different desirability functions $d_i(Y_i)$ can be used. For example, if a response is of the "target is best" kind, then its individual desirability function is

$$d_i(Y_i) = \begin{cases} 0 & \text{if } Y_i(x) < L_i \\ \left(\frac{Y_i(x) - L_i}{T_i - L_i} \right)^s & \text{if } L_i \leq Y_i(x) < T_i \\ \left(\frac{Y_i(x) - U_i}{T_i - U_i} \right)^t & \text{if } T_i \leq Y_i(x) < U_i \\ 0 & \text{if } Y_i(x) > U_i \end{cases} \quad (2)$$

If a response should be maximized, the individual desirability function is defined as:

$$d_i(Y_i) = \begin{cases} 0 & \text{if } Y_i(x) < L_i \\ \left(\frac{Y_i(x) - L_i}{T_i - L_i} \right)^s & \text{if } L_i \leq Y_i(x) < T_i \\ 1 & \text{if } Y_i(x) > T_i \end{cases} \quad (3)$$

If a response should be minimized, the individual desirability function is defined as:

$$d_i(Y_i) = \begin{cases} 1 & \text{if } Y_i(x) < T_i \\ \left(\frac{Y_i(x) - U_i}{T_i - U_i} \right)^s & \text{if } T_i \leq Y_i(x) < U_i \\ 0 & \text{if } Y_i(x) > U_i \end{cases} \quad (4)$$

where L_i , U_i and T_i be the lower, upper, and target values, respectively, that are desired for response $Y_i(x)$, with $L_i \leq T_i \leq U_i$. The exponents s and t determining how important it is to hit the target value. For $s = t = 1$, the desirability function increases linearly towards T_i ; for $s < 1$, $t < 1$, the function is convex, and for $s > 1$, $t > 1$, the function is concave.

It should be considered that, in spite of quite spread, this approach present some difficulties (Derringer and Suich, 1980): *i*) dependence of choice of individual desirability functions; *ii*) the increase of the non-linearity of D and of number of responses can lead to location of optimal; *iii*) the quality of solution obtained is affected drastically by definition of L_i , T_i and U_i .

3. MULTI-OBJECTIVE OPTIMIZATION

When dealing with MOOP, the notion of optimality needs to be extended. The most common one in the current literature is that originally proposed by Edgeworth (Edgeworth, 1881) and later generalized by Pareto (Pareto, 1896). This notion is called Edgeworth-Pareto optimality, or simply Pareto optimality, and refers to finding good tradeoffs among all the objectives. This definition leads us to find a set of solutions that is called the Pareto optimal set, whose corresponding elements are called non-dominated or non-inferior. The concept of optimality in single objective is not directly applicable in MOOPs. For this reason a classification of the solutions is introduced in terms of Pareto optimality, according to the following definitions (Deb, 2001):

- *Definition 1 - The Multi-objective Optimization Problem (MOOP) can be defined as:*

$$f(x) = (f_1(x), f_2(x), \dots, f_m(x)), m = 1, \dots, M \quad (5)$$

subject to

$$h(x) = (h_1(x), h_2(x), \dots, h_i(x)), i = 1, \dots, H \quad (6)$$

$$g(x) = (g_1(x), g_2(x), \dots, g_j(x)), j = 1, \dots, J \quad (7)$$

$$x = (x_1, x_2, \dots, x_n), n = 1, \dots, N, x \in X \quad (8)$$

where x is the vector of design (or decision) variables, f is the vector of objective functions and X is denoted as the design (or decision) space. The constraints h and g (≥ 0) determine the feasible region.

- *Definition 2 - Pareto Dominance: For any two decision vectors u and v , u is said to dominate v , if u is not worse than v in all objectives and u is strictly better than v in at least one objective.*
- *Definition 3 - Pareto Optimality: When the set P is the entire search space, or $P = S$, the resulting non-dominated set P' is called the Pareto-optimal set. Like global and local optimal solutions in the case of single-objective optimization, there could be global and local Pareto-optimal sets in multi-objective optimization.*
- *Definition 4 - Non-dominated Set: Among a set of solutions P , the non-dominated set of solutions P' are those that are not dominated by any member of the set P .*

In the multi-objective context, various Multiple-Objective Evolutionary Algorithms (MOEAs) can be found. This group of algorithms conjugates the basic concepts of dominance described in the later section with the general characteristics of evolutionary algorithms. MOEAs are able to deal with non-continuous, non-convex and/or non-linear spaces, as well as problems whose objective functions are not explicitly known (Deb, 2001). Basically, the main features of these MOEAs are:

- *Mechanism of adaptation assignment in terms of dominance*: between one non-dominated solution and another dominated, the algorithm will favor the non-dominated one. Moreover, when both solutions are equivalent in dominance, the one located in a less crowded area will be favored. Finally, the extreme points, (i.e. the solutions that have the best value in one particular objective) of the non-dominated population are preserved and their adaptation is better than any other non-dominated point, to allow maximum front expansion.
- *Incorporation of elitism*: the elitism is commonly implemented using a secondary population of non-dominated solutions previously stored. When performing recombination (selection-crossover-mutation), parents are taken from this archive in order to produce the offspring.

3.1. Differential Evolution Algorithm

Differential Evolution (DE) (Price and Storn, 1997) is an improved version of the Goldberg's Genetic Algorithm (GA) (Goldberg, 1989) for faster optimization and presents the following advantages: simple structure, easiness of use, speed and robustness (Babu and Anbarasu, 2005; Price *et al.*, 2005). The crucial idea behind DE is a scheme for generating trial parameter vectors. Basically, DE adds the weighted difference between two population vectors to a third vector. The key parameters of control in DE are: N the population size, CR the crossover probability, and F the weight applied to random differential (scaling factor). Price and Storn (1997) have given some simple rules for choosing the key parameters of DE for any given application. Normally, N should be about 5 to 10 times the dimension (number of parameters in a vector) of the problem. As for F , it lies in the range 0.4 to 1.0. Initially $F = 0.5$ can be tried then F and/or N is increased if the population converges prematurely.

DE has been successfully applied to various fields such as digital filter design (Storn, 1995), estimation of heat transfer parameters in a bed reactor (Babu and Sastry, 1999), synthesis and optimization of heat integrated distillation system (Babu and Singh, 2000), parameter estimation in fed-batch fermentation process (Wang *et al.*, 2001), optimization of thermal cracker operation (Babu and Angira, 2001), engineering system design (Lobato and Steffen, 2007), apparent thermal diffusivity estimation during the drying of fruits (Mariani *et al.*, 2008), estimation of the parameters of Page's equation and heat loss coefficient by using experimental data from a realistic rotary dryer (Lobato *et al.*, 2008), estimation of space-dependent single scattering albedo in radiative transfer problems (Lobato *et al.*, 2010), and other applications (Storn *et al.*, 2005).

3.2. Multi-Objective Optimization Differential Evolution

Due to success obtained by DE in different applications in science and engineering, several attempts to extend the DE to solve multi-objective problems can be found in the literature. In this work the MODE (Multi-objective Optimization Differential Evolution) algorithm proposed by Lobato (2008) is used. This approach is based on DE algorithm and has the following structure: an initial population of size N is randomly generated. All dominated solutions are removed from the population through the operator Fast Non-Dominated Sorting. In this way, the population is sorted into non-dominated fronts F_j (sets of vectors that are non-dominated with respect to each other). This procedure is repeated until each vector is member of a front. Three parents are selected at random in the population. A child is generated from the three parents (this process continues until N children are generated). Starting from population P_1 of size $2N$, neighbors are generated for each one of the individuals of the population, in the following way (Hu *et al.*, 2005):

$$\chi(x) = [x - D_k(g)/2, x + D_k(g)/2] \quad (9)$$

where

$$D_k(g) = k/R [U - L] \quad (10)$$

$D_k(g)$ is a vector in R^n and a function of the generation counter g . R is the number of pseudo fronts defined by the user and the initial maximum neighborhood size in a population is $D_k(0)=[U - L]$, where L and U represent the lower and upper bounds of the variables. The pre-defined number of individuals in each pseudo front is given by (Hu *et al.*, 2005):

$$n_k = rn_{k-1}, \quad k = 2, \dots, R \quad (11)$$

where n_k is the number of individuals in the k -th front and r (<1) is the reduction rate. For a given population with N individuals, n_k can be calculated as

$$n_k = N \frac{1-r}{1-r^R} r^{k-1} \quad (12)$$

According to Hu *et al.* (2005), if $r < 1$, the number of individuals in the first pseudo front is the highest and each pseudo front has an exponentially reducing number of solutions, this emphasizing a local search. On the contrary, a greater r results in more solutions in the last pseudo front and hence emphasizes the global search.

This way, the neighbors generated are classified according to the dominance criterion and only the neighbors non-dominated ($P2$) will be put together with $P1$ to form $P3$. The population $P3$ is then classified according to the dominance criterion. If the number of individuals of the population $P3$ is larger than a number defined by the user, it is truncated according to the criterion named the Crowding Distance (Deb, 2001). The Crowding Distance describes the density of solutions surrounding a vector. To compute the Crowding Distance for a set of population members the vectors are sorted according to their objective function value for each Objective Function. To the vectors with the smallest or largest values an infinite Crowding Distance (or an arbitrarily large number for practical purposes) is assigned. For all other vectors the Crowding Distance is calculated according to:

$$dist_{x_i} = \sum_{j=0}^{m-1} \frac{f_{j,i+1} - f_{j,i-1}}{|f_{j,max} - f_{j,min}|} \quad (13)$$

where f_j corresponds to the j -th objective function and m equals the number of objective functions.

4. METHODOLOGY

The methodology proposed in this work consists of following steps:

- i. Defining the problem: to identify the responses to be analyzed and the variables relevant in the process;
- ii. Assemble the table of experimental design;
- iii. Obtain the corresponding response surfaces: usually is adopted a polynomial approximation:

$$Y = \beta_o + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j \geq 1}^k \beta_{ij} x_i x_j + \varepsilon \quad (14)$$

where $\beta_o, \beta_1, \dots, \beta_k$ and β_{ij} are unknown parameters and ε represent the systematic errors.

- iv. The next step consists in using an optimization technique to obtain the best configuration. In this case two approaches will be used: mono and multi-objective optimization using the Differential Evolution algorithm.

In order to use the methodology proposed above data from a previous work developed by Ramos *et.al*, (2003) were used. The scheme shown in figure 1, illustrates the system analyzed in that work with the purpose of process optimization. The cutting speed, feed per tooth and depth of cut were the input data that had their values varied. The analyzed responses were: tool life and cutting forces indirectly obtained by measuring the relation between the electric current of the chain electrical motor and the cutting speed used by Ramos *et. al* (2003).

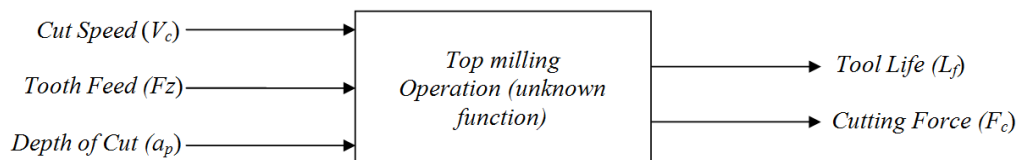


Figure 1 – Relation between the input factors and the observed output response

Using the 2nd order mathematical model of responses (Central Compounded Planning – PCC) for experiment planning, equations related to cutting force and tool life were obtained as a function of cutting speed, feed per tooth, and depth of cut. The functional relation between the responses of this operation and the independent variables investigated can be represented by equations:

$$T = K \cdot v_c^k \cdot f_z^l \cdot a_p^m \quad (15)$$

$$\frac{I}{v_c} = M \cdot v_c^k \cdot f_z^l \cdot a_p^m \quad (16)$$

where T , is the tool life given by feed length (cm), the I/v_c term (electric current of the electrical motor by cutting speed) or cut effort (Amperes/m/min) and v_c, f_z e a_p mean cutting speed (m/min), feed per tooth (mm/tooth) and depth of cut (mm) respectively. x is the level value (code) for each factor corresponding to its value x_n , that is: x_{n1} is the value of the level +1 and x_{n0} corresponds to the zero level value. Numeric input factors were obtained (cut conditions) encoded by levels ($\pm 2, 0, \pm 1$), as shown in Table 1:

Table 1- Levels of the independent variables and identification by code (Ramos et. al, 2003)

Independent variables	Levels in code form				
	-2 (Too Low)	-1 (low)	0 (central)	+1 (high)	+2 (Too high)
v_c , m/min (x_1)	87.07	94.46	115	0.12	151.88
f_z , mm/tooth (x_2)	0.077	0.083	0.10	1.5	0.129
a_p , mm (x_3)	0.56	0.67	1	140	1.77

To obtain these conditions, it was first chosen the values for zero “central” and +1 “high” levels, from pre-tests considering cut limitations (milling machine capability, inserts manufacturer recommendations, etc). Then, values related to other levels were calculated, according to the equations below:

$$x_1 = \frac{\ln(v_c) - \ln(115)}{\ln(140) - \ln(115)} \quad (17)$$

$$x_2 = \frac{\ln(f_z) - \ln(115)}{\ln(140) - \ln(115)} \quad (18)$$

$$x_3 = \frac{\ln(a_p) - \ln(115)}{\ln(115) - \ln(115)} \quad (19)$$

Therefore, x_1 is the encoded value of cutting speed for v_c , x_2 is the encoded value for the corresponding value f_z , and x_3 for the a_p value.

$$\frac{I}{v_c} = \frac{F_c}{\eta u} \quad (20)$$

Where, F_c is the main cutting force, η is the machine-tool efficiency and u is the electric tension in the main electrical motor terminals. But, during cut procedure, it can be considered that tension remains constant for a given rotation of the electrical motor, varying the current only. Considering also, that the machine efficiency remains constant with cutting conditions variation, it can be said that cutting force is directly proportional to the relation I/v_c .

The process used for the tests was the slot milling (channels), with dry cut in a ROMI INTERACT IV CNC, milling machine with 15 cv of power. The machinability tests were carried out over a rectangular bar of stainless steel ABNT 420, in agreement with the ISO/R 683-3, made by Villares Metals S/A.

It was used a toroidal tool with 32 mm of diameter, with three interchangeable inserts and screw fixation (Sandvik (1999)]. Inserts for the milling operation with corner facing of 90° entering into the workpiece with a ramp angle $\alpha=3,6^\circ$. The inserts have Vickers hardness of HV3=1500HV and the following chemical composition: 10.5 wt-% Co (cobalt) and the remaining of WC (tungsten carbide). This carbide is coated with a TiAlN and TiN layer of 2 - 6 micrometer of thickness by physical vapor deposition technique (PVD) (Sandvik, 1999).

The electric current sensing of the three-phase motor was done by a Hall effect current sensor, manufactured by Newtronic, with board ampere band of 0 to 50 A, and output signal from 0 to 5 VDC. The signal is sent to an analogical-digital acquisition board managed by a computer using the LabView 5.1 software, from National Instruments. The acquisition of the signal was done with a sample rate of 5 kHz during 20s of each pass. Each value of current, having a relation of 0.0968V/A, that is, the real value of current consumed by the motor found multiplying the

output signal by this conversion factor. The end of tool life criterion is recommended by ISO8688-1 standard, 1989, for tools life test in end milling. Therefore, the uniform flank wear of the tool ($v_B=0,35$ mm) was taken as the end of tool life criterion.

5. RESULTS AND DISCUSSIONS

Table 2 shows the cutting conditions obtained by experimental planning and the results obtained for tool life and cutting force.

Table 2 - Cutting conditions obtained by experimental planning (Ramos et. all, 2003)

Experiment	Cutting Speed (x_1)	Feed per Tooth (x_2)	Depth of Cut (x_3)	Tool Life (Y_1) [cm]	Cutting Force expressed by I/v_c (Y_2) [A/m/min] $\times 10^{-2}$
1	-1	-1	-1	2562.3	4.013
2	1	-1	-1	403.7	2.658
3	-1	1	-1	2562.3	4.088
4	1	1	-1	298.4	3.024
5	-1	-1	1	1316.3	5.188
6	1	-1	1	280.8	3.804
7	-1	1	1	965.3	6.252
8	1	1	1	140.4	4.32
9	0	0	0	368.6	3.751
10	0	0	0	351	3.745
11	0	0	0	386.1	4.051
12	0	0	0	386.1	3.958
13	-1.41	0	0	1895.4	4.629
14	1.41	0	0	140.4	3.021
15	0	-1.41	0	403.7	3.815
16	0	1.41	0	245.7	4.103
17	0	0	-1.41	1614.6	2.934
18	0	0	1.41	228.2	5.537

Table 3 shows the Coefficients estimated for each response Y_i .

Table 3- Coefficients estimated for each response Y_i (p is the confidence level and R^2 is the correlation coefficient)

	Y_1	p	Y_2	p
β_0	346.75	0.000483	3.85489	0.000000
β_1	-1462.48	0.000000	-1.33636	0.000006
β_{11}	727.28	0.000050	0.01241	0.939445
β_2	-136.85	0.107030	0.40532	0.013728
β_{22}	29.93	0.754874	0.14722	0.379810
β_3	-848.14	0.000004	1.57834	0.000002
β_{33}	630.20	0.000137	0.42537	0.027677
β_{12}	26.33	0.782596	-0.06425	0.694474
β_{13}	640.53	0.000119	-0.22425	0.192944
β_{23}	-96.53	0.325948	0.28475	0.108706
R^2	0.9881	-	0.9722	-

For evaluating the methodology proposed in this work, some practical points regarding the application of this procedure should be emphasized:

- The objectives are: to maximize the tool life and to minimize the cutting force.
- The parameters used by DE algorithm (mono-objective optimization): 20 individuals, 100 generations, perturbation rate and crossover probability equal to 0.8 and DE/rand/1/bin strategy for the generation of potential candidates. For the considered parameters, the number of objective function evaluations is 2020.
- The parameters used by MODE algorithm (multi-objective optimization): 50 individuals, 50 generations, perturbation rate and crossover probability equal to 0.8 and DE/rand/1/bin strategy for the generation of potential candidates, reduction rate and number of pseudo-curves equals to 0.9 and 10, respectively. For the considered parameters, the number of objective function evaluations is 5050.
- Stopping criterion: a given number of generations is defined to interrupt the procedure.

In order to evaluate the methodology proposed in this work, three test cases are presented.

5.1. Test Case 1: Target Values (T_i) constants (Mono-objective)

In this first case, lower, upper, and target values for each response surface are considered (see Table 4). It should be emphasized that these values are chosen through experiment planning presented in Table 4. In this table it also is showed the average values obtained by the DE algorithm in 20 runs.

Table 4. Average results obtained using the DE algorithm (in parenthesis is presented the standard deviation in each run).

	L_i	T_i	U_i	x_i	D	Y_1	Y_2
(1)	{140 3}	{1000 4}	{2600 6}	{-0.54 -0.94 -0.11}	0.996 (0.02)	999.99	4.00
(2)		{1500 4.5}		{-1.02 -1.23 0.168}	0.999 (2e-4)	1500.03	4.49
(3)		{2000 4.8}		{-1.37 -0.29 0.08}	0.999 (2e-4)	1998.25	4.79
(4)	{140 3.2}	{1000 4}	{2200 5}	{-0.54 -1.34 -0.03}	0.981 (0.02)	1000.00	4.03
(5)		{1500 4.5}		{-1.01 0.09 -0.06}	0.999 (1e-4)	1499.91	4.49
(6)		{2000 4.8}		{-1.34 0.19 -0.01}	0.997 (1e-3)	1994.27	4.79
(7)	{500 3.5}	{1000 4}	{2600 5}	{-0.55 -1.41 0.08}	0.901 (0.03)	961.82	4.11
(8)		{1500 4.5}		{-0.96 0.65 -0.19}	0.999 (1e-4)	1500.88	4.49
(9)		{2000 4.8}		{-1.24 1.19 -0.27}	0.998 (1e-2)	2001.26	4.79

In this table it is possible to observe that the results obtained by the DE algorithm are dependent of L_i , U_i , and T_i , and consequently in objective function value D .

5.2. Test Case 2: Target Values (T_i) calculated by DE algorithm (Mono-objective)

As observed in test case 1, the objective function value is dependent of L_i , U_i , and T_i . In this test case, the values of L_i and U_i are fixed and target values (T_i) are calculated by DE algorithm. The target values are defined according to the following interval for each response: $[140 3] \leq T_i \leq [2600 6]$.

Table 5 presents the results obtained using the DE algorithm.

Table 5. Results obtained considering T_i as design variable.

x_1	x_2	x_3	T_1	T_2	D	Y_1	Y_2
0.87	-1.08	-0.37	140.00	3.00	0.999 (0.01)	141.35	3

In this table it is important to observe that in this case T_i values were obtained in a way that D assume its maximum, that is, equal to 1. Besides, it is noticed that the values of code variables tend to assume their maximum values.

5.3. Test Case 3: Pareto's Curve (Multi-objective)

In this case, the target values are defined according to the following equation for each response i : Table 6 presents the best point, in terms of to maximize tool life (point B, see Figure 2) and minimize cutting force (point A, see Figure 2), respectively.

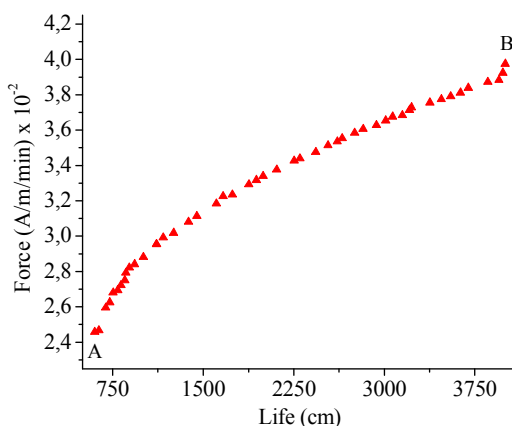


Figure 2. Pareto's Curve (Force versus Life).

Table 6. Points belong to Pareto's Curve.

	x_1	x_2	x_3	Y_1	Y_2
A	1.40	0.24	-1.37	602.65	2.45
B	-1.40	-1.39	-1.40	4002.60	3.97

6. CONCLUSIONS

In this work it was studied the treatment of multi-response surface using the desirability function approach and multi-objective optimization associated with the Differential Evolution algorithm.

The proposed algorithm is applied to machinability of stainless steel ABNT420 using a model that foresees the responses of tool life and cutting forces in terms of cutting speed, feed per tooth and depth of cut. The effects of these variables in responses were investigated crossing information contained in bound surfaces of material removal rate and cutting force.

The results obtained show that the methodology used represents an interesting approach to the treatment of the optimization problem formulated.

Finally, it is important to observe that the methodology proposed in this work eliminates the necessity of transforming an original multi-response problem into a similar with single-response, i.e., it solves the original multi-objective problem.

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