USE OF BAYESIAN INFERENCE FOR THERMOPHYSICAL PROPERTIES ESTIMATION WITH THE TRANSIENT LINE HEAT SOURCE PROBE METHOD

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Abstract. *The transient line heat source probe technique is widely used for the measurement of the thermal conductivity of nanofluids. In this work, we present a mathematical model that takes into account the probe and the surrounding nanofluids and thus permits the simultaneous estimation of the nanofluid and the probe thermal properties. Since several parameters appearing in the formulation are not deterministicaly known, a technique within the Bayesian framework is used for the solution of the inverse problem. To ensure minimum variance in the estimated parameters, the D-optimal approach is used, together with the analysis of the sensitivity coefficients for the design of the experiment. Simulated temperature measurements of a sensor located close to the probe surface, together with those of the probe, are used in the inverse analysis. Due to the high sensitivty of the inverse problem solution to this sensor position, and the difficulty which may arise from its measurement, it is estimated together with the other parameters of interest.*

Keywords*: Nanofluids, Thermophysical Properties, Line heat source probe, Bayesian statistics, MCMC method*

1. INTRODUCTION

The thermal loads are increasing in a wide variety of applications like microelectronics, transportation, etc. Micromechanical systems (MEMS) technology and nanotechnology are also rapidly emerging as a new revolution in miniaturization. Hence the management of high thermal loads in these systems offers challenges and the thermal conductivity of heat transfer fluids have become vital. Traditional heat transfer fluids such as water, engine oil, and ethylene glycol (EG) usually have low thermal conductivity (Chandrasekar and Suresh, 2009). Metals in the solid form have thermal conductivities larger by three orders-of-magnitude than usual heat transfer fluids. For example, the thermal conductivity of water at room temperature is 0.6 W/m K, while for copper is 386 W/m K. These three orders of magnitude between the thermal conductivity of fluids and metals makes one consider since one hundred years ago, the possible enhancement of thermal conductivity of liquids by suspending metal particles in them (Patel et al, 2003). However, early investigations have concerned millimeter or micrometer-sized particles, and the resulting solutions suffered from severe sedimentation (Murshed et al., 2008).

Following this idea, Choi (1995) imagined the application of the emerging nanotechnology to thermal engineering to create a new generation of fluids with enhanced thermal properties. This new class of fluids, named as nanofluids, is engineered by dispersing metal-oxides or metals nanoparticles or nanotubes in traditional heat transfer fluids. In contrast to micrometer-sized particles, nanoparticles, due to their high surface to volume ratio, can remain in suspension and thereby reduce erosion and channels obstructions. Furthermore, due to their small size, nanoparticles are more appropriate for use in micro-systems (Murshed et al., 2008). Most of the published works on nanofluids shows that they present enhanced thermal properties than the pure base fluids. As thermal conductivity is one of the most important parameter responsible for enhancing heat transfer, numerous experiments in nanofluids were concerned with its determination. The literature survey shows that the obtained results are dispersed, and many factors such as particles size and shape, particles volume fraction, temperature, base-fluid, dispersion, etc. are supposed to have an influence on the nanofluid's thermal properties. On the other hand, several mechanisms and models of enhanced conductivity have been proposed, but none has gained universal support (Choi, 2009).

Most of the nanofluid thermal conductivity measurements reported in the literature have been conducted by using the transient hot wire method, or the line heat source probe. However, in the classical use of this technique, some ideal hypotheses like the non-participation of the probe in heat transfer and the infinite size of the sample, can lead to errors in the resulting estimates. Recently, inverse parameter estimation which involves accurate modeling has been successfully applied to the line heat source probe, allowing the simultaneous estimation of more than one parameter, in addition to the thermal conductivity (André et al., 2003; Thomson and Orlande, 2006; Banaszkiewicz et al., 1997; Carvalho and Neto, 1999). A common feature of these works is that the solution of the inverse problem was retrieved deterministically. Therefore, they provide no means to quantify the uncertainties associated with the supposedly "known" parameters. On the other hand, Bayesian inference approaches to inverse problems offer a way to cope with the uncertainties of all parameters appearing in the formulation, by using a complete probabilistic description via prior modeling. This approach provides a natural framework for uncertainties quantification.

In this work, we present a mathematical model for a line heat source probe that takes into account the probe and the surrounding material. In a previous work, parameters of the mathematical model were estimated by assuming available only measurements of a sensor within the probe, which was of high volumetric heat capacity (Lamien and Orlande, 2010). In the present work it is examined the case of a probe with low volumetric heat capacity. In this case, temperature measurements of a second sensor are needed, in order to obtain estimates of the desired parameters. A technique within the Bayesian framework, known as the Markov Chain Monte Carlo based on Metropolis-Hastings algorithm (MCMC-MH) is used for the solution of the inverse problem. Such technique enables to account for the uncertainties on the a priori known parameters, including the second sensor position, which may not be accurately known.

2. LINE HEAT SOURCE PROBE

The classical Hot-Wire technique (Blackwell, 1954) consists in a constant heat power generation by Joule effect through a thin cylindrical wire placed inside the material that is assumed to be a semi-infinite medium (no heat losses). The temperature rise of the wire is then measured. At longer times, the Hot-Wire temperature evolution is shown to be a linear function of the logarithm of time. Knowing the heat power dissipation, the thermal conductivity of the material is computed according to the following formulae, where m is the slope of the curve (Blackwell, 1954):

$$k = \frac{Q}{4\pi m} \tag{1}$$

This work is based on a commercial line heat source probe, the Hukseflux TP-02 which is shown in figure 1. It consists of a needle of stainless steel with 150 mm length and 1.5 mm external diameter, connected to a stainless steel base with 50 mm length and 10 mm external diameter. Inside the needle there's a heating wire, as well as two K type thermocouples connected in a way to provide the temperature difference between the probe and the medium. Inside the base there's a PT-1000 temperature sensor for the measurement of the temperature of the cold joints of the thermocouples**.**

Figure 1: TP02 line heat source probe from Hukseflux

3. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem under analysis consists of a long and thin cylinder (the probe) of radius r_s and high thermal conductivity k_s, assumed as a lumped system. The probe is inserted into the fluid with unknown properties, which is contained in a cylindrical cell. The fluid and the cell are both considered to be hollow cylinders with internal radius r_s, r_{int} and external radius r_{int}, r_{ext}, respectively. We assume heat conduction in homogeneous and isotropic media, with constant thermal properties. Furthermore, it is assumed a perfect thermal contact at both probe/liquid, and liquid/cell interfaces, while for t > 0, the surface at $r = r_{ext}$ exchange heat with a surrounding liquid in a thermostatic bath. The system is assumed to be initially in thermal equilibrium, at the temperature T_0. For the time scale of interest, t > 0, the probe is uniformly heated with a time dependent heat source g(t). By neglecting end-effects, the physical problem under picture can be formulated as one dimensional. The proposed mathematical formulation is divided into two parts. The first one corresponds to the heating period and the second one to the non-heating period. For the sake of brevity, it is presented below only the mathematical formulation for the heating period.

$$C_s^* \frac{d\,\Theta_s(\tau)}{d\tau} = 1 + 2K_f^* \left.\frac{\partial \Theta_f(R,\tau)}{\partial R}\right|_{R=1} \quad , \; for \quad \tau > 0 \tag{2}$$

$$\frac{1}{\alpha_f^*}\frac{\partial \Theta_f(R,\tau)}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Theta_f(R,\tau)}{\partial R}\right) \; , \; in \;\; 1 < R < \mathrm{R}_{\mathrm{int}} \; , \; for \;\; \tau > 0 \tag{3}$$

$$\frac{1}{\alpha_m^*}\frac{\partial \Theta_m(R,\tau)}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Theta_m(R,\tau)}{\partial R}\right) \; , \; in \;\; \mathrm{R}_{\mathrm{int}} < R < \mathrm{R}_{ext} \; , \; for \; \tau > 0 \tag{4}$$

$$\Theta_s(\tau) = \Theta_f(R,\tau) \quad at \;\; R = 1, \;\; \tau > 0 \tag{5}$$

$$\Theta_f(R,\tau) = \Theta_m(R,\tau) \;\; at \;\; R = R_{\mathrm{int}} \; , \; \tau > 0 \tag{6}$$

$$K_f^* \frac{\partial \Theta_f(R,\tau)}{\partial R} = K_m^* \frac{\partial \Theta_m(R,\tau)}{\partial R} \quad at \;\; R = R_{\mathrm{int}} \; , \; \tau > 0 \tag{7}$$

$$K_m^* \frac{\partial \Theta_m(R,\tau)}{\partial R} + Bi\Theta_m(R,\tau) = 0 \;\; at \;\;\; R = \mathrm{R}_{ext} \; , \; \tau > 0 \tag{8}$$

$$\Theta_s(\tau) = \Theta_f(R,\tau) = \Theta_m(R,\tau) = 0 \quad in \; 1 \le R \le R_{ext}, \; at \; \tau = 0 \tag{9}$$

where the subscripts *f*, *m* and *s* denote the fluid, the cell and the probe, respectively. The following dimensionless parameters were introduced:

$$\Theta(R,\tau) = \frac{T(r,t) - T_0}{\dfrac{g(t) r_s^2}{k_{ref}}}; \;\; \tau = \frac{k_{ref}\, t}{C_{ref}\, r_s^2}; \;\; \alpha^* = \frac{\alpha}{\alpha_{ref}}; \;\; R = \frac{r}{r_s}; \;\; K^* = \frac{k}{k_{ref}}; C_s^* = \frac{C_s}{C_{ref}}; \;\; Bi = \frac{h\, r_s}{k_{ref}} \tag{10.a-i}$$

where the subscript *ref* denotes a reference value.

4. DIRECT PROBLEM AND INVERSE PROBLEM

The *direct problem*, associated with the formulation given above for the physical problem, consists in determining the temperature distribution from the knowledge of the heating and final times, the geometry and the thermal properties of the probe and of the surrounding materials. The solution of the *direct problem* was obtained with an implicit finite volume discretization, which yields a tridiagonal matrix system of equations (Patankar, 1980). Grid refinement was performed by using as reference an analytical solution for a heat conduction problem in a single medium with a time-dependent heat source (probe) of strength g(t) along its axis (ASME V&V, 2009). In order to match the lumped body assumption for the probe, it was considered as a highly conductive material. This problem was solved with the *Classical Integral Transform Technique* (Ozisik, 1993). The numerical solution graphically matched the analytical one for 250 and 16 volumes, respectively, for the fluid and the cell.

The *inverse problem* of concern aims at the simultaneous estimation of the probe's volumetric heat capacity (C^*_s), the liquid's thermal properties (K_f^* and α_f^*), the cell's thermal properties (K_m^*, α_m^*), and the Biot number (*Bi)* at the surface of the cell, as well as the position of the second sensor (X_p), which are regarded as unknown. In addition, temperature measurements of the probe and of a second sensor are supposed available. Such measurements may contain random errors, which are assumed to be additive, uncorrelated, normally distributed with zero mean and with known and constant standard deviation.

For the solution of the *inverse problem*, a technique within the Bayesian framework is used. Such technique is described next.

5. MARKOV CHAIN MONTE CARLO (MCMC) METHODS

In the Bayesian approach to statistics, an attempt is made to utilize all available information in order to reduce the amount of uncertainty present in an inferential or decision-making problem. As new information is obtained, it is combined with any previous information to form the basis for statistical procedures. The formal mechanism used to combine the new information with the previously available information is known as Bayes' theorem (Lee, 2004). Therefore, the term Bayesian is often used to describe the so-called statistical inversion approach, which is based on the following principles (Kaipio and Somersalo, 2004):

1. All variables included in the model are modeled as random variables.
2. The randomness describes the degree of information concerning their realizations.
3. The degree of information concerning these values is coded in probability distributions.
4. The solution of the inverse problem is the posterior probability distribution.

Consider, for the sake of generality, the vector of parameters appearing in the physical model formulation as

$$\boldsymbol{P}^T = [P_1, P_2, \ldots, P_N] \tag{11}$$

and the vector of the available measurements as

$$\boldsymbol{Y}^T = [Y_1, Y_2, \ldots, Y_I] \tag{12}$$

where N is the number of parameters and I is the number of measurements. Bayes' theorem can then be stated as (Kaipio and Somersalo, 2004):

$$\pi_{posterior}(\boldsymbol{P}) = \pi(\boldsymbol{P}|\boldsymbol{Y}) = \frac{\pi_{prior}(\boldsymbol{P})\pi(\boldsymbol{Y}|\boldsymbol{P})}{\pi(\boldsymbol{Y})} \tag{13}$$

where $\pi_{posterior}(\boldsymbol{P})$ is the posterior probability density, that is, the conditional probability of the parameters $\boldsymbol{P}$ given the measurements $\boldsymbol{Y}$; $\pi_{prior}(\boldsymbol{P})$ is the prior density, that is, the coded information about the parameters prior to the measurements; $\pi(\boldsymbol{Y}|\boldsymbol{P})$ is the likelihood function, which expresses the likelihood of different measurement outcomes $\boldsymbol{Y}$ with $\boldsymbol{P}$ given; and $\pi(\boldsymbol{Y})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant.

In practice such normalizing constant is difficult to compute and numerical techniques like Markov Chain Monte Carlo are required in order to obtain samples that accurately represent the posterior probability density. In order to implement the Markov Chain, a density $q(\boldsymbol{P}^*, \boldsymbol{P}^{(t-1)})$ is required, which gives the probability of moving from the current state in the chain $\boldsymbol{P}^{(t-1)}$ to a new state $\boldsymbol{P}^*$.

The Metropolis-Hastings algorithm (Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004) was used in this work to implement the MCMC method. It can be summarized in the following steps:

1. Sample a *Candidate Point* $\boldsymbol{P}^*$ from a jumping distribution $q(\boldsymbol{P}^*, \boldsymbol{P}^{(t-1)})$
2. Calculate:

$$\alpha = \min\left[1, \frac{\pi(\boldsymbol{P}|\boldsymbol{Y})\, q(\boldsymbol{P}^{(t-1)}, \boldsymbol{P}^*)}{\pi(\boldsymbol{P}^{(t-1)}|\boldsymbol{Y})\, q(\boldsymbol{P}^*, \boldsymbol{P}^{(t-1)})}\right] \tag{14}$$

3. Generate a random value U which is uniformly distributed on (0,1).
4. If $U \leq \alpha$, define $\boldsymbol{P}^{(t)} = \boldsymbol{P}^*$; otherwise, define $\boldsymbol{P}^{(t)} = \boldsymbol{P}^{(t-1)}$
5. Return to step 1 in order to generate the sequence $\{\boldsymbol{P}^{(1)}\ \boldsymbol{P}^{(2)}, \ldots, \boldsymbol{P}^{(n)}\}$.

In this way, we get a sequence that represents the posterior distribution and inference on this distribution is obtained from inference on the samples $\{\boldsymbol{P}^{(1)}, \boldsymbol{P}^{(2)}, \ldots, \boldsymbol{P}^{(n)}\}$. We note that values of $\boldsymbol{P}^{(i)}$ must be ignored until the chain has not converged to equilibrium. For more details on theoretical aspects of the Metropolis-Hastings algorithm and MCMC methods, the reader should consult references (Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004).

We assume in this work that the errors in the measured variables are additive, uncorrelated, normally distributed, with zero mean and known constant standard-deviation. Hence, the likelihood function is given by (Beck and Arnold, 1977, Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004):

$$\pi(\boldsymbol{Y}|\boldsymbol{P}) = (2\pi)^{-M/2}\, |\boldsymbol{W}|^{-1/2} \exp\left\{-\frac{1}{2}[\boldsymbol{Y} - \Theta(\boldsymbol{P})]^T \boldsymbol{W}^{-1} [\boldsymbol{Y} - \Theta(\boldsymbol{P})]\right\} \tag{15}$$

where $\boldsymbol{\Theta}$ is the vector of estimated dimensionless temperatures obtained from the solution of the direct problem with an estimate for the parameters $\boldsymbol{P}$, and $\boldsymbol{W}$ is the covariance matrix of the measurements.

Generally, before the solution of an inverse problem is investigated, a sensitivity analysis together with a D-optimal design of the experiment needs to be performed. Such examinations give an indication of the best sensor location and measurements times to be used in the inverse analysis which correspond to linearly independent sensitivity coefficients with large absolute values and large magnitudes of the information matrix determinant (Ozisik and Orlande, 2000).

6. SENSITIVY ANALYSIS AND D-OPTIMAL DESIGN OF THE EXPERIMENT

The sensitivity coefficient J_{ij} is defined as the first derivative of the temperature at time τ_i, with respect to the unknown parameter P_j, that is,

$$J_{ij} = \frac{\partial \Theta_i}{\partial P_j} \tag{16}$$

Together with the information matrix determinant $(|\boldsymbol{J}^T\boldsymbol{J}|)$, the analysis of the sensitivity coefficients plays an important role in inverse parameter estimation. As a whole, it is desirable to obtain linearly independent sensitivities coefficients with large magnitudes. Such conditions enable the solution of the inverse problem to be less sensitive to measurements errors. For L sensors, the sensitivity matrix is given by (Ozisik and Orlande, 2000):

$$\boldsymbol{J} = \left[\frac{\partial\, \Theta^T}{\partial\, \boldsymbol{P}}\right]^T = \begin{bmatrix} \begin{bmatrix} \dfrac{\partial \Theta_{l,1}}{\partial P_1} \\ \vdots \\ \dfrac{\partial \Theta_{L,1}}{\partial P_1} \end{bmatrix} & \cdots & \begin{bmatrix} \dfrac{\partial \Theta_{l,1}}{\partial P_N} \\ \\ \dfrac{\partial \Theta_{L,1}}{\partial P_N} \end{bmatrix} \\ \vdots & \cdots & \vdots \\ \begin{bmatrix} \dfrac{\partial \Theta_{l,I}}{\partial P_1} \\ \vdots \\ \dfrac{\partial \Theta_{L,I}}{\partial P_1} \end{bmatrix} & \cdots & \begin{bmatrix} \dfrac{\partial \Theta_{l,I}}{\partial P_N} \\ \\ \dfrac{\partial \Theta_{L,I}}{\partial P_N} \end{bmatrix} \end{bmatrix} \tag{17}$$

Optimum experiments can be designed by maximizing the determinant of the information matrix ($max|\boldsymbol{J}^T\boldsymbol{J}|$). Such an analysis is performed, to ensure minimum variance for the estimates, and experimental variables such as the heating time, the duration of the experiment, location and number of sensors, the measurements methods can be chosen based on this analysis (Ozisik and Orlande, 2000).

For a case involving L sensors, each element of the sensitivity matrix $F_{m,n}$, m,n=1,…,N, of the matrix $\boldsymbol{F}=\boldsymbol{J}^T\boldsymbol{J}$ is given by (Ozisik and Orlande, 2000):

$$F_{m,n} = \left[\boldsymbol{J}^T \boldsymbol{J}\right]_{m,n} = \sum_{l=1}^{L}\sum_{i=1}^{I}\left(\frac{\partial \Theta_{l,i}}{\partial P_m}\right)\left(\frac{\partial \Theta_{l,i}}{\partial P_n}\right) \quad m,n = 1,...,N \tag{18}$$

where I is the number of measurements and N is the number of unknown parameters.

7. RESULTS AND DISCUSSIONS

In order to carry out the simulations, the following material properties were selected: steel for the cell container (k_m= 43.2 W/m°C; α_m=11.8x10^{-6}), water as the fluid whose thermal properties are to be determined (k_f=0.6 W/m°C, α_f =1.4x10^{-7} m^2/s), and a material with 10% of the volumetric heat capacity of steel was considered to simulate a probe with low heat capacity. We assume a heat transfer coefficient h = 20 W/m^2 °C from the cell to the external medium. The chosen reference material properties are those of the base-fluid; such a choice enables to get directly, in the case of nanofluids, a relative enhancement in thermophysical properties if such is the case. For the case involving the estimation of the thermal properties of water, we have the following dimensionless parameters: K_m^*= 72; α_m^*=82.2067; Bi =0.02; K_f^*=1; α_f^*=1 and C^*_s =0.0876. The diameter of the probe is taken as 1.5 mm, the sample dimension (40 mm) was chosen, as specified in the probe user's manual, and a cell with 4 mm of thickness.

We make use of reduced sensitivity coefficients in the analysis presented below. The reduced sensitivity coefficients are defined as the sensitivity coefficients multiplied by their corresponding parameters, and, therefore, can have the temperature as a basis of comparison. In this work, the sensitivities coefficients were computed with forward finite differences.

As previously mentioned, temperature measurements of a second sensor are needed together with those of the probe in order to reduce the linear dependence between the fluid's parameters, and get their estimates. The analysis of the sensitivity coefficients for different positions has shown that, such a sensor must be placed close to the probe, because the magnitudes of the sensitivity coefficients are important for these positions. Figures 2 and 3 present the transient behavior of the reduced sensitivity coefficients with respect to the parameters appearing in the formulation, for a temperature sensor within the probe, and for a sensor located at a distance of 3 mm from the probe's surface, respectively. The variations of the dimensionless temperature of the probe and of the second sensor were included in these figures, as well. These figures show that the reduced sensitivity coefficients of the parameters of the fluid under study have the same order of magnitude of the dimensionless temperatures and tend to be linearly dependent. However, after the heating is ceased, we can notice a rapid decrease of the sensitivity coefficients with respect to the thermal conductivity and thermal diffusivity of the fluid. The sensitivity coefficient of the probe's volumetric heat capacity is very close to zero, so this one may be not accurately estimated. In addition, it can be notice that the sensitivity coefficients with respect to the parameters of the material of the cell and the Biot number are null. Consequently, these parameters cannot be estimated with measurements taken at such sensors positions.

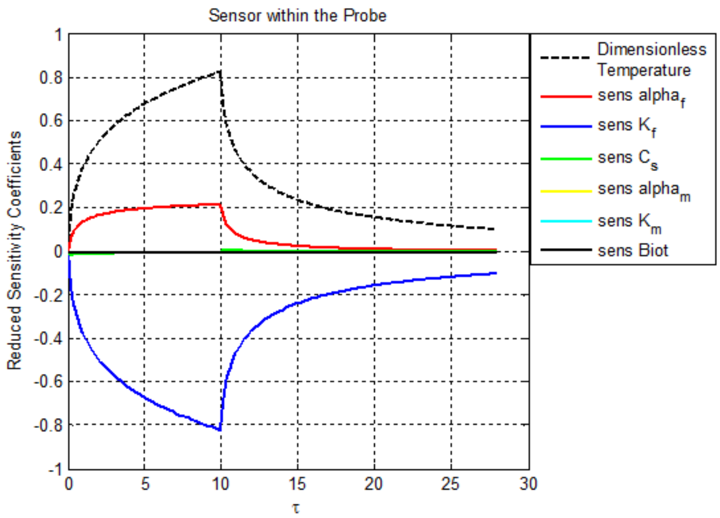


Figure 2: Sensitivity coefficients analysis
for a Sensor within the Probe

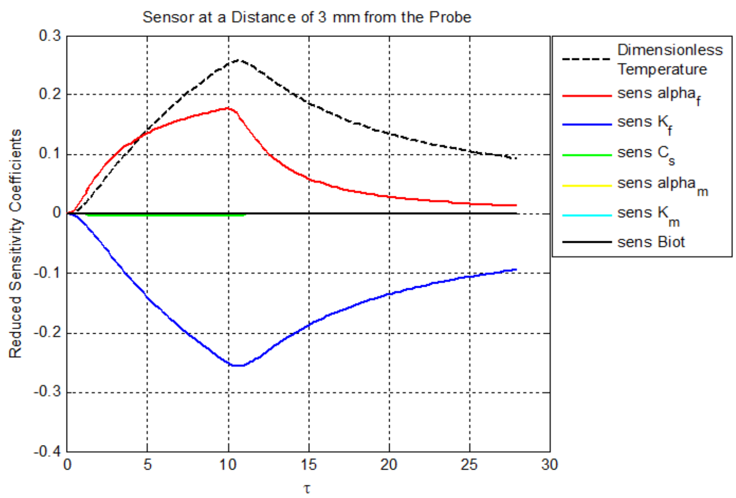


Figure 3: Sensitivity coefficients analysis
for a sensor at a distance of 3 mm from the Probe Surface

Figure 4 presents the results of the D-optimal design, by considering that the temperature measurements are collected with a fixed frequency of one measurement every 0.5 second. It can be noticed in this figure that, when the heating time is taken equal to the duration of the experiment, the maximization of the determinant yields a relatively lower value as compared to the cases in which the heating is ceased before the end of the experiment. Thus, more informative experiments can be obtained by choosing a heating time smaller than the duration of the experiment. As shown in this figure, by increasing the final time, the determinant increases, but less significantly after 275 s. Hence, the duration of the experiment can be chosen equal to 275 s and the heating time to 200 s, as this value yields the maximum determinant of the information matrix.

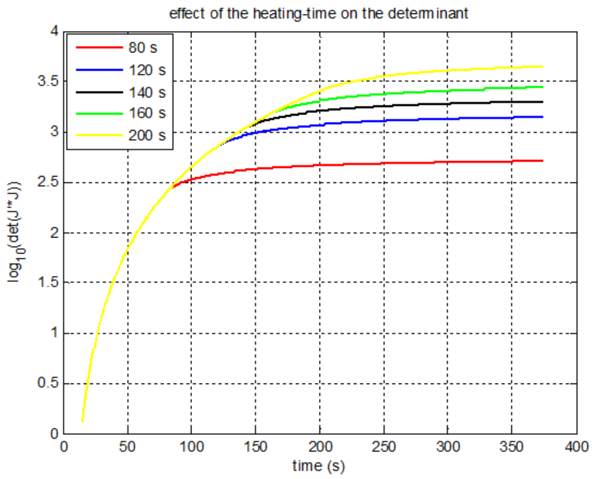


Figure 4: Effect of heating-time on the determinant

Based on the results obtained from the D-optimal design of the experiment presented above, as well as of the previously defined thermal and geometrical parameters, simulated measurements were generated for a sensor within the probe and another one at 3 mm from the probe's surface, by solving the *direct problem*. As the measurements may contain errors, simulated measurements errors are added, which are supposed to be Gaussian random values, with a standard deviation of 1% of the maximum temperature.

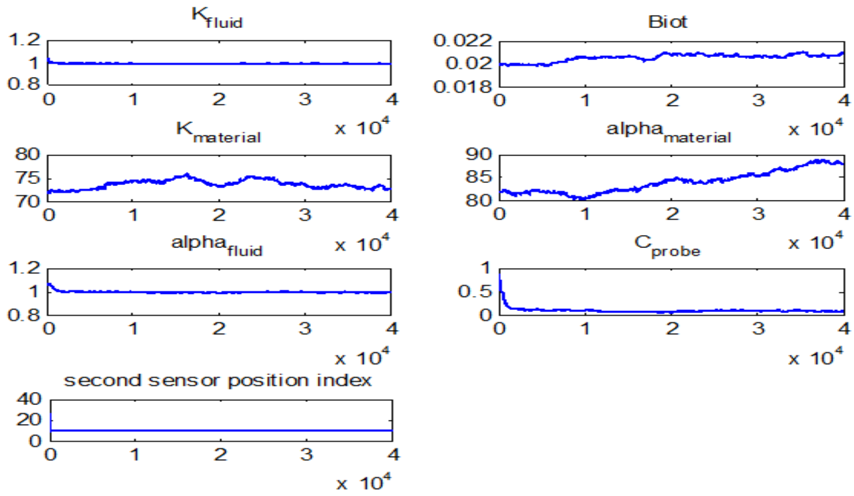
The solution of the inverse problem was considered in two steps, as follows. Firstly, we consider a fluid with known thermal properties where the objective of the inverse problem is to get an estimate of the volumetric heat capacity of the probe. As mentioned above, accurate estimates of the cell container parameters and the Biot number cannot be obtained in the time interval used in the analysis, thus it is assumed that they are known from another experiment, like the flash method for the thermal properties and from available correlations of natural convection for the Biot number. In order to solve the inverse problem for this first step, the prior distributions presented in Table 1 were assumed for each parameter. As in practice it may occur measurement error on the distance between the probe and the second sensor, it is assumed a normal prior with known mean differing of 1 mm from the exact one used to generate the simulated measurements, and with ± 2 mm of 99% confidence interval for a Gaussian distribution. Due to the discrete nature of the *direct problem* solution's, the position of the second sensor is estimated through its discretization index.

Table 1: Prior Distributions for the first step of the inverse problem solution

Dimensionless Parameter	Prior Distribution	
C_s^* Volumetric Heat Capacity of the Probe	Uniform	Lower Bound : $C_s^* > 0$ Upper Bound : C^*_{steel}
K_f^* Fluid Thermal Conductivity	Normal	Mean : K_f^* Standard Deviation: $5\% * K_f^*$
α_f^* Fluid Thermal Diffusivity	Normal	Mean: α_f^* Standard Deviation: $5\% * \alpha_f^*$
K_m^* Cell Container Material Thermal Conductivity	Normal	Mean : K_m^* Standard Deviation: $5\% * K_m^*$
α_m^* Cell Container Material Thermal Diffusivity	Normal	Mean : α_m^* Standard Deviation: $5\% * \alpha_m^*$
Bi Biot Number	Normal	Mean : Bi Standard deviation: 5%*Bi
Xp Second Sensor Pcsition Index	Normal	Mean : 15 Standard Deviation: 4.6584

Figures 5 show the states of the Markov Chain for each parameter, and also the burn-in period of roughly of 2000 samples, which are required for the Markov Chain corresponding to the probe's volumetric heat capacity to reach equilibrium. It must be noticed in this figure, the rapid convergence (roughly 200 samples) of the Markov Chain corresponding to the second sensor position index to its exact value, despite the misspecification on the prior mean. Also, one can note in this figure that the Markov Chains corresponding to the cell container material properties and to the Biot number display an oscillatory behavior. From the posterior distribution, we compute the mean value of the parameters, by discarding the first 2000 samples. Table 2 presents the obtained results with their associated standard deviation and 99% confidence interval. It is worth noting in this table, the excellent estimate of the probe's volumetric heat capacity, despite the use of a non-informative prior, and the initial state of the Markov Chain far from its exact value.

As shown in this table, the thermal properties of the fluid and the volumetric heat capacity of the probe are more accurately estimated than the thermal properties of the cell and the Biot number. Such a result was expected from the sensitivity analysis.



Figures 5: States of the Markov Chain

Table 2: Estimated parameters with their related statistics – first step

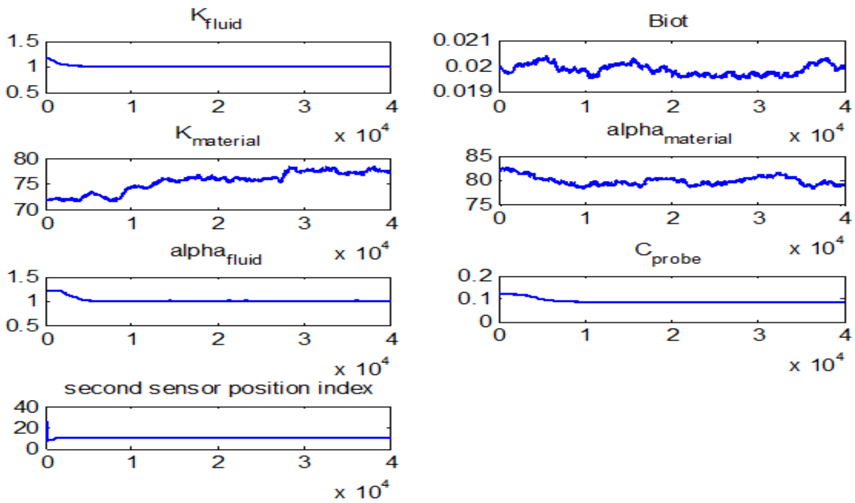
Parameters	K_f^*	α_f^*	Bi	K_m^*	α_m^*	C_s^*	Xp
Exact	1.0000	1.0000	0.0200	72.0000	82.2067	0.0876	10
Initial State of the Markov Chain	1.0500	1.0500	0.0200	72.0000	82.2067	0.8759	27
Estimates	0.9981	0.9935	0.0206	73.8595	84.2349	0.0841	10
Standard Deviation	0.0010	0.0035	0.0002	0.7818	2.3971	0.0147	0
Confidence Interval (99 %)	0.9956 1.0006	0.9845 1.0025	0.0200 0.0212	71.8456 75.8734	78.0599 90.4099	0.0462 0.1220	

The estimation of the probe volumetric heat capacity obtained from the previous step in the inverse problem solution was then used in a second step as an informative normal prior. The second step was concerned with the estimation of the thermal properties of the fluid and, thus, non-informative uniform priors were assumed for these parameters. Again, in order to account for the uncertainties on the second sensor position, it is estimated together with the model's thermal parameters. Table 3 presents the specified prior distributions used for the second step. The prior distributions for the cell container thermal properties, the Biot number, and the second sensor position index remain the same as previously.

Figures 6 show the states of the Markov Chain for each parameter. The burning period was roughly of 5000 samples. Note again the rapid convergence of the Markov Chain corresponding to the second sensor position index, to its exact value. The parameters were computed from the mean value of the posterior distribution, by discarding the samples of the burning-period. Table 4 shows the obtained results and their related statistics. As shown in this table, accurate estimates for the fluid thermal properties and for the probe volumetric heat capacity can be obtained from the proposed methodology.

Table 3: Prior Distributions for the second step of the inverse problem solution

Dimensionless Parameter	Prior Distribution	
C_s^* Volumetric Heat Capacity of the Probe	Normal	Mean & Standard Deviation: Resulting of the previous Inverse Solution
K_f^* Fluid Thermal Conductivity	Uniform	Lower Bound: K_f^* - 20%*K_f^* Upper Bound: K_f^*+20%*K_f^*
α_f^* Fluid Thermal Diffusivity	Uniform	Lower Bound: α_f^*-20%* α_f^* Upper Bound: α_f^*+20%* α_f^*
K_m^* Cell Container Material Thermal Conductivity	Normal	Mean: K_m^* Standard Deviation: 5%*K_m^*
α_m^* Cell Container Material Thermal Diffusivity	Normal	Mean: α_m^* Standard deviation: 5%*α_m^*
Bi Biot Number	Normal	Mean: Bi Standard deviation: 5%*Bi
Xp Second Sensor Position Index	Normal	Mean: 15 Standard Deviation: 4.6584



Figures 6: States of the Markov Chain

Table 4: Estimated parameters with their related statistics – second step

Parameters	K_f^*	α_f^*	Bi	K_m^*	α_m^*	C_s^*	Xp
Exact	1.0000	1.0000	0.0200	72.0000	82.2067	0.0876	10
Initial State of the Markov Chain	1.2	1.2	0.0200	72.0000	82.2067	0.1220	27
Estimates	1.0014	1.0044	0.0199	75.9527	79.6808	0.0846	10
Standard Deviation	0.0011	0.0037	0.0002	1.6538	0.6898	0.0029	0
Confidence Interval (99 %)	0.9987 1.0042	0.9950 1.0139	0.0193 0.0204	71.6924 80.2130	77.9038 81.4578	0.0772 0.0920	

8. CONCLUSIONS

In this paper a Bayesian estimation technique was used for the thermal characterization of fluids, by using transient temperature measurements of a sensor within a line heat source probe together with those a second sensor. The Markov Chain Monte Carlo method, coded in the form of the Metropolis-Hastings algorithm, was used to obtain the posterior probability density for the parameters. Simulated temperature measurements were used in the inverse analysis. The proposed methodology enables to get accurate and simultaneous estimates of the thermal conductivity and thermal diffusivity of nanofluids, and the volumetric heat capacity of the probe.

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