STRUCTURAL DAMAGE ASSESSMENT USING OPTIMIZATION TECHNIQUES AND SENSITIVITY ANALYSIS

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Abstract. *Structural systems in a variety of applications including aerospace vehicles, automobiles and engineering structures such as tall buildings, bridges and offshore platforms, accumulate damage during their service life. In several situations, such damage may not be visually observable. From the standpoint of both safety and performance, it is desirable to monitor the occurrence, location and extent of such damage. System identification methods, which may be classified in a general category of non-destructive evaluation techniques, can be employed for this purpose. Using experimental data, such as eigenmodes and static displacements, and an analytical structural model, parameters of the structures can be identified. The approach used in the present work is one where the structural properties of the analytical model are varied to minimize the difference between the analytically predicted and empirically measured response. This is an inverse problem where the structural parameters are identified. For the damage assessment problem a finite element model of the structural system is available and the model of the damaged structure will be identified. Damage will be represented by a reduction in the elastic stiffness properties of the structure, introducing a damage variable that varies from 0 to 1. The problem described above is a nonlinear unconstrained problem that can be solved using classical gradient based methods such as the Levenberg-Marquardt . Depending on the number of design variables the resulting design space can be very nonconvex and probably will present several local minima, which means that damage in some members may not be detected. An alternative to solve this problem is the use of a global optimization method such as the Differential Evolution. This results in a very large computational cost. To circumvent this a combination of a global and a local optimization methods, keeping the best features of each method is used in this paper. To use a gradient based method a sensitivity analysis is necessary. It also indicates which members will have possible damages more difficult to detect. The methodology was applied to a simple planar truss structure. A reduced set of eigenmodes is used as experimental response.*

Keywords*: structural damage assessment, inverse problems, optimization methods, sensitivity analysis, hybrid methods*

1. INTRODUCTION

In a typical load bearing structure, degradation of structural properties due to damage manifests itself as a change in the static and dynamic structural response. A correlation of the measured response with that obtained from an analytical model of the undamaged structure, allows for the possibility of determining a modified model that predicts the altered response. This process can be broadly categorized in the realm of system identification methods. The output error method of system identification, wherein the analytical model is refined to minimize the difference between the predicted and measured response of the structure was used in the present work (Stavroulakis, 2001). Damage is represented by reduction in the elastic extensional modulus of the element introducing in the element stiffness matrix a factor varying from 0 to 1 designated as design variables in the resulting optimization problem (Hajela and Soeiro, 1990a) and (Hajela and Soeiro, 1990b). Numerical evidence clearly indicates that when eigenmodes alone are used for identification, the location and extent o damage predicted by the optimization approach is dependent on the number of modes used to match the measured and the predicted response. Higher modes are difficult to determine and to measure. In this work incomplete sets of modes are used to simulate real measured data. The approach of defining one design variable per element produces a large dimensionality problem. This results in a very nonconvex design space probably with several local minima. The gradient-based nonlinear programming algorithms may have difficulties to find the global optimum. A sensitivity analysis indicates that some members will have their possible damage very difficult to detect. The combination of a global and a local optimization methods will be a good alternative to solve the damage assessment problem.

2. STRUCTURAL DAMAGE ASSESSMENT

In a finite element formulation, structural characteristics are defined in terms of the stiffness, damping, and mass matrices [K], [C] and [M], respectively. The governing equation of equilibrium for a dynamical system involves each of these matrices, and can be written as,

$$[M]\ddot{x}+[C]\dot{x}+[K]x=P(t) \tag{1}$$

Where {x} is the displacement vector and P(t) is the vector of applied loads. The static load-deflection relation only involves the system stiffness matrix,

$$[K](x)=\{P\} \tag{2}$$

The analytical model describing the eigenvalue problem for an undamped system can be stated in terms of the system matrices defined above, the i-th eigenvalue ω_i^2, and the corresponding eigenmode X_i as follows:

$$\left([K]-\omega_i^2[M]\right)\{X_i\}=\{0\} \tag{3}$$

It is clear from these equations that a change in the system matrices results in a different response, and this difference can be related to changes in specific elements of the system matrices. Since internal structural damage typically does not result in a loss of material, it will be assumed that the mass matrix is constant. The stiffness matrix can be expressed as a function of the thickness 't', the length 'L', the cross-sectional area 'A', the Young's modulus 'E' and the flexural and torsional stiffness EI and GJ, respectively,

$$[K]=[K(t,L,A,E,EI,GJ)] \tag{4}$$

In the present work, changes in these quantities are lumped into a damage coefficient d_i, that is used to multiply the stiffness matrix of a particular element. The coefficients d_i constitute the design variables for the damage assessment problem and vary from 0 (undamaged element) to 1 (completely damaged element). The values of the coefficients d_i give the location and the extent of damage in the structure.

The approach was applied to simple truss structures with different levels of damage. The stiffness matrix of the truss element modified to include the damage coefficient is

$$K_i^{(.)}=\frac{(1-d_i)E_iA_i}{L_i}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \tag{5}$$

Where $C = \cos\alpha$ and $S = \sin\alpha$, and the truss element is shown in Fig. 1.

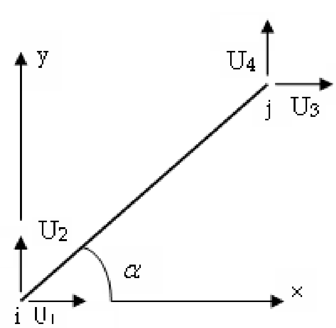

Figure 1. Truss element

If the measured and analytically determined static displacements or vibration modes are denoted by $\{Y_m\}$ and $\{Y_a\}$, respectively, the optimization problem can be formulated as determining the vector of design variables d_i that minimize the scalar objective representing the Euclidean norm of the difference between the analytical and experimental response and stated as follows:

$$F = \left\| Y_m^{ij} - Y_a^{ij} \right\| = \sum_i \sum_j (Y_m^{ij} - Y_a^{ij})^2 \quad (6)$$

where i represents the degree of freedom and j denotes a static loading condition or a particular vibration mode.

One important advantage of this approach is that the complete set of modes or displacements is not needed since the objective function involves only the difference between components of those vectors. Some of the components may be neglected according to its importance in the behavior of the structure. In this paper only the first four modes and the respective eigenvalues (natural frequencies) were used in the objective function.

3. SOLUTION OF THE INVERSE PROBLEM

In the present work a combination of a deterministic gradient based method, the Levenberg-Marquardt method, and a global optimization method, the Differential Evolution, is considered for the minimization of the objective function $F(\vec{d})$ given by Eq. (6) (Silva Neto and Moura Neto, 2005).

3.1. A Deterministic Local Optimizer – Levenberg-Marquardt (LM)

In order to minimize the objective function $F(\vec{d})$ we first write the critical point equation

$$\frac{\partial F}{\partial \vec{d}} = \frac{\partial}{\partial \vec{d}} \left(\vec{R}^T \vec{R} \right) = 0 \quad (7)$$

where $\vec{R} = \vec{Y}_m - \vec{Y}_a$

Using a Taylor's expansion and keeping only the terms up to the first order results

$$R\left(\vec{d}^{\,n+1}\right) = R\left(\vec{d}^{\,n} + \Delta\vec{d}^{\,n}\right) + J\big|_{\vec{d}^{\,n}} R\left(\vec{d}^{\,n}\right) \quad (8)$$

where the Jacobian matrix elements are given by

$$J_{ij} = \frac{\partial Y_{a_i}}{\partial d_j} \quad (9)$$

Adding a damping parameter λ one obtains from Eqs. (7) and (8) the Levenberg-Marquardt formulation

$$[(J^T)^n J^n + \lambda^n I]\Delta\vec{d}^{\,n} = -J^T \vec{R}(\vec{d}^{\,n}) \quad (10)$$

where I is the identity matrix, n is the iteration counter, and λ is reduced along the iterative procedure according the original proposition made by Marquardt (1963).

The iterative procedure starts with an initial guess $\vec{d}^{\,0}$, and new estimates are obtained with

$$\vec{d}^{\,n+1} = \vec{d}^{\,n} + \Delta\vec{d}^{\,n} \quad (11)$$

where the vector $\Delta\vec{d}^{\,n}$ is obtained from Eq.10.

The iterative procedure of sequentially calculating $\Delta\vec{d}^{\,n}$ and $\vec{d}^{\,n+1}$ from Eqs. 10 and 11, respectively, is continued until a convergence criteria such as $\left| \Delta d_j^n / d_j^n \right| < \varepsilon$, is satisfied, where ε is a small number.

The method described above requires the calculation of derivatives (Eq. 9). This is called sensitivity analysis and it seeks to determine the effect of the variation of a given variable on the objective function. The sensitivity analysis can also be a useful tool in different areas of engineering to determine the significance of one variable against another. The parameter estimates and an examination of the sensitivity coefficients of a function can provide significant information about a problem, to identify the variables that will have greater success in their determination. It is obvious that any object of study is always a factor of uncertainty and sensitivity analysis is sought to minimize these influences.

The sensitivity coefficients represent the first derivative of the objective function with respect to each design variable. Such information may be relevant to any project. As in any optimization problem it is best to perform a sensitivity analysis before applying the methodology of identification of damage in structures (Kleiber et al., 1997). In case of detection of damage in structures the sensitivity coefficients are defined as:

$$S_i \equiv \frac{\partial F(d_i)}{\partial d_i} \tag{12}$$

where F is the value of the objective function described in Eq. 6 and d_i are the damage variable corresponding to each structure bar. To determine the sensitivity coefficients, the damage of each element will be changed by 1%, while other elements will remain unchanged without damage. This information can be shown in a graph (sensitivity factor versus each element) indicating which damaged member will significantly influence the objective function.

3.2. Stochastic Global Optimizer - Differential Evolution

The Differential Evolution (DE) was proposed by Storn and Price (1997) as an algorithm to solve global optimization problems of continuous variables. The main idea behind DE is how possible solutions taken from the population of individuals are set, recombined and chosen to evolve the population to the next generation.

In a population of individuals, a fixed number of vectors are randomly initialized, and then evolved over the optimization task to explore the design space and hopefully to locate the optimum of the objective function. At each iteration, new vectors are generated by the combination of vectors randomly chosen from the current population. This operation is called "mutation" and a mutant population is created. The outcoming vectors are then mixed with a predetermined target vector. This operation is called "crossover" or "recombination" and produces a "trial vector". Finally, the "trial vector" is accepted for the next generation if it yields a reduction in the value of the objective function. This last operation is referred to as "selection." As can be seen, the basic algorithm preserves some common aspects of the traditional simple Genetic Algorithm (GA), specially the nomenclature of selection, crossover and mutation. A population of individuals can be expressed as a matrix:

$$P = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} \\ x_{12} & x_{22} & \cdots & x_{2j} \\ \cdots & \cdots & \cdots & \cdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} \end{bmatrix} \tag{13}$$

where i is the number of individuals of the population and j is the number of design variables.

As described before, the "mutation" operator adds the weighted difference between two individuals to a third individual (base vector). There are several ways to mutate a variable and the equation below shows a possible mutation scheme, among others.

$$v_i = x_{r1} + F\left(x_{r2} - x_{r3}\right) \tag{14}$$

where $\boldsymbol{v_i}$ is the mutant vector, x_{r1}, x_{r2} *and* x_{r3} are random integer indexes and mutually different, F is a real constant factor *[0,2]* which controls the amplification of the differential variation and x_{best} is the best individual of the current population.

The next operation is "crossover". Each mutant vector is combined with a target vector $\boldsymbol{x_i}$. This operation is performed by swapping the contents of the mutant vector with the correspondent component of the target vector based on a crossover probability *CR*. The resulting vector is denominated "trial vector". At the sequence of the *DE* algorithm, the selection operator decides whether or not the new vector $\boldsymbol{x_{trial}}$ should become a member of the next generation. This is decided by the objective function value of all new individuals $f(\boldsymbol{x_{trial}})$ which are compared with the one of the target vector $f(\boldsymbol{x_{target}})$. If there is an improvement, $\boldsymbol{x_{trial}}$ is selected to be part of the next generation, otherwise, $\boldsymbol{x_{target}}$ is kept.

According to Storn and Price (2009) it is recommended that the population has a size of 10 times the number of design variables, the crossover probability, *CR*, usually is chosen in the range *[0,1]* and the weight factor *F* is usually chosen in the interval *[0,2]*. In this paper, the *CR* was set as *0.8* and *F* was set as *0.9*.

3.3. Combination of a Gradient Based Local Optimizer (LM) and a Stochastic Global Optimizer (DE)

Due to the complexity of design space, if convergence is achieved with a gradient based method it may in fact lead to local minima. Therefore, global optimization methods are required in order to reach the global minimum. The main disadvantage of these methods is that the number of function evaluations is high, becoming sometimes prohibitive from the computational point of view.

Trying to keep the best features of each method, LM and DE methods were combined (Silva Neto and Soeiro, 2000, 2001). The stochastic method is allowed to run for a while, say for 10 or 20 iterations, obtaining an initial guess for the LM. If this initial guess is in the region of convergence to the global optimum the LM will reach this point in a small number of iterations. This procedure can be repeated. If the same solution is obtained, then this is probably the global optimum and the correct solution of the damage assessment problem.

4. DISCUSSION OF RESULTS

The procedure for damage assessment described in preceding sections was implemented using the MATLAB Optimization Toolbox (MATLAB, 2008). The function *lsqnonlin* which uses a local, gradient-based optimization method developed for nonlinear least squares problems based on the LM method was employed as the main subroutine in the problem solution. The Differential Evolution method used in the hybrid methodology was also developed in MATLAB language. The method was applied to a simple truss structure with 15 bars (Fig. 2). The first four measured and analytically predicted eigenmodes were used to detect the extent and location of damage. The experimental data were synthetic obtained from the finite element model (ANSYS, 2005) with the introduction of damage in some members.

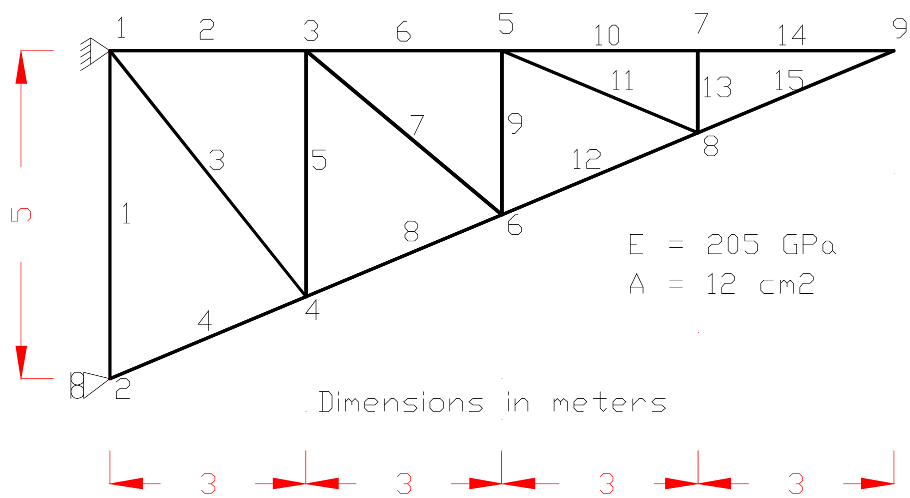

Figure 2. Fifteen bar trus

Prior to solve the damage assessment problem a sensitivity analysis was performed to verify which members would have a possible damage easier or more difficult to detect. The sensitivity analysis is shown in Fig. 3. One can see that member 3 has the highest sensitivity coefficient which indicates that damage in this member would be more easily detected. On the other hand, damage in member 13 will be the most difficult to be detected since the corresponding sensitivity coefficient is very small. The influence of the sensitivity coefficient can be seen in Figs. 4 and 5. First a damage of 50% was introduced in member 3. Figure 4 shows that the location and extent of damage was correctly detected. The initial point used by the optimizer was the undamaged structure where all the damage variables are made equal to zero. In a few iterations the correct solution was obtained. If damage is introduced in member 13, with very small sensitivity coefficient, one can see that the optimizer converges to the wrong solution. The design space becomes very nonconvex and probably presents several local optima. The results are shown in Fig. 5. The extent of damage in member 13 was not correctly detected and another member also presents some damage.

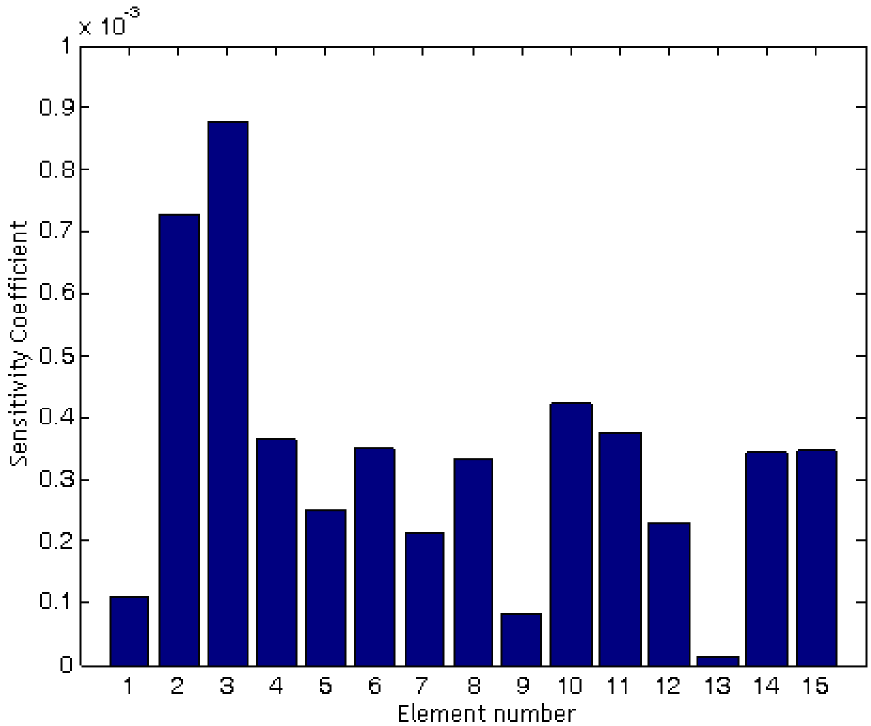

Figure 3. Sensitivity Analysis

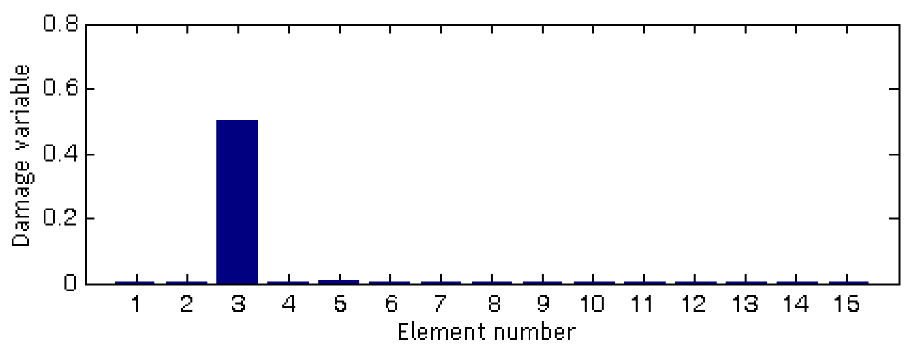

Figure 4. Damage of 50% in member 3

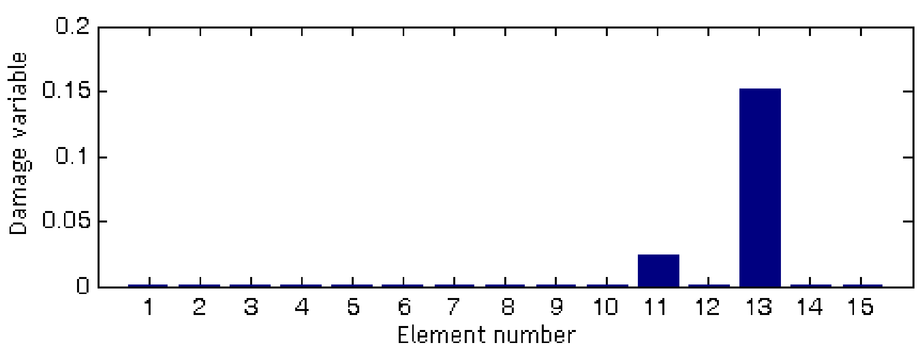

Figure 5. Damage of 50% in member 13

To circumvent this problem a hybrid method can be used. It combines the good features of a local and a global method. The global method converges to the global optimum with a large computational cost whereas the local method is faster e more accurate than the global one. The global method is left running only for a few iterations which usually is enough to bring the solution to a point in the region of convergence to the global optimum. Then, starting from that point, the local method is used to get a more accurate solution. Figure 6 shows the final solution for the same problem of 50% damage in member 13. The DE was set to run for 30 iterations obtaining a solution that was used as the starting point for the LM (function *lsqnonlin* of MATLAB). The right solution was obtained as can be seen in Fig. 6.

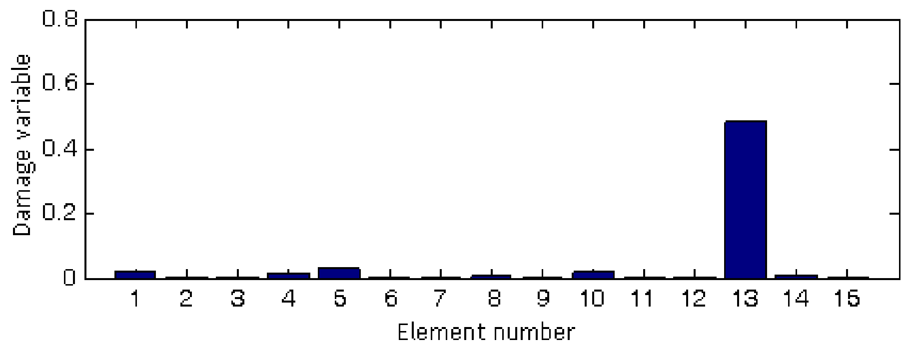

Figure 6. Damage of 50% in member 13 using a hybrid method (DE-LM)

Another representative test was the detection of damage in two members. Again the two members used previously were used with a small damage of 10% in each one. Again only damage in the member with higher sensitivity coefficient was correctly detected as shown in Fig. 7. The hybrid method (DE-LM) was able to detect damage in both members correctly. The results are shown in Fig. 8.

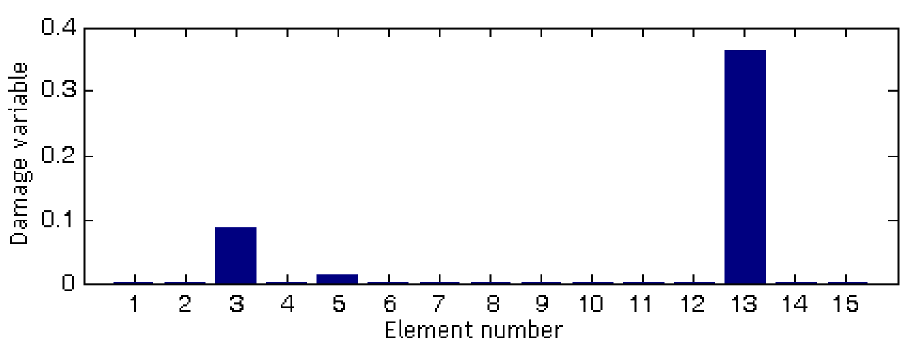

Figure 7. Damage of 10% in members 3 and 13

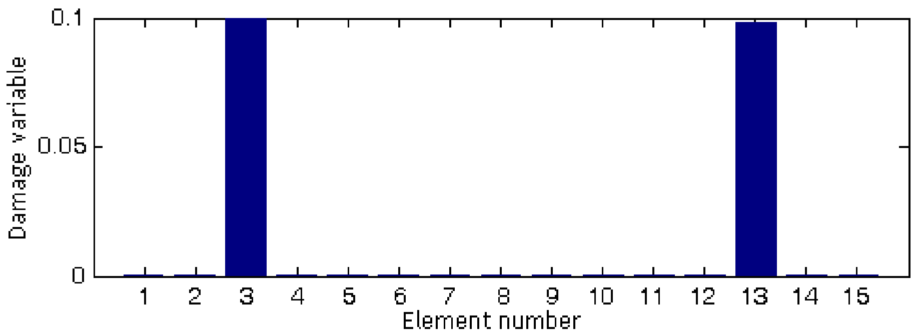

Figure 8. Damage of 10% in members 3 and 13 using a hybrid method (DE-LM)

5. CONCLUSIONS

This paper presents an approach to damage detection in structures based in iterative methods of optimization. Eigenmodes are used in the inverse problem as experimental data. Gradient based methods are faster and more accurate. Before using these methods is recommended that a sensitivity analysis is performed. The aim is to verify if some of the design variables have low sensitivity. In this case a hybrid method can be a good alternative. The exploration of the study of the sensitivity analysis in the solution of the damage assessment problem and the application of this approach to larger structures is the continuation of this research.

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