ANALYZING THE DYNAMICS AND ATTITUDE CONTROL OF FLEXIBLE SOLAR SAILS

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Abstract. *Space exploration has increasingly attracted the interest of researches from several countries. What certainly can be said is that a major objective in common is to lower the cost of the missions and provide more autonomy to the spacecrafts. Solar Sails are a promising low-cost option for space exploration for it uses for propulsion an abundant resource in space: solar radiation. In such a system radiation pressure from sunlight is reflected by the structure so as to propel the spacecraft. Whereupon, a solar sail must have a very large surface area and also be very light. When it concerns large surface areas and low weight it is unavoidable to think of flexible structures, or, in the present case, a flexible Flat Solar Sail (FSS). Any spacecraft needs an attitude control system (ACS) with good performance, autonomy and robustness for navigation. FSSs are not an exception, as their control is performed by tilting the surface in respect to the Sun direction. The objective of this paper is to present an approach for analyzing the dynamics and attitude control of flexible solar sails. The simplified model for the FSS consists of a central spherical hub with four flexible booms with tip masses attached to each one of them. The modeling was carried out by following the Hamiltonian approach and the investigated control methods were Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG), for these control methods have their particular characteristics. This investigation aims to highlight the advantages of each method from computer simulation to control algorithm implementation point of view. All over the controller design one takes into account the FSSs flexibility influence on its performance and robustness.*

Keywords: *flexible solar sail, flexibility influence, attitude control system.*

1. INTRODUCTION

In the space exploration, the idea of using a solar sail as a primary propulsion device is becoming very popular (Chiaramonti et all., 2009). Sailing vessels on Earth navigate with wind sails, which divert a small portion of the massive momentum flux present in moving bodies of air. In space, a vehicle called a solar sail or light sail can achieve the same by diverting a small portion of the massive flux of electromagnetic energy put out by the sun as light using large and lightweight mirrored sails (Herbeck et all., 2002).

Sunlight has long been known to carry momentum. To be exact, the momentum of the electromagnetic energy from the Sun pushes the spacecraft and from Newton´s second law momentum is transferred when the energy strikes and when it is reflected (Macdonald and McInnes, 2010). Sails can be made highly reflective 85-90%, so that the net force vector has a direction that is close to the vector normal to the sail surface. Therefore, the force acting on the sail is a function of the orientation of the sail normal vector with respect to the incident sunlight. Solar sail trajectories are then determined by the time history of the sail normal vector (Diedrich, 2001).

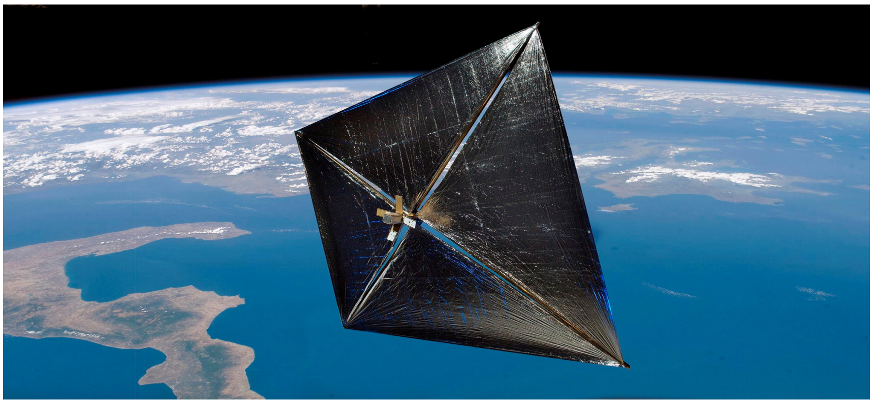

Figure 1. Solar Sail

For many years, solar sailing has been studied as a novel propulsion system for space missions. The basic idea behind solar sailing is very simple so there are difficult engineering problems to solve. From the time when the NASA Comet Halley mission studies a large number of solar sail mission concepts have been devised and promoted by solar sail proponents. As such, this range of mission applications and concepts enables technology requirements derivation and a technology application pull roadmap to be developed based on the key features of missions which are enabled, or significantly enhance, through solar sail propulsion (Macdonald and McInnes, 2010).

This technology appears as a promising form of advanced spacecraft propulsion which can enable exciting new space-science mission concepts such as solar system exploration and deep space observation (Simo and McInnes, 2010). Solar sails have the capability to provide cost effective, propellantless propulsion that enables longer mission lifetime, increased scientific payload mass fraction, and access to previously inaccessible orbits (Wie, 2007). To access inaccessible orbits and permit longer missions lifetime with low cost, the solar sails in the last days have shown great potential for both application and research. So with the recent progress in lightweight and ultralightweight instruments the solar-sail missions will become a reality.

Japanese researchers (Japan Aerospace Exploration Agency - JAXA) have sailed through space. The project name for the first mission is IKAROS (Interplanetary Kite-craft Accelerated by Radiation of the Sun) showed in Fig.2. This craft was launched on May 21, 2010 together with the Venus Climate Orbiter, AKATSUKI. This is the world's first solar powered sail craft employing both photon propulsion and thin film solar power generation during its interplanetary cruise (Ikaros, 2010).

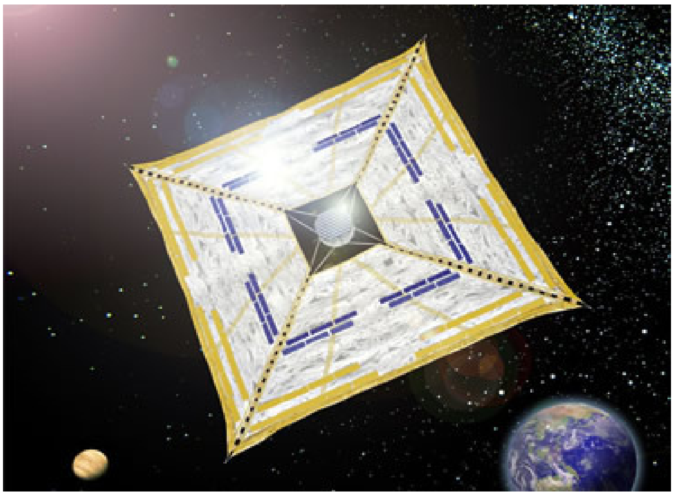

Figure 2. Ikaros Solar Sail

The spacecraft attitude control system (ACS) performance for a solar sailing mission is strongly attached with propulsion efficiency and trajectory goals, so the ACS designer has the paired tasks of control over orientation of the sailcraft and the thrust vector. All practical spacecraft control designs are subject to the physical limits of actuators, sensors, spacecraft structural rigidity, and other mission constraints. A gimbaled control boom and/or sail control vanes (thrusters, reaction wheels, and magnetic torques) are to be employed as primary actuators for active three-axis attitude control of solar sail spacecraft, there exist a variety of practical issues to be resolved (Wie, 2004).

To control attitude, the spacecraft must have the ability to: determine the current attitude, verify the error between the current and desired attitudes, and apply proper torques to remove the error (Murphy and Wie, 2004). A flexible sailcraft demands an ACS with good performance, autonomy and robustness. The ACS must take into account flexibility effects, or a control designed based on linear dynamic model can lead to instability or, at least, poor performance. The importance of structural flexibility in the attitude stability and control of spacecraft has been acknowledged since the earliest artificial earth satellites (Hughes, 1974).

Junkins and Kim (1993) investigated the reason why the flexibility in vibration control should be considered in controller design. In the work of the Murphy and Wie (2004), several ACS approaches were analyzed in an effort to determine design drivers and compare and contrast performance differences to find an ACS method that is truly robust, and that is also low mass, low cost, low power, and low risk. Wie and Liu (1993) investigated the solar array vibration on telescope pointing jitter using classical and $H\infty$ control design.

The Linear Quadratic Regulator (LQR) controller has been studied in (Diedrich, 2001) where was utilized for the angular position of the sail with respect to the incident sunlight. In this work concluded what LQR control is able to provide effective control for the sail even when it is commanded to rotate to an angular position far from the equilibrium point about which the LQR controller was found.

In most cases, the finite element method is used in modeling flexible space structures. But for that is accurate follows: (i) It has high order system matrix, (ii) not easy to get the physical insights due to design changes, and (iii) can be constructed only after all structural features have been decided (Chae and Park, 2003).

The Timoshenko beam theory along with the assumed modes method were used in (Karkoub et all., 1999) to derive reference equations of motion for the flexible manipulator. Eltimsahy and Chen (1992) developed a dynamic model of two coordinating robots having flexible links using Hamilton's Approach, in this paper showed suitable for control and optimization studies.

In this paper a model for a flexible Flat Solar Sail (FSS) is developed using Hamilton´s Extended Principle (HEP) and two different methods for its control are studied, Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG).

2. SYSTEM MODEL

In this paper a square FSS is considered. As an approximation, the model for the sail consists of a central spherical hub with four flexible booms with tip masses attached to each one of them. The scheme of the FSS is shown in Fig.3. HEP was used to develop the model for such a structure.

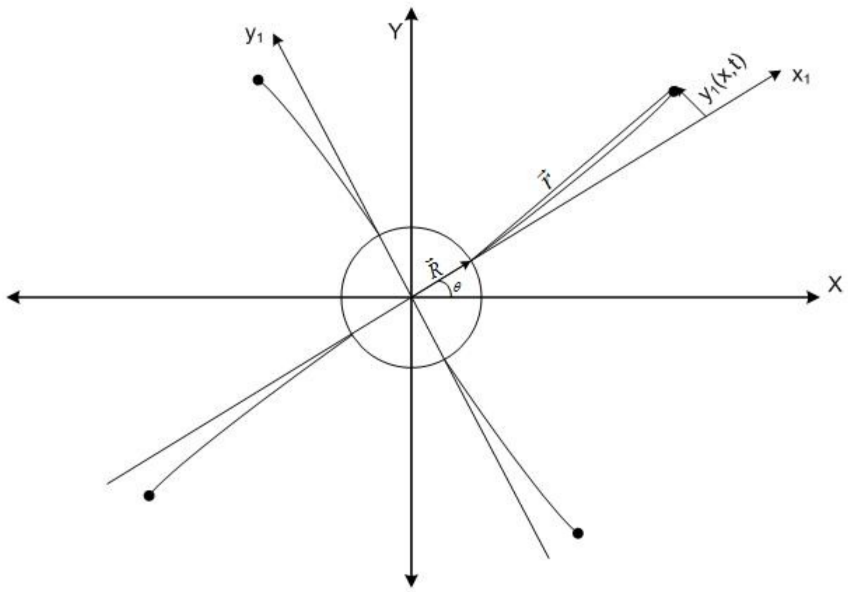

Figure 3. Square Flat Solar Sail scheme

To model the flexible solar sail, it was considered the following terms: a) Gravitational potential energy is negligible; b) The center of mass of the FSS is fixed; - control is performed by applying five independent torques on the sail: a central one acting at the hub and each of the other four at the extremes of the booms; c) The booms have the same length and linear density of mass; d) The tip masses are equal; e) The booms cannot be extended in symmetry axes; f) Transversal displacement (deformation) is small; g) The beams suffer no shear deformation or warping; h) Control is performed by applying five independent torques on the sail.

2.1. Hamilton's Extended Principle

The law governing the motion of mechanical system is the extended Hamilton's principle which can be written as follows:

$$\delta\int_{t_1}^{t_2} L\,dt = \delta\int_{t_1}^{t_2} (T - V)dt = 0 \tag{1}$$

Hamilton's principle has two primary advantages. First, it is extremely succinct. This means, for example, that, once the exact Hamiltonian has been replaced by a discrete numerical analogue, then the numerical analyst is relieved of all further opportunities to exercise his bias: the principle of least action dictates the evolution equations for the discrete dependent variables. Secondly, there exists a well-known connection between the symmetry properties of the Hamiltonian and the conservation laws of the dynamical system (Salmon, 1983).

2.2. Assumed Modes Method

The flexible space structures are most meticulously modeled as distributed parameter systems; their motion is described by a coupled system of ordinary and partial differential equations. Since coupled partial differential equation models are difficult to deal with both analytically and computationally, approximate finite-dimensional equations of motion are usually used. In the assumed method, the deflection of continuous elastic structures is modeled by a finite series of space-dependent functions that are multiplied by specified time-dependent amplitude functions (Junkins and Kim, 1993).

To approximate the partial differential equation by ordinary differential equations, a solution of the flexible motion is assumed to be a linear combination of admissible functions multiplied by time dependent generalized coordinates. The approximation is then described by

$$y_i(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t) \tag{2}$$

where $\phi_i(x)$ denotes ith assumed mode shape, $q_i(t)$ denotes the ith generalized coordinate, and N denotes the number of terms in the approximation. For clamped-free appendages, an admissible base function is given by Junkins and Kim (1993):

$$\phi_k(x) = 1 - \cos\left(\frac{k\pi x}{L}\right) + \frac{1}{2}(-1)^{k+1}\left(\frac{k\pi x}{L}\right)^2 \tag{3}$$

This base function satisfies both geometric and physical boundary conditions for such booms clamped to the hub in one end, with the other end free.

2.3. Mathematical Equations

The kinetic energy T of the system is given by:

$$T = T_{hub} + T_{\text{beams}} + T_{tipmasses} \tag{4}$$

$$T_{hub} == \frac{1}{2} J_{hub} \dot{\theta}^2 \tag{5}$$

$$T_{beams} = \frac{1}{2} \rho_i \sum_{i=1}^{4} \int_0^L (y_i^2 \dot{\theta}^2 + \dot{y}_i^2 + 2x_i \dot{y}_i \dot{\theta} + x_i^2 .\dot{\theta}^2) dx_i \tag{6}$$

$$T_{tipmasses} = \frac{1}{2} \sum_{i=1}^{4} \left[m_i (\dot{y}_{iL}^2 \dot{\theta}^2 + \dot{y}_{iL}^2 + 2L\dot{y}_{iL}\dot{\theta} + L^2\dot{\theta}^2) + J_{mi}(\dot{\theta} + \dot{y}'_{iL})^2 \right] \tag{7}$$

where J_{hub} is moment of inertia of the central body, L is the length of each boom, m and J_{mi} are respectively the tip masses and their moments of inertia. The elastic potential energy of the system is:

$$2V = \sum_{i=1}^{4} EI (y_i'')^2 dx_i \tag{8}$$

EI is the flexural rigidity of the booms. The Lagrangean $\mathcal{L}$ for the system is given by

$$2\mathcal{L} = \left[J_T + \sum_{i=1}^{4} \left(\int_0^L \rho_i y_i^2 dx_i + m_i y_{iL}^2 \right) \right] \dot{\theta}^2 + \left[\sum_{i=1}^{4} \left(2\int_0^L \rho_i x_i \dot{y}_i \, dx_i + 2m_i L \dot{y}_{iL} + 2\dot{y}'_{iL} \right) \right] \dot{\theta} + \sum_{i=1}^{4} \left(\int_0^L \rho_i \dot{y}_i^2 dx_i + m_i \dot{y}_{iL}^2 + J_{mi} \dot{y}'^2_{iL} - EI_i (y_i'')^2 dx_i \right) \tag{9}$$

ρ is the linear mass distribution of the booms and y_i are give details in Eq.2. As a result, according to Hamilton's Extended Principle, replacing Eq. (9) in Eq. (1):

$$
\begin{aligned}
\delta\int_{t_1}^{t_2} \mathcal{L}\,\mathrm{dt} = \frac{1}{2}\int_{t_1}^{t_2} \Bigg\{ & 2\left[J_T + \sum_{i=1}^{4} \left(\int_0^L \rho_i y_i^2 dx_i + m_i y_{iL}^2 \right) \right] \dot{\theta}\delta\dot{\theta} + \left[2\sum_{i=1}^{4} \left(\int_0^L \rho_i \dot{y}_i\, \delta y_i dx_i + m_i y_{iL} \right) \right] \dot{\theta}^2 + \\
& \left[2\sum_{i=1}^{4} \left(\int_0^L \rho_i x_i \dot{y}_i\, \delta y_i dx_i + m_i L \dot{y}_{iL} + \dot{y}'_{iL} \right) \right] \delta\dot{\theta} + \left[2\sum_{i=1}^{4} \left(\int_0^L \rho_i x_i \dot{y}_i\, \delta\dot{y}_i dx_i + m_i L \delta\dot{y}_{iL} + \delta\dot{y}'_{iL} \right) \right] \dot{\theta} \\
& + \sum_{i=1}^{4} \left(2\int_0^L \rho_i \dot{y}_i\, \delta\dot{y}_i\, dx_i + m_i \dot{y}_{iL} \delta\dot{y}_{iL} + J_{mi} \dot{y}'_{iL} \delta\dot{y}'_{iL} - EI_i \int_0^L (y_i'')\delta y_i''\, dx_i \right) \Bigg\} + dt
\end{aligned} \tag{10}
$$

After several calculations, boundary conditions, integrating the variational terms and simplifying notation. The equation that describes the motion of the system for one mode of vibration is:

$$
\ddot{q}_k + \ddot{\theta}\left(\int_0^L x\phi_k dx\right) + w_k^2 q_k - \dot{\theta}^2 q_k = 0 \tag{11}
$$

3. CONTROL METHODS

3.1 Linear Quadratic Regulator

Linear Quadratic Regulator (LQR) is a control method in which the control law can be written as a time-varying function of the system states (Kirk, 1970). This method was used to keep both position and velocity of the flexible booms as close as possible to nominal position using reasonable values for control expenditure. Given a linear plant:

$$
\dot{\underline{x}}(t) = A(t)\underline{x}(t) + B(t)\underline{u}(t) \tag{12}
$$

The control law is given by

$$
\underline{u}(t) = -R^{-1}B^T P(t)\underline{x}(t) \tag{13}
$$

where *P(t)* is the solution for the Riccati Equation:

$$
-P(t) = A^T(t)P(t) + P(t)A(t) + Q - P(t)BR^{-1}B^T P(t) \tag{14}
$$

where Q is matrix symmetric, definite and semi-positive and R matrix is symmetric, definite and positive. Q and R are weight matrices for states and control, respectively.

3.2 Linear Quadratic Gaussian

Linear Quadratic Gaussian (LQG) is a control method for plants considering uncertainties at the model (random disturbances) and using noisy sensors (Kwakernaak and Sivan, 1972). Let the plant be the following stochastic system:

$$
\begin{aligned}
\dot{\underline{x}}(t) &= A(t)\underline{x}(t) + B(t)\underline{u}(t) + G(t)w(t) \\
\underline{y}(t) &= C(t)\underline{x}(t) + v(t)
\end{aligned} \tag{15}
$$

where $\underline{y}(t)$ representing the measured outuputs and *w(t)* and *v(t)* being Gaussian noises with null mean and covariances given by

$$
\begin{aligned}
E\{w(t)w^T(t)\} &= W(t) \geq 0 \\
E\{v(t)v^T(t)\} &= V(t) \geq 0 \\
E\{w(t)v^T(t)\} &= 0
\end{aligned} \tag{16}
$$

As suggested in Kwakernaak and Sivan (1972) let the control law is found by treating the problem in two different subproblems, as if there was a Kalman Filter coupled to a LQR controller. The first sub-problem is the Kalman Filter, given by a state estimator of the form

$$
\dot{\hat{\underline{x}}} = (A(t) - K_f(t)C(t))\underline{x}(t) + B(t)\underline{u}(t) + K_f(t)\underline{y}(t) \tag{17}
$$

The control law $\underline{u}(t) = -K(t)\hat{\underline{x}}(t)$ is based on the estimate state vector $\hat{\underline{x}}$ and the Kalman filter gain is given by

$$K_f(t) = P_k(t) + C(t)^T V(t)^{-1} \tag{18}$$

where PK satisfies the algebraic Riccati equation

$$0 = A(t)P_k(t) + P_k(t)A(t)^T + G(t)W(t)G(t) - P_k(t)C(t)^T V(t)^{-1} C(t)P_k(t) \tag{19}$$

Once the estimate states are obtained, the problem falls on getting an optimal control law based on the LQR method.

4. NUMERICAL SIMULATIONS

In this paper were considered 5 controls, providing torque to the hub for central and the forces apply in the beams:

$$\vec{\tau} = J_T \ddot{\theta} = u_1 \tag{20}$$
$$\vec{F} = m\ddot{\vec{R}} = u_1 \tag{21}$$

According to Wie (2004), typical values for sail structures are boom length L=30m; hub radius r_H=0.3m, hub mass m_H=19.6 Kg, boom linear density ρ=70g/m; flexural rigidity EI=2000Nm^2; and tip masses were considered to be m=0.8Kg. Initial states for both controllers were chosen randomly and position of the four booms from their nominal positions are, respectively, 2.0m, 1.2m, -1.0m, 1.0m and their transversal velocities -1.0m/s, 0.0m/s, -5m/s, 3m/s. In this work, these values are the perturbations of the system. And the simulations the performance of control law are analyzed.

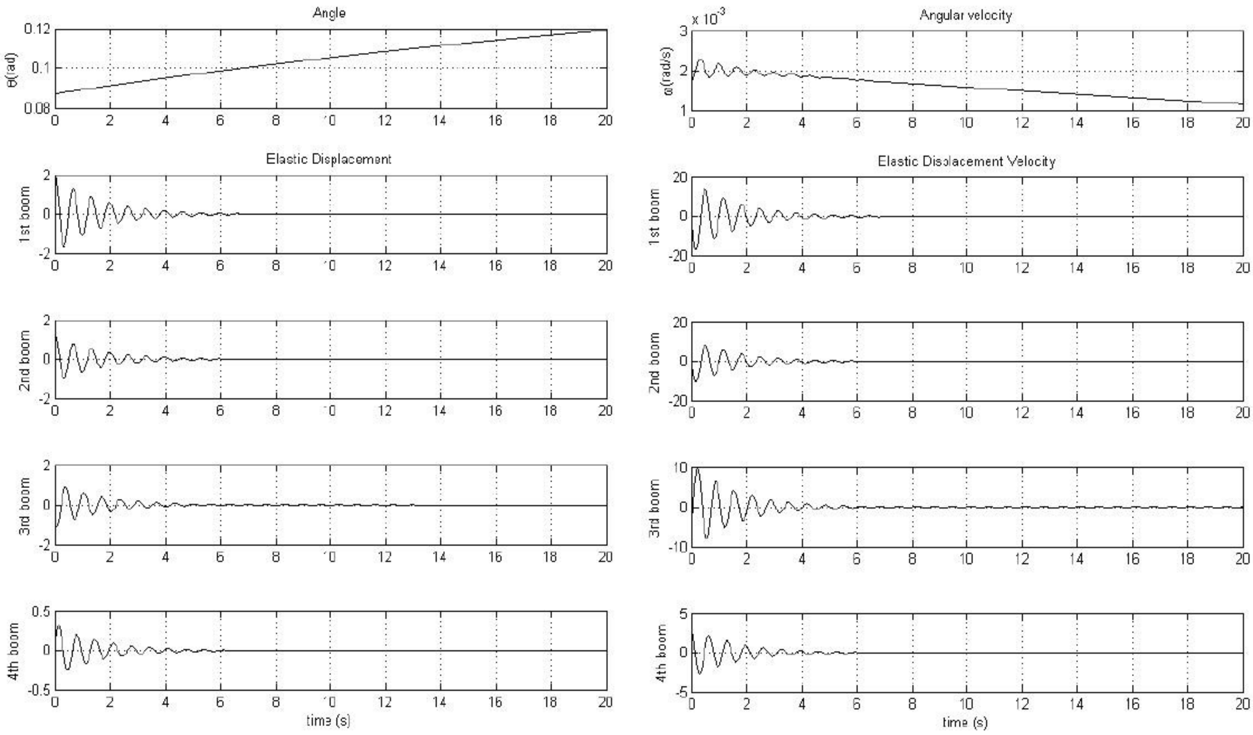

Figure 4. Performance of FSS with LQR Control

Analyzing the results in Fig. 4, we can see the time to stability for the booms is around 6 seconds. Good performance to the first mode of vibration and control using the LQR. For other hand, the central hub has other conduct, more slowly than the booms. In this paper not include the other modes of vibration of the system, which can be excited when the control and applied, including leading the system to instability.

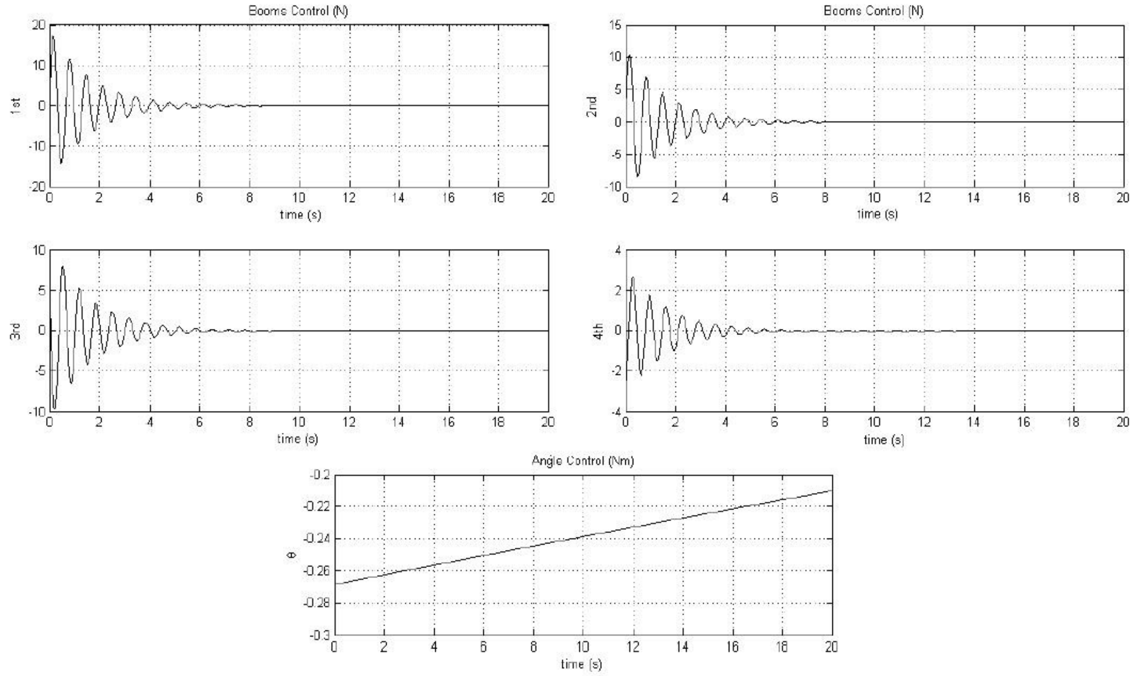

Figure 5. Signals Control - LQR

In Fig. 5 shows the control rods control a high demand, and varying a large part in a short space of time. After 8 seconds the control practically stopped to act once the rods are already stabilized.

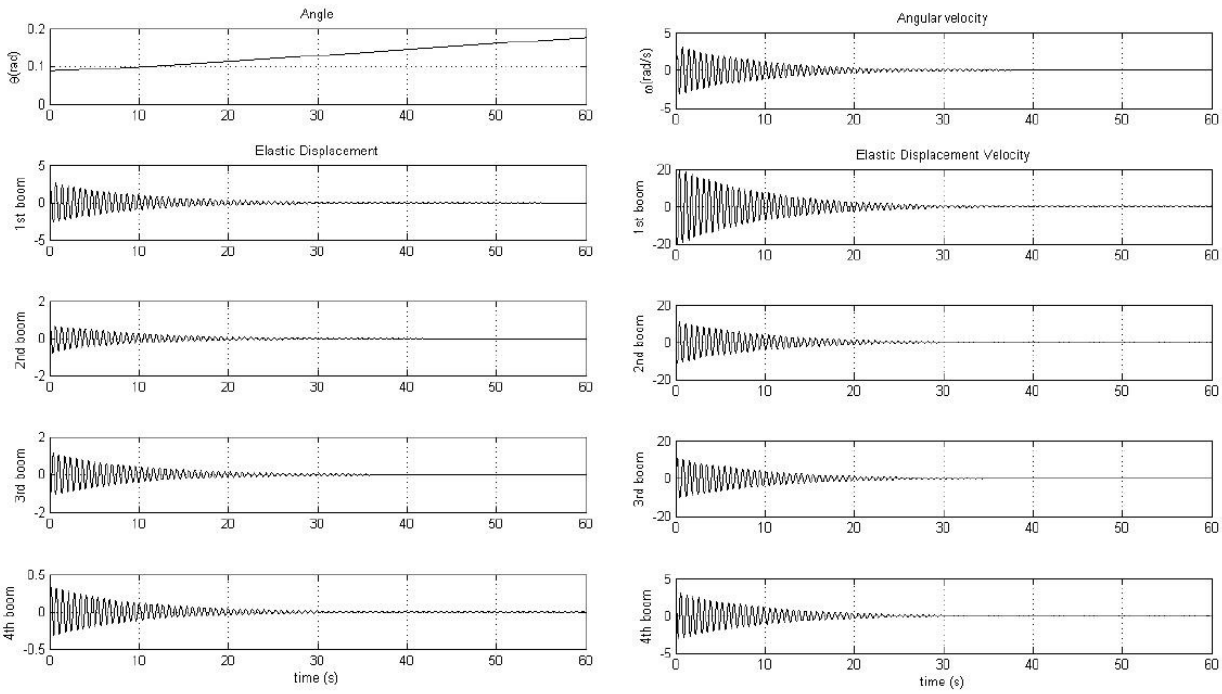

Figure 6. Performance of FSS with LQG Control

The results in Fig.6 show that compared to the LQR, Fig. 4, the stabilization time of the booms are also controlled, however a longer time. The system still oscillates until it stabilizes. The results for central hub show that tends to stabilize, but slower compared to the results of the LQR, displaying a degree of oscillation in angular velocity.

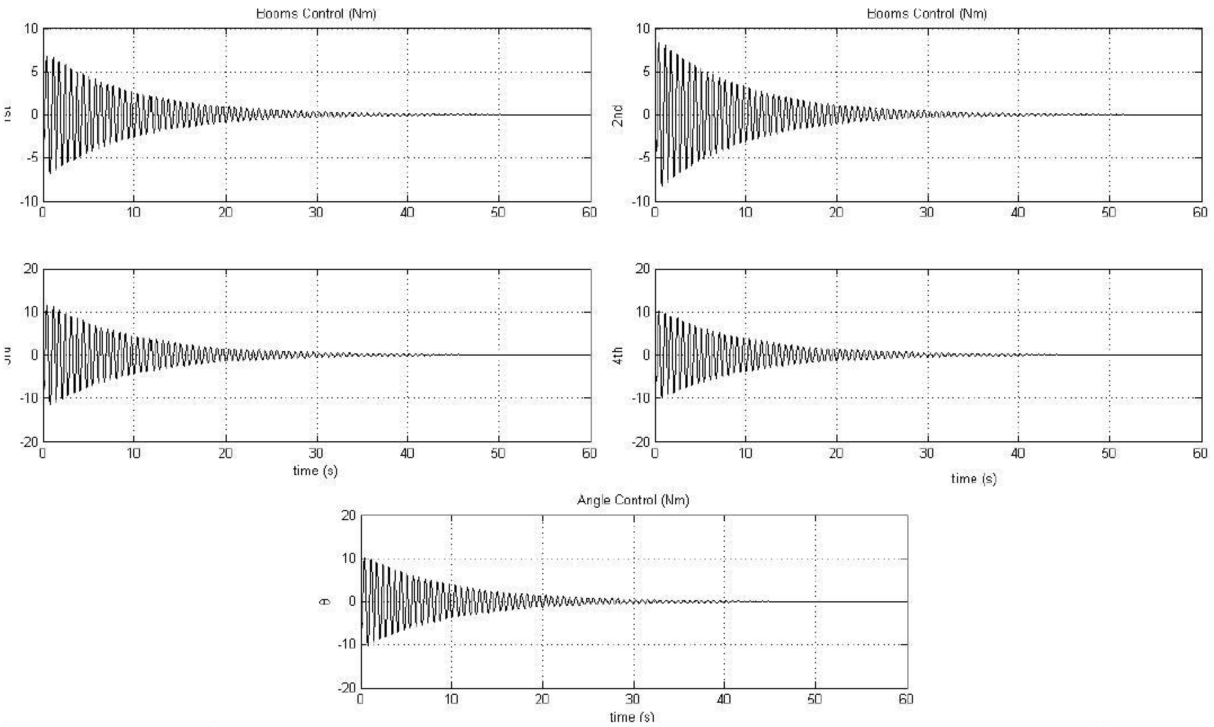

Figure 7. Signals Control - LQG

The controllers shown in Fig.7 we can see that the maximum values of the controls are not very different from the values obtained for the LQR, Fig.5, but the control work long time, besides showing greater variation.

5. CONCLUSIONS

In this paper were investigated the methods of control, LQR and LQG, in ACS performance of the FSS. This investigation use HEP and Assumed Modes Method for describes flexible structures. There preliminary methods could control the sail when considering one vibration mode. Next studies should take into account more vibration modes, and other robust control techniques as H-Infinity and μ-Synthesis should be implemented. The study of solar sailing offers striking advantages for Solar System exploration; it is a subject that covers various areas of space engineering. So, this paper is the beginning for the knowledge and application in this area.

6. ACKNOWLEDGMENTS

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