MODELING AND SIMULATION OF GAS, OIL AND WATER FLOW IN A PIPELINE-RISER SYSTEM

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Abstract. *A mathematical model and dynamic simulations of gas, oil and water flow in a pipeline-riser system are presented. The model considers continuity equation for gas, oil and water phases and a simplified momentum equation without inertia terms for the mixture. Oil and water phases are considered to have the same velocity and are homogenized. A drift-flux model, evaluated for the local conditions in the riser, is used as a closure law. Mass transfer between the oil and gas phases is calculated using the black oil approximation. A choke valve is located at the top of the riser, being possible to control the pressure at this location. The properties of fluids are calculated by analytical correlations based on experimental results and field data. As result of the dynamic simulations, pressure, volume fractions and superficial velocities for the phases along the tubes are estimated for different inflow boundary conditions.*

Keywords: *Multiphase flow, petroleum production systems, no-pressure-wave model, black oil approximation, choke valve.*

1. INTRODUCTION

In petroleum production systems, depending on the piping topography and flow rates, it is possible the occurrence of flow instabilities. These instabilities lead to production losses and some phenomena, as the severe slugging, can even cause a shutdown, stopping the production for long periods of time (Wordsworth *et al.*, 1998).

The modeling and simulation of production systems is an important way of predicting the occurrence and verifying mitigation alternatives for such instabilities. Many models were proposed in order to investigate the system stability. Taitel *et al.* (1990), Sarica and Shoham (1991) and Baliño *et al.* (2010) are some of the authors that studied the severe slugging characteristics, using air and water as flowing fluids.

Although basic mechanisms of severe slugging can be investigated using air-water systems, there are many limitations when trying to extrapolate these results to petroleum production systems:

- Pipeline lengths and riser heights in petroleum production systems are much bigger (order of kilometers long) than the values for air-water experimental facilities. The high pressure ratios between the bottom and top of the riser give rise to important expansion effects in the gas phase.
- Petroleum is a multicomponent system in which both liquid and gas phases coexist at operating conditions (McCain, 1990). Mass transfer between the phases are dependent on pressure and temperature through the PVT curve. With the high pressure variations in the riser, mass transfer effects cannot be ignored. Besides, the fluid coming from the reservoir has a water content, so three phases can coexist in the general case.
- Most of the experiments in air-water systems were realized keeping a constant separation pressure as a boundary condition. A few experiments investigated the effect of a choking valve at the top of the riser. Because of the low pressures involved, the valve operated in subcritical conditions. In petroleum production systems, a choke valve located at the top of the riser normally operates in critical conditions.

In this paper a one-dimensional model for gas, oil and water flow in a pipeline-riser system is presented. This model is based on the previous work of Baliño *et al.* (2010), incorporating aspects that are important in petroleum production systems, such as the evolution of the gas as a real gas, mass transfer between the oil and gas phases and a choke valve at the top of the riser that can operate in the critical or subcritical state.

2. SYSTEM MODEL

The system model is composed of two subsystems, namely the pipeline and the riser. Assuming the same flow passage area for the pipeline and riser, the pressure and superficial velocities are continuous at the bottom of the riser, so that it is possible the coupling between the subsystem models.

2.1 Riser model

The model is based on one-dimensional three-phase isothermal flow. It considers continuity equation for gas, oil and water phases and a simplified momentum equation without inertia terms for the phases flowing together. Oil and water

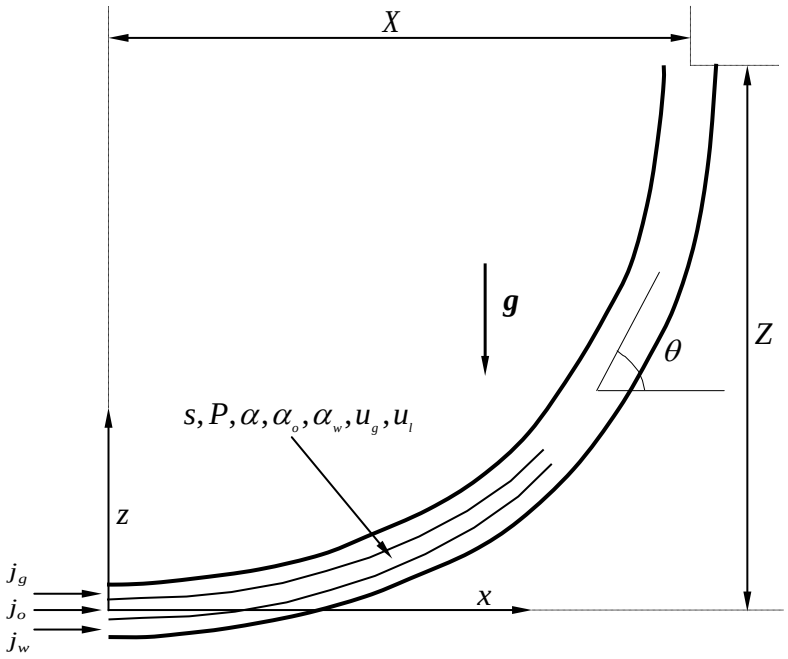


Figure 1. Variables at the riser.

phases are considered to have the same velocity and are homogenized. Slip between the liquid and gas phases are taken into account by using a drift flux model. Mass transfer between the oil and gas phases are calculated using the black oil model. The liquid and gas phases are assumed to be compressible and the gas behaves as a real gas. Solubility of gas and vaporization are neglected for water.

2.1.1 Riser geometry

The catenary geometry is characterized by the coordinates X and Z, corresponding to the abscissa and the height of the top of the riser (see Fig. 1). It is assumed that the inclination angle at the bottom is zero.

The local height z of a point belonging to catenary can be written as:

$$z = \varphi \left[\cosh\left(\frac{x}{\varphi}\right) - 1\right] \tag{1}$$

where the dimensional catenary constant φ is obtained from the solution of the following transcendental equation:

$$Z = \varphi \left[\cosh\left(\frac{X}{\varphi}\right) - 1\right] \tag{2}$$

The local position s along the catenary results:

$$s = \varphi \sinh\left(\frac{x}{\varphi}\right) \tag{3}$$

The local inclination angle θ can be written as:

$$\theta = \arctan\left[\sinh\left(\frac{x}{\varphi}\right)\right] \tag{4}$$

Knowing the position s, the local abscissa x can be calculated from Eq. (3):

$$x = \varphi \operatorname{arcsinh}\left(\frac{s}{\varphi}\right) \tag{5}$$

2.1.2 Conservation equations

Considering continuity equations for the phases oil, gas and water, we get:

$$\frac{\partial}{\partial t}\left(\rho_g\, \alpha\right) + \frac{\partial j_g}{\partial s} = \Gamma \tag{6}$$

$$\frac{\partial}{\partial t}\left(\rho_o\,\alpha_o\right)+\frac{\partial j_o}{\partial s}=-\Gamma \tag{7}$$

$$\frac{\partial}{\partial t}\left(\rho_w\,\alpha_w\right)+\frac{\partial j_w}{\partial s}=0 \tag{8}$$

where s is the coordinate along the flow direction, t is time, ρ_g, ρ_o and ρ_w are the densities of the phases (correspondingly gas, oil and water), j_g, j_o and j_w are the superficial velocities, α, α_o and α_w are the volume fractions and Γ is the vaporization source term.

In most of the transients occurred in oil and gas transport, for instance in severe slugging, the response of the system proves to be relatively slow, showing that pressure waves do not have a strong effect on the initiation and transport of void waves. In the no-pressure-wave (NPW) model (Masella *et al.*, 1998), acoustic waves are ruled out by neglecting inertia terms from the momentum equation, resulting an algebraic relation for the pressure gradient:

$$\frac{\partial P}{\partial s}=-\frac{4\,\tau_w}{D}+\rho_m\,g_s \tag{9}$$

$$\rho_m=\rho_g\,\alpha+\rho_o\,\alpha_o+\rho_w\,\alpha_w \tag{10}$$

where P is pressure, ρ_m is the density of the mixture, D is the pipe diameter, g_s is the gravity component in the s-direction and τ_w is the mean shear stress at the pipe wall. The volume fractions are related by:

$$\alpha_o+\alpha_w+\alpha=1 \tag{11}$$

2.1.3 Closure laws

In order to close mathematically the problem, some simplifications must be made.

Homogenization of liquid phases Assuming equal velocities for oil and water, we obtain:

$$j_o=j_l\,\frac{\alpha_o}{1-\alpha} \tag{12}$$

$$j_w=j_l\,\frac{\alpha_w}{1-\alpha} \tag{13}$$

$$j_l=j_o+j_w=u_l\,(1-\alpha) \tag{14}$$

where j_l and u_l are correspondingly the superficial velocity and the velocity of the liquid (oil plus water) phase.

Shear stress at the wall The shear stress at the wall is estimated using a homogeneous two-phase model and the correlation from Chen (1979) for the Fanning friction factor f, resulting the following relations:

$$\tau_w=\frac{1}{2}\,f_m\,\rho_m\,j\;|j| \tag{15}$$

$$f_m=f\left(Re_m,\,\frac{\epsilon}{D}\right) \tag{16}$$

$$f\left(Re,\,\frac{\epsilon}{D}\right)=\left\langle -4\log_{10}\left\{\frac{1}{3,7065}\frac{\epsilon}{D}\;\;-\frac{5,0452}{Re}\log_{10}\left[\frac{1}{2,8257}\left(\frac{\epsilon}{D}\right)^{1,1098}+\frac{5,8506}{Re^{0,8981}}\right]\right\}\right\rangle^{-2} \tag{17}$$

$$Re_m=\frac{\rho_m\,D\;|j|}{\mu_m} \tag{18}$$

$$\mu_m=\mu_o\,\alpha_o+\mu_w\,\alpha_w+\mu_g\,\alpha \tag{19}$$

$$j=j_o+j_w+j_g \tag{20}$$

where Re_m and μ_m are correspondingly the Reynolds number and dynamic viscosity of the mixture, μ_o, μ_w and μ_g are the viscosities of the phases, ϵ is the pipe roughness and j is the total superficial velocity.

Real gas As the pressures involved are high, the constitutive relation for the gas phase is considered as:

$$\rho_g = \frac{\gamma_g \, M_a}{\Lambda \, T} \, \frac{P}{Z} \tag{21}$$

where $\gamma_g = \dfrac{M_g}{M_a}$ is the gas specific gravity, M_g and $M_a = 28.966$ are respectively the molar masses of gas and air, Z is the gas compressibility factor (dependent on pressure, temperature and gas composition) and $\Lambda = 8.314 \, m^2 s^{-2} K^{-1}$ is the gas universal constant.

Drift flux model The superficial velocities for the liquid and gas phases are determined by using a drift flux model (Zuber and Findlay, 1965):

$$j_g = \alpha \left(C_d \, j + U_d \right) \tag{22}$$

$$j_l = \left(1 - \alpha \, C_d \right) j - \alpha \, U_d \tag{23}$$

$$j = j_l + j_g \tag{24}$$

where the parameters C_d and U_d depend on the local geometric and flow conditions (Bendiksen, 1984; Chexal *et al.*, 1992). In a general form, it will be assumed that $C_d = C_d \left(\alpha, \, P, \, j, \, \theta \right)$ and $U_d = U_d \left(\alpha, \, P, \, j, \, \theta \right)$, where θ is the local inclination angle of the pipe.

Black oil model The vaporization term can be calculated by using the black oil model (McCain, 1990). According to this model, the gas specific gravity does not change with variations of pressure and temperature:

$$\gamma_g \cong \gamma_{g\,0} \tag{25}$$

$$\gamma_{dg} \cong \gamma_{g\,0} \tag{26}$$

where γ_g is the gas specific gravity at local conditions, $\gamma_{g\,0}$ is the gas specific gravity at standard conditions and γ_{dg} is the dissolved gas specific gravity.

In this way, many properties corresponding to the phases at operating conditions can be estimated based on parameters at standard condition (1 atm and 60 ^oF for API, American Petroleum Institute) and a set of correlations depending on pressure, temperature and composition, which will be considered as locally and instantaneously valid.

The vaporization term can be expressed as:

$$\Gamma = -\frac{\rho_{g\,0} \, \alpha_o}{B_o} \left(\frac{\partial R_s}{\partial t} + \frac{j_o}{\alpha_o} \, \frac{\partial R_s}{\partial s} \right) \tag{27}$$

where $\rho_{g\,0}$ is the gas density at standard condition, B_o is the oil formation volume factor and R_s is the solution gas-oil ratio. It is worth noting that for $\Gamma > 0$ must be $\alpha_o > 0$, while for $\Gamma < 0$ must be $\alpha > 0$.

2.1.4 Choke valve

The choke valve model is based on the work of Sachdeva *et al.* (1986), which is valid for both critical and subcritical regime. The valve is located at the top of the riser. By means of the Sachdeva's formulation, it is possible to evaluate the pressure upstream of the valve as follows:

$$P_{up} = f(\dot{m}_{g\,up}, \dot{m}_{o\,up}, \dot{m}_{w\,up}, \alpha_{o\,up}, \alpha_{w\,up}, P_{sep}, D_{riser}, D_{choke}, T) \tag{28}$$

where $\dot{m}_{g\,up}$, $\dot{m}_{o\,up}$, $\dot{m}_{w\,up}$ are the mass flow rates of the gas, oil and water, respectively, P_{sep} is the pressure at the separator, D_{riser} is the riser diameter, D_{choke} is the choke diameter, T is the temperature and the index up indicates that the variable is evaluated upstream of the valve.

2.1.5 Well-posedness and method of characteristics

For a model to describe physical phenomena correctly it must be well-posed, this is, the solution must exist, must be uniquely determined and must depend in a continuous fashion on the initial and boundary conditions (Drew and Passman, 1999). This property is particularly important in multiphase flows, where partial differential equations of hyperbolic nature can be found; in this case, well-posedness implies that the characteristic values (eigenvalues or characteristic wave velocities) must be real.

The characteristic values of the presented system of conservation equations are given by (Nemoto and Baliño, 2009):

$$e_1 = \frac{\partial j_g}{\partial \alpha} \quad e_2 = \frac{j_o}{\alpha_o} = u_l \quad e_3 = \infty \quad e_4 = \infty \tag{29}$$

where u_l is the liquid velocity. If the parameters C_d and U_d are not dependent of α, i.e. $C_d = C_d(P, j, \theta)$ and $U_d = U_d(P, j, \theta)$ (as in the correlation developed by Bendiksen (1984)) we have:

$$\frac{\partial j_g}{\partial \alpha} = \frac{j_g}{\alpha} = u_g \tag{30}$$

where u_g is the gas velocity.

There exists an algebraically-double eigenvalue equal to ∞, these eigenvalues are related to the pressure wave velocities. The pressure wave is propagated in negative and positive directions with an infinite velocity, meaning that any pressure change is felt by the entire system instantaneously.

The method of characteristics will be applied to solve the system of equations. This method is the natural numerical procedure for hyperbolic systems. By an appropriate choice of coordinates, the original system of hyperbolic partial differential equations can be replaced by a system of ordinary differential equations expressed in the characteristic coordinates. Characteristic coordinates are the natural coordinates of the system in the sense that, in terms of these coordinates, differentiation is simpler (Ames, 1992).

The resulting system of equations in the characteristic coordinates, or compatibility conditions, is given by:

$$b^*_{11} \frac{D_g \alpha}{Dt} + b^*_{13} \frac{D_g P}{Dt} + d^*_1 = 0 \tag{31}$$

$$b^*_{21} \frac{D_l \alpha}{Dt} + b^*_{22} \frac{D_l \alpha_o}{Dt} + b^*_{23} \frac{D_l P}{Dt} = 0 \tag{32}$$

where the coefficients b^*_{11}, b^*_{13}, b^*_{21}, b^*_{22}, b^*_{23} and d^*_1 are function of the state variables and dependent variables.

From Eqs. (6), (7) and (8), the equation that follows can be obtained to calculate the evolution of the total superficial velocity:

$$\frac{\partial j}{\partial s} = -\frac{\alpha}{\rho_g} \frac{\partial \rho_g}{\partial P} \frac{D_{ug} P}{Dt} + \frac{D_{ul} P}{Dt} \left[\frac{\rho_{dg\,0}\,\alpha_o}{B_o} \frac{\partial R_s}{\partial P} \left(\frac{1}{\rho_o} - \frac{1}{\rho_g} \right) - \frac{\alpha_o}{\rho_o} \frac{\partial \rho_o}{\partial P} - \frac{\alpha_w}{\rho_w} \frac{\partial \rho_w}{\partial P} \right] \tag{33}$$

The directional derivatives are defined as:

$$\frac{D_g}{Dt} = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial s} \tag{34}$$

$$\frac{D_l}{Dt} = \frac{\partial}{\partial t} + u_l \frac{\partial}{\partial s} \tag{35}$$

2.2 Pipeline model

The pipeline model is based on one-dimensional three-phase isothermal flow. It considers continuity equation for gas, oil and water phases and a momentum balance equation evaluated at the stationary state (Taitel and Dukler, 1976), which is used to determine the void fraction in the pipeline:

$$\tau_{w\,g} \frac{S_g}{\alpha_p} - \tau_{w\,l} \frac{S_l}{1-\alpha_p} + \tau_i S_i \left(\frac{1}{1-\alpha_p} + \frac{1}{\alpha_p} \right) + (\rho_l - \rho_g)\, A\, g\, \sin\beta = 0 \tag{36}$$

where S_g, S_i and S_l are respectively the gas, interfacial and liquid wetted perimeters, τ_{wg}, τ_i and τ_{wl} are respectively the wall-gas, interface and wall-liquid shear stresses and ρ_l and ρ_g are the average density of the liquid and the density of the gas.

It is assumed that water and oil phases have the same velocity and are homogenized, the flow pattern is the smooth stratified, the pressure is constant in the gas cavity, mass transfer between the oil and gas phases is calculated using the black oil model and variations in the void fraction α_p are neglect during the transient.

The pipeline model is composed of two control volumes (see Fig. 2). The control volume $\forall 1$ is fixed at the pipeline inlet and deformable with the liquid accumulation front and the control volume $\forall 2$ is also deformable with the liquid accumulation front, but fixed at the pipeline outlet.

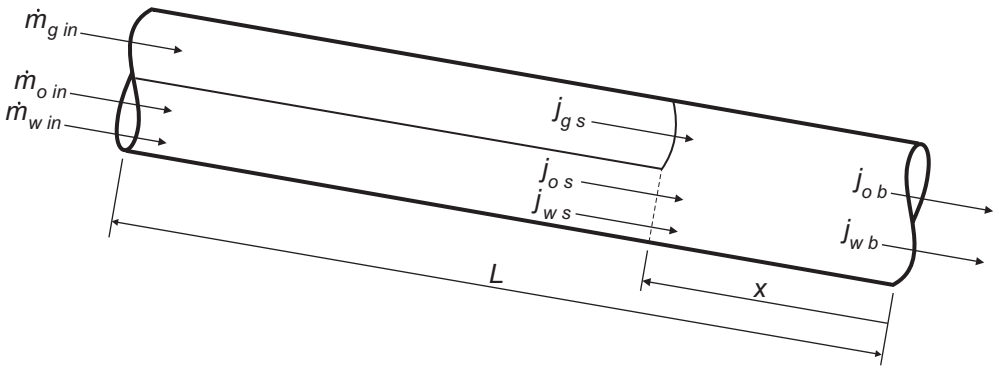


Figure 2. Control volumes in the pipeline model.

2.2.1 Condition $x = 0$

The condition $x = 0$ is observed when there is no liquid penetration in the pipeline. In this case, there exist only one control volume in the pipeline, the control volume $\forall 1$. The state equations are obtained by applying continuity equations for the gas, oil and water phases:

$$j_{g\,b} = \frac{\dot{m}_{g\,in}}{\rho_{g\,b}\,A} - \frac{\partial P_b}{\partial t}\left(\alpha_p\,\frac{\partial \rho_{g\,b}}{\partial P} + \frac{\rho_{dg\,0}\,\alpha_{o\,p}}{B_{o\,b}}\frac{\partial R_{s\,b}}{\partial P}\right)\frac{L}{\rho_{g\,b}} \tag{37}$$

$$j_{o\,b} = \frac{\dot{m}_{o\,in}}{\rho_{o\,b}\,A} - \frac{\partial P_b}{\partial t}\left(\alpha_{o\,p}\,\frac{\partial \rho_{o\,b}}{\partial P} - \frac{\rho_{dg\,0}\,\alpha_{o\,p}}{B_{o\,b}}\,\frac{\partial R_{s\,b}}{\partial P}\right)\frac{L}{\rho_{o\,b}} \tag{38}$$

$$j_{w\,b} = \frac{\dot{m}_{w\,in}}{\rho_{w\,b}\,A} - \frac{\partial P_b}{\partial t}\,\alpha_{w\,p}\,\frac{\partial \rho_{w\,b}}{\partial P}\,\frac{L}{\rho_{w\,b}} \tag{39}$$

where L is the pipeline length, $\dot{m}_{g\,in}$, $\dot{m}_{o\,in}$, $\dot{m}_{w\,in}$ are the mass flow rate of gas, oil and water phases at the pipeline inlet, A is the flow passage area, the index b represents the bottom of the riser and the index p evidences that the variable refers to the pipeline.

2.2.2 Condition $x > 0$

Whenever the gas superficial velocity at the bottom of the riser becomes zero, commutation exists between the set of equations determined by the condition $x = 0$ to the set of equations determined by the condition $x > 0$. In this case, two control volumes are taken into account and the liquid front penetrates the pipeline.

The pressure in the control volume $\forall 1$, P_s, is constant throughout its length and the pressure in the control volume $\forall_2$ depends on position. In order to evaluate fluid properties and its derivatives, a representative pressure P_m is used:

$$P_s = P_b - \rho_l\,g\,x\,\sin\,\beta \tag{40}$$

$$P_m = \frac{P_b + P_s}{2} \tag{41}$$

where ρ_l is the density of the liquid phase, g is the gravity, β is the pipeline inclination, x is the liquid penetration length, the index s refers to the interface between the control volumes and the index m refers to the middle of the control volume $\forall 2$.

Volume control $\forall_1$ From the continuity equations for the gas, oil and water, we obtain:

$$\frac{d\,x}{d\,t} = -\frac{\dot{m}_{g\,in}}{A\,\rho_{g\,s}\,\alpha_p} + \frac{\partial P_s}{\partial t}\left(\alpha_p\,\frac{\partial \rho_{g\,s}}{\partial P} + \frac{\rho_{dg\,0}\,\alpha_{o\,p}}{B_{o\,s}}\,\frac{\partial R_{s\,s}}{\partial P}\right)\frac{(L-x)}{\rho_{g\,s}\,\alpha_p} \tag{42}$$

$$j_{o\,s} = \frac{\dot{m}_{o\,in}}{A\,\rho_{o\,s}} - \frac{\partial P_s}{\partial t}\left(\alpha_{o\,p}\,\frac{\partial \rho_{o\,s}}{\partial P} - \frac{\rho_{dg\,0}\,\alpha_{o\,p}}{B_{o\,s}}\,\frac{\partial R_{s\,s}}{\partial P}\right)\frac{(L-x)}{\rho_{o\,s}} - \alpha_{o\,p}\,\frac{d\,x}{d\,t} \tag{43}$$

$$j_{w\,s} = \frac{\dot{m}_{w\,in}}{A\,\rho_{w\,s}} - \frac{\partial P_s}{\partial t}\,\alpha_{w\,p}\,\frac{\partial \rho_{w\,s}}{\partial P}\,\frac{(L-x)}{\rho_{w\,s}} - \alpha_{w\,p}\,\frac{d\,x}{d\,t} \tag{44}$$

Volume control $\forall_2$ From the continuity equations for the gas, oil and water, we get:

$$j_{o\,b} = \frac{1}{\rho_{o\,b}} \left[\rho_{o\,s}\, j_{o\,s} + \alpha_{o\,p}\, \rho_{o\,s}\, \frac{d\,x}{d\,t} - x\, \alpha_{o\,\forall 2} \frac{\partial P_m}{\partial t}\, \frac{\partial \rho_{o\,m}}{\partial P} \right] \tag{45}$$

$$j_{w\,b} = \frac{1}{\rho_{w\,b}} \left[\rho_{w\,s}\, j_{w\,s} + \alpha_{w\,p}\, \rho_{w\,s}\, \frac{d\,x}{d\,t} - x\, \alpha_{w\,\forall 2}\, \frac{\partial P_m}{\partial t}\, \frac{\partial \rho_{w\,m}}{\partial P} \right] \tag{46}$$

$$u_{l\,b} = \frac{1}{\rho_{l\,b}} \left[\rho_{l\,s} \left(j_{o\,s} + j_{w\,s} + \alpha_{l\,p}\, \frac{d\,x}{d\,t} \right) - x\, \alpha_{l\,\forall 2}\, \frac{\partial \rho_{l\,m}}{\partial P}\, \frac{\partial P_m}{\partial t} \right] \tag{47}$$

where α_l is the liquid holdup and the index $\forall 2$ relates the variable to the control volume $\forall 2$.

3. FLUID PROPERTIES

The properties of fluids are calculated by analytical correlations based on experimental results and field data.

3.1 Gas formation volume factor and gas density

The gas formation volume factor is calculated by the following expression:

$$B_g = \frac{P_0}{T_0}\, \frac{Z\,T}{P} \tag{48}$$

where P_0 is the pressure at standard conditions, T_0 is the absolute temperature at standard conditions, Z is the compressibility factor and P and T are the pressure and the absolute temperature at local conditions.

The compressibility factor is determined using the correlation of Dranchuk and Abu-Kassem (1975), which correlates the results of the chart of Standing and Katz (1942). For the evaluation of the compressibility factor is also necessary to calculate the pseudocritical temperature and pressure, which can be determined using the correlation of Standing (1981), that was based on the charts of Brown *et al.* (1948).

Considering the black oil approximation, which assumes a approximately constant gas specific gravity, it can be shown that Eq. (21) reduces to:

$$\rho_g \cong \frac{\rho_{g\,0}}{B_g} \tag{49}$$

3.2 Water formation volume factor and water density

The correlation for water formation volume factor is presented in the work of McCain (1990). Water density at local condition is determined by:

$$\rho_w = \frac{\rho_{w\,0}}{B_w} \tag{50}$$

3.3 Gas-oil solubility and bubble point pressure

If the local pressure is above the bubble point pressure, the gas-oil solubility is equal to the GOR, otherwise the gas-oil solubility is calculate according to the correlation of Standing (1981), based on the charts of Standing (1947).

The bubble point pressure is determined based on the correlation of Velarde *et al.* (1999).

3.4 Oil formation volume factor and oil density

Based on the definition of oil formation volume factor:

$$B_o = \frac{\upsilon_o}{\upsilon_{o\,0}} \tag{51}$$

where υ_o and $\upsilon_{o\,0}$ is, respectively, the oil volume of a particle at local conditions and at standard conditions, the following material balance relation results:

$$B_o = \frac{\rho_{o\,0} + \dfrac{P_0\, M_a}{\Lambda\, T_0}\, R_s\, \gamma_{dg}}{\rho_o} \tag{52}$$

Assuming that the black oil approximation is valid and substituting Eq. (26) in Eq. (52), we obtain:

$$B_o \cong \frac{\rho_{o\,0} + \rho_{g\,0}\, R_s}{\rho_o} \tag{53}$$

The oil density is calculated based on the correlation of Velarde *et al.* (1999).

3.5 Gas, oil and water viscosities

The gas viscosity is calculated using the correlation of Lee *et al.* (1966).

The dead oil viscosity at standard pressure is calculated using the correlation of Ng and Egboah (1994), which was based on the charts of Beal (1946). The dead oil viscosity is necessary to calculate the saturated and subsaturated oil viscosity. The former is calculated using the correlation of Beggs and Robinson (1975), which was based on the charts of Chew and Connaly Jr (1959), while the latter is calculated using the correlation of Vasquez and Beggs (1980), based on the work of Beal (1946).

The water viscosity is calculated using the results of Collins (1987). The first step is the determination of the water viscosity at standard pressure, then it is possible to evaluate the water viscosity at local conditions.

4. SIMULATIONS

Based on the presented model, a computational program for transient simulations was developed.

The program calculates the stationary state by setting to zero the time derivatives in the dynamic equations. To evaluate the transient state, the initial conditions are taken from the stationary state results.

A moving grid method was adopted, in which node i ($1 \leq i \leq N-1$) moves with the gas characteristic velocity. Last node N moves with the liquid velocity if the liquid level falls bellow the top of the riser, or remain fixed at the top otherwise. The time step is calculated as the time demanded to node $N-1$ to intersect node N. As the gas velocity is positive, a node disappears at the liquid level or top of the riser and a node is created at the bottom of the riser, keeping constant the number of nodes.

Table 1 presents the input data used to simulate a case, in which the flow is unstable and reaches a limit-cycle. The following figures show the transient response of important variables: void fraction at the bottom of the riser (Fig. 3(a)), oil holdup at the bottom of the riser (Fig. 3(b)), water holdup at the bottom of the riser (Fig. 3(c)), gas (red), oil (black) and water (blue) superficial velocity at the bottom of the riser (Fig. 3(d)), gas (red), oil (black) and water (blue) superficial velocity at the liquid level in the riser (Fig. 3(e)), pressure at the bottom of the riser (Fig. 3(f)), position of liquid accumulation front at pipeline (Fig. 3(g)) and height of the liquid level at the riser (Fig. 3(h)). Observe that the pressure at the top of the riser, or at the separator, is constant. It is also possible to simulate cases in which the pressure is controlled by a choke valve.

Table 1. Input data for simulation.

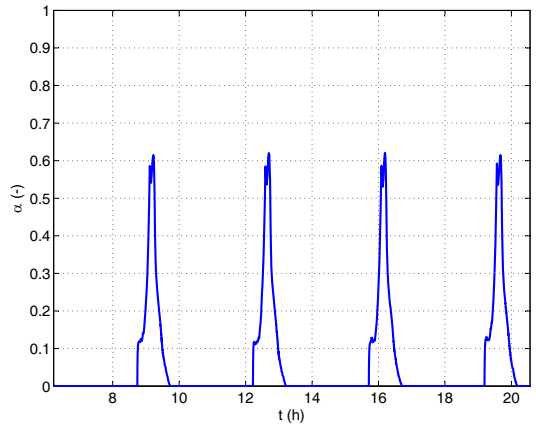
Symbol	Variable	Values
API	API-grad	19
γ_g	Gas specific gravity	0.6602
$Q_{g\,0}$	Gas volumetric flow rate at standard conditions	$0.1\,Sm^3/s$
GOR	Gas-oil ratio	145
WOR	Water-oil ratio	0.3
T	Temperature	$323K$
D	Inner diameter	4”
X	Horizontal length of the top of the riser	$845\,m$
Z	Height of the top of the riser	$1300\,m$
ϵ	Roughness	$4,6\,.\,10^{-5}\,m$
P_{sep}	Pressure at the separator	$25\ bara$
Y	Salinity	0
N	Number of nodes	101

5. CONCLUSIONS

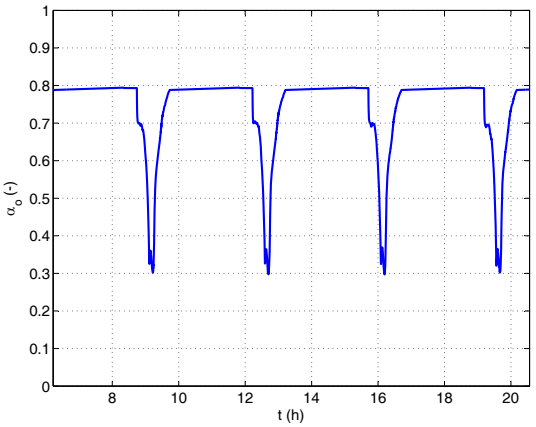
A mathematical model was presented and a computational program was developed. The program is able to simulate the three-phase gas-oil-water flow in a pipeline-riser system and allows to verify if the flow is stable, reaching the steady state, or unstable, reaching a limit-cycle.

By simulating systematically the flow with controlled inputs, its possible to construct stability maps, which shows graphically the region where the flow is expected to be stable.

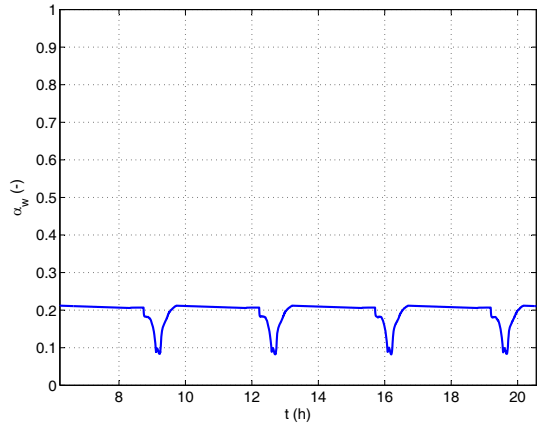
The model takes into account the gas vaporization, so that it is possible to investigate the influence of this phenomenon on the stability of the system. Besides, the model considers a choke valve at the top of the riser and the effects of this device on the flow stability can also be verified. This is a work in progress.



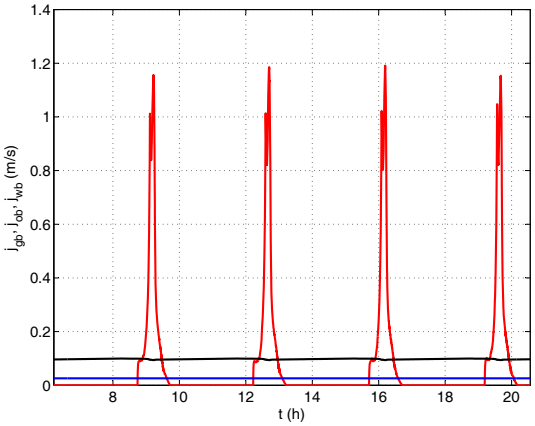
(a)



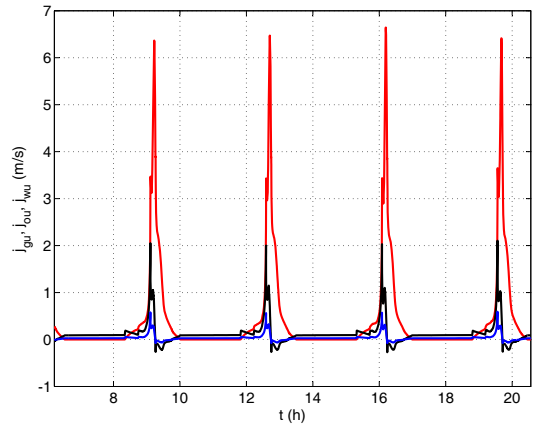
(b)



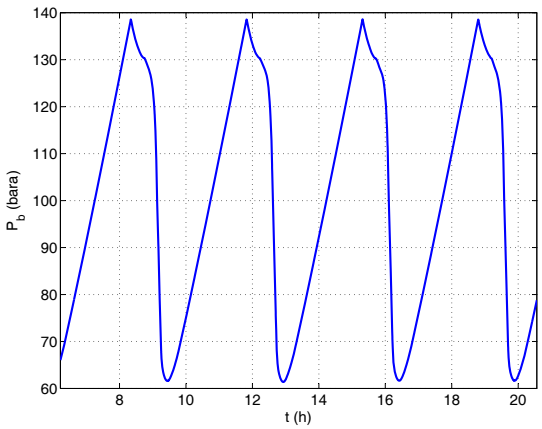
(c)



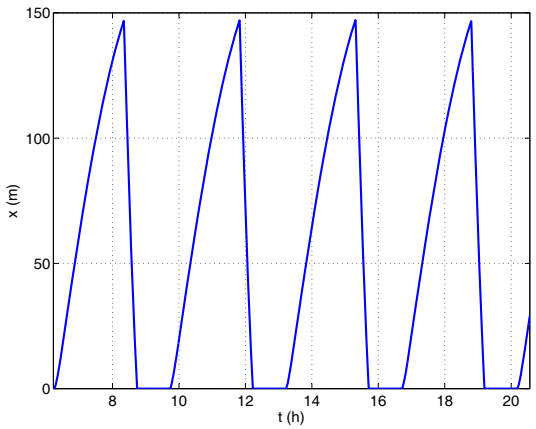
(d)



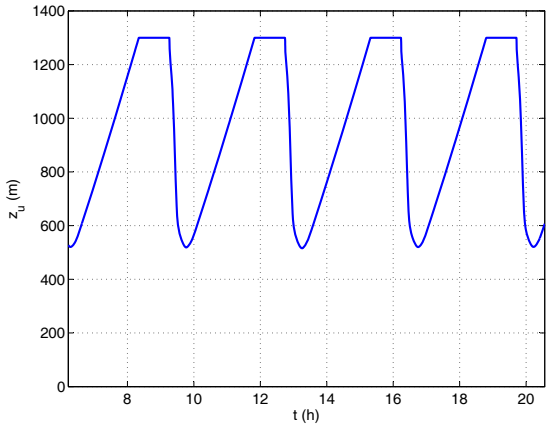
(e)



(f)



(g)



(h)

Figure 3. Simulation results.

6. ACKNOWLEDGEMENTS

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