Effect of Bearing Pad Deformation on the Performance of Finite Journal Bearing

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Abstract. *The load support and other performance characteristics of any bearing depend on its film thickness profile which can be affected by elastic deformations. In this work, finite volume method is used to solve the classical equation of journal bearings considering elastic deformation of bearing pad as well. Presence of cavitation in the journal bearing is considered. Results of static performance are compared with bearing with no deformations showing that elastic deformations cannot be neglect in some situations.*

Keywords*: Journal bearing, cavitation, finite volume method.*

1. INTRODUCTION

In their simplest form, a journal bearing and its bearing consist of two eccentric cylinders. The outer cylinder (bearing) is usually held stationary while the inner cylinder (journal) is made to rotate (Szeri, 2005). The clearance gap between those cylinders is fulfilled by some lubricant. In journal bearings, where the journal is eccentric with the bearing, Eq. (1) is used to describe the film thickness, h:

$$h = c\left(1+\varepsilon\cos\theta\right) \tag{1}$$

where c is the radial clearance, $0 \le \varepsilon < 1$ is the eccentricity ration, θ is the angular coordinates measured from lines of cylinders centers (Szeri, 2005). Equation (1) may not describe accurately the film thickness in many situations because it considers both cylinders (bearing and journal) as rigid cylinders. It is possible that any small variation of the film thickness caused by the film pressure at the bearing surface (bearing pad) could affect the film pressure itself.

The classical Reynolds theory of lubrication has been used to develop equations to model the lubricant film flow in several types of bearings, including journal bearings. This equation can be written as (Prata & Ferreira, 1990; Szeri, 2005):

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\mu R\omega\frac{\partial h}{\partial x} \tag{2}$$

where p is the pressure, μ the viscosity, ω shaft angular velocity and R is the radius of the shaft.

As can be seen, the film thickness is presented in Eq. (2) and possibly plays an important role in this equation, therefore to achieve accurate values of p the film thickness must be evaluated correctly.

In this work, the journal was not considered rigid and the effects of this hypothesis has been analyzed for different D/L ratios, where $D = 2R$ is the bearing diameter and L is the bearing length.

2. ANALYSIS

In dimensionless form Eq. (2) can be written as:

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial\overline{P}}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\overline{Z}}\left(H^3\frac{\partial\overline{P}}{\partial\overline{Z}}\right)=12\pi\frac{\partial H}{\partial\theta} \tag{3}$$

where

$$\theta=\frac{x}{R}\ ;\ \overline{Z}=\frac{2z}{L}\ ;\ \overline{P}=\frac{P}{\mu N\left(\frac{R}{c}\right)^2} \tag{4}$$

and Eq. (1)

$$H=\frac{h}{c}=1+\varepsilon\cos\theta \tag{5}$$

To complete the statement it is necessary to define the boundary conditions of this problem:

$$\bar{P}\left(\theta,\bar{Z}=\pm 1\right)=0\ ,\ \bar{P}\left(\theta=0,\bar{Z}\right)=0 \tag{6}$$

Under normal operating conditions a lubricant film of converging-diverging geometry is expected to cavitate within the diverging part of the clearance (Szeri, 2005). But, the position where is the film-cavity interface is not known *a priori*. Therefore the boundary condition for this interface can be written as:

$$\bar{P}\left(\theta=\pi+\alpha,\bar{Z}\right)=\frac{\partial\bar{P}}{\partial\theta}\left(\theta=\pi+\alpha,\bar{Z}\right)=0 \tag{7}$$

where the position $\theta=\pi+\alpha\left(\bar{Z}\right)$ is the film-cavity interface and $0\le\alpha\left(\bar{Z}\right)<\pi$ is not known and must be evaluated using some methodology. In this work was used a methodology proposed by Prata and Ferreira (1990). To deal with this problem, the solution domain was transformed into a simple shape using a new coordinate system, η, in the θ direction defined as

$$\theta=\left(\pi+\alpha\right)\eta \tag{8}$$

Equation (3) is now rewritten for this new coordinate system as

$$\left(\frac{1}{\pi+\alpha}\right)^2\frac{\partial}{\partial\eta}\left(H^3\frac{\partial\bar{P}}{\partial\eta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{Z}}\left(H^3\frac{\partial\bar{P}}{\partial\bar{Z}}\right)=\left(\frac{12\pi}{\pi+\alpha}\right)\frac{\partial H}{\partial\eta} \tag{9}$$

It should be noted that in this new coordinate system, the domain used to calculate the pressure do not change, therefore the boundary condition is defined as:

$$\bar{P}\left(\eta,\bar{Z}=\pm 1\right)=0\ ,\ \bar{P}\left(\eta=0,\bar{Z}\right)=0 \tag{10}$$

$$\bar{P}\left(\eta=1,\bar{Z}\right)=0\ ,\ \frac{\partial\bar{P}}{\partial\eta}\left(\eta=1,\bar{Z}\right)=0 \tag{11}$$

As can be seen in Eq. (10) and (11), now the boundary condition is not dependent of α. This variable which its value is unknown now appears just in the governing equation, Eq. (9).

As stated before, Eq. (1) or its dimensionless form, Eq. (5), can be used only for rigid bearings. In this work it was introduced a new term in the film thickness equation, Eq. (5), given by

$$H=1+\varepsilon\cos\left(\left(\pi+\alpha\right)\eta\right)+\beta\bar{F}_n \tag{12}$$

where $\bar{F}_n$ is the dimensionless force at the journal surface produced by $\bar{P}$ and β is the flexibility coefficient. An additional difficulty is introduced by Eq. (12). The deflection of the pad depends on de pressure distribution in the bearing gap, while that pressure is influenced by the geometry of the bearing gap. This nonlinear interaction needs to be carefully considered by the numerical method employed to solve the Reynolds equation.

Similar equation were proposed by Higginson (1965) , Gethin (1985) or Jain *et alli* (1990), but in those works cavitation were not considered.

3. FINITE VOLUME EQUATIONS

The finite volume method (Patankar, 1980; Maliska, 1994) was used to solve numerically all differential equations presented herein. The domain is divided into non-overlapping small control volumes and the differential equation is integrated over each one of these volumes. Using the classical finite volume notation, all terms of Eq. (9) becomes:

$$\int_{\bar{Z}_s}^{\bar{Z}_n}\int_{\eta_w}^{\eta_e}\left(\frac{1}{\pi+\alpha}\right)^2\frac{\partial}{\partial\eta}\left(H^3\frac{\partial\bar{P}}{\partial\eta}\right)d\eta d\bar{Z}=\int_{\bar{Z}_s}^{\bar{Z}_n}\left(\frac{1}{\pi+\alpha}\right)^2\left\{H^3\left.\frac{\partial\bar{P}}{\partial\eta}\right|_{\eta_e}-H^3\left.\frac{\partial\bar{P}}{\partial\eta}\right|_{\eta_w}\right\}d\bar{Z}$$
$$=\left(\frac{1}{\pi+\alpha_P}\right)^2\left\{H_e^3\frac{\bar{P}_E-\bar{P}_P}{\delta\eta_e}-H_w^3\frac{\bar{P}_P-\bar{P}_W}{\delta\eta_w}\right\}\Delta\bar{Z}_P \tag{13}$$

$$\int_{\eta_w}^{\eta_e}\int_{\bar{Z}_s}^{\bar{Z}_n}\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{Z}}\left(H^3\frac{\partial\bar{P}}{\partial\bar{Z}}\right)d\bar{Z}d\eta=\left(\frac{D}{L}\right)^2\int_{\eta_w}^{\eta_e}\left\{H^3\left.\frac{\partial\bar{P}}{\partial\bar{Z}}\right|_{\bar{Z}_n}-H^3\left.\frac{\partial\bar{P}}{\partial\bar{Z}}\right|_{\bar{Z}_s}\right\}d\eta$$
$$=\left(\frac{D}{L}\right)^2\left\{H_P^3\frac{\bar{P}_N-\bar{P}_P}{\delta\bar{Z}_n}-H_P^3\frac{\bar{P}_P-\bar{P}_S}{\delta\bar{Z}_s}\right\}\Delta\eta_P \tag{14}$$

$$\int_{\bar{Z}_s}^{\bar{Z}_n}\int_{\eta_w}^{\eta_e}\left(\frac{12\pi}{\pi+\alpha}\right)\frac{\partial H}{\partial\eta}d\eta d\bar{Z}=\int_{\bar{Z}_s}^{\bar{Z}_n}\left(\frac{12\pi}{\pi+\alpha}\right)\left\{H_e-H_w\right\}d\bar{Z}$$
$$=\left(\frac{12\pi}{\pi+\alpha_P}\right)\left\{H_e-H_w\right\}\Delta\bar{Z}_P \tag{15}$$

Equation (9) is now rewritten as:

$$\left[\frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\left\{\frac{H_e^3}{\delta\eta_e}+\frac{H_w^3}{\delta\eta_w}\right\}+\left(\frac{D}{L}\right)^2H_P^3\left(\frac{\Delta\eta_P}{\delta\bar{Z}_n}+\frac{\Delta\eta_P}{\delta\bar{Z}_s}\right)\right]\bar{P}_P=$$
$$\left[\frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\frac{H_e^3}{\delta\eta_e}\right]\bar{P}_E+\left[\frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\frac{H_w^3}{\delta\eta_w}\right]\bar{P}_W+$$
$$\left[\left(\frac{D}{L}\right)^2H_P^3\frac{\Delta\eta_P}{\delta\bar{Z}_n}\right]\bar{P}_N+\left[\left(\frac{D}{L}\right)^2H_P^3\frac{\Delta\eta_P}{\delta\bar{Z}_s}\right]\bar{P}_S+\left(\frac{12\pi}{\pi+\alpha_P}\right)\left(H_e-H_w\right)\Delta\bar{Z}_P \tag{16}$$

or, in an usual form (Patankar, 1980):

$$A_P\bar{P}_P=A_E\bar{P}_E+A_W\bar{P}_W+A_N\bar{P}_N+A_S\bar{P}_S+S_P \tag{17}$$

where

$$A_P=\frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\left(\frac{H_e^3}{\delta\eta_e}+\frac{H_w^3}{\delta\eta_w}\right)+\left(\frac{D}{L}\right)^2H_P^3\left(\frac{1}{\delta\bar{Z}_n}+\frac{1}{\delta\bar{Z}_s}\right)\Delta\eta_P$$
$$=A_N+A_S+A_E+A_W \tag{18}$$

$$A_N=\left(\frac{D}{L}\right)^2H_P^3\frac{\Delta\eta_P}{\delta\bar{Z}_n} \tag{19}$$

$$A_S=\left(\frac{D}{L}\right)^2H_P^3\frac{\Delta\eta_P}{\delta\bar{Z}_s} \tag{20}$$

$$A_E=\frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\frac{H_e^3}{\delta\eta_e} \tag{21}$$

$$A_W=\frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\frac{H_w^3}{\delta\eta_w} \tag{22}$$

$$S_P=\left(\frac{12\pi}{\pi+\alpha_P}\right)\left(H_e-H_w\right)\Delta\bar{Z}_P \tag{23}$$

Equation (16) is valid for all internal volumes used to discretize the domain. This equation should be slight modified to those volumes located at the boundary. For example, volumes located at right side of the domain (cavitation boundary) have the following equations:

$$A_N = \left(\frac{D}{L}\right)^2 H_P^3 \frac{\Delta\eta_P}{\delta\bar{Z}_n} \; ; \; A_S = \left(\frac{D}{L}\right)^2 H_P^3 \frac{\Delta\eta_P}{\delta\bar{Z}_s} ; \; A_E = 0 ; \; A_W = \frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\frac{H_w^3}{\delta\eta_w} \tag{24}$$

$$A_P = A_N + A_S + \frac{\Delta\bar{Z}_P}{\left(\pi+\alpha_P\right)^2}\frac{H_e^3}{\delta\eta_e} + A_W \tag{25}$$

$$S_P = \left(\frac{12\pi}{\pi+\alpha_P}\right)\left(H_e - H_w\right)\Delta\bar{Z}_P \tag{26}$$

The film thickness, H , and α are dependent of $\bar{P}$ thus usual techniques employed to solve nonlinear equations must be used to accelerate the convergence rates.

4. CAVITATION INTERFACE

The cavitation interface is not known *a priori*, thus one equation should be proposed to evaluate α_P . In this work it was used a methodology proposed by Prata and Ferreira (1990). Equation (17) is rewritten as:

$$S_P = A_P\bar{P}_P - A_W\bar{P}_W - A_N\bar{P}_N - A_S\bar{P}_S = \left(\frac{12\pi}{\pi+\alpha_P}\right)\left(H_e - H_w\right)\Delta\bar{Z}_P \tag{27}$$

then

$$H_e = \frac{A_P\bar{P}_P - A_W\bar{P}_W - A_N\bar{P}_N - A_S\bar{P}_S}{\left(\frac{12\pi}{\pi+\alpha_P}\right)\Delta\bar{Z}_P} + H_w \tag{28}$$

Since $H_e = 1+\varepsilon\cos\left(\left(\pi+\alpha_P\right)\eta_e\right)+\beta\bar{F}_n$ and at the cavitation boundary, $\eta_e = 1$, $\bar{P}\left(\eta=1,\bar{Z}\right)=0$:

$$H_e = 1-\varepsilon\cos\left(\alpha_P\right) \tag{29}$$

and

$$\alpha_P = \cos^{-1}\left(1-H_e\right) \tag{30}$$

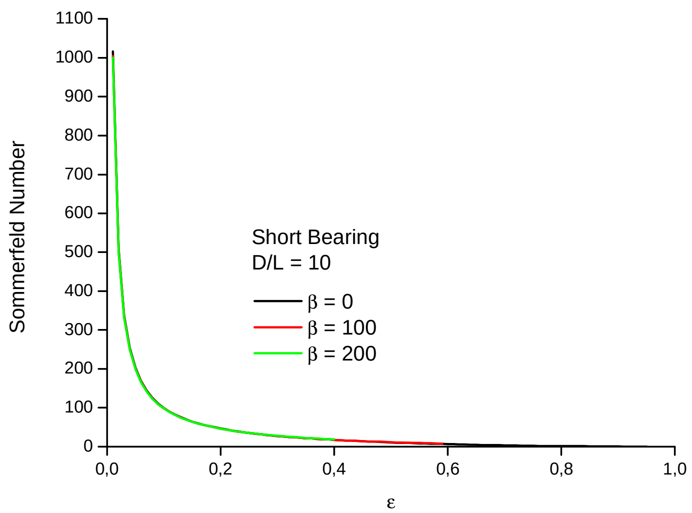

Figure 1 – Sommerfeld Number - $D/L = 10$

5. RESULTS AND DISCUSSION

All results presented herein were obtained using a grid with (41 x 81) $(\eta, \bar{Z})$ volumes. The linear system was solved using Jacob Method (Maliska, 1994).

5.1. Short Bearing

The first result presented in this work, Figure 1, has been obtained for short bearing, which $D/L > 4$. The Sommerfeld number is shown for three different flexibility coefficients (0, 100 and 200). In this work, the Sommerfeld number is defined by the Eqs. (31) to (34).

$$W = \sqrt{W_n^2 + W_t^2} \tag{31}$$

$$W_n = \mu\omega\left(\frac{R}{c}\right)^2\left(\frac{DL}{4}\right)\int_{-1}^{1}(\pi+\alpha)\int_{0}^{1}\bar{P}\cos\left((\pi+\alpha)\eta\right)d\eta d\bar{Z} \tag{32}$$

$$W_t = \mu\omega\left(\frac{R}{c}\right)^2\left(\frac{DL}{4}\right)\int_{-1}^{1}(\pi+\alpha)\int_{0}^{1}\bar{P}\,\mathrm{sen}\left((\pi+\alpha)\eta\right)d\eta d\bar{Z} \tag{33}$$

$$\frac{1}{So} = \frac{\left(\dfrac{W}{DL}\right)}{\mu\omega\left(\dfrac{R}{c}\right)^2} \tag{34}$$

Figure 1 shows that there is not an important influence of the flexibility coefficients on the Sommerfeld number for short bearing, because it works with small values of pressure, therefore the influence of the flexibility term in Eq. (12) seems to be irrelevant.

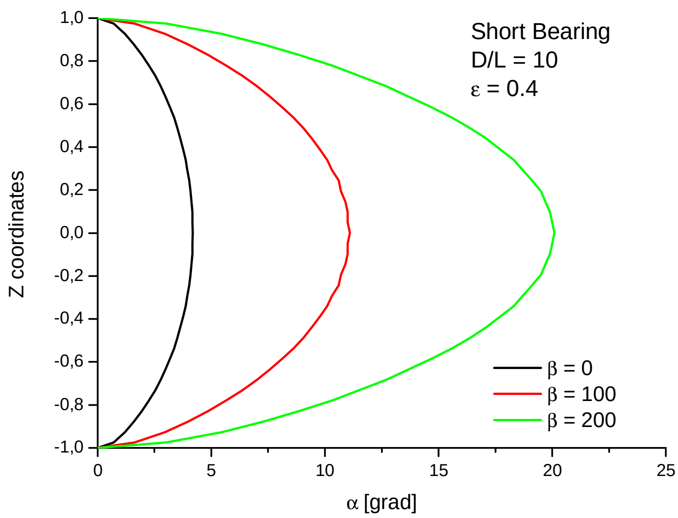

Figure 2 – Cavitation pattern

Figure 2 illustrates the effect of the flexibility coefficients at the cavitation boundary for $\varepsilon = 0.4$. As can be seen, the cavitation pattern is clearly affected by the elastic deformation in the pad. This effect will be higher for higher values ε . Figure 3 shows the pressure field for the same value of ε . The pressure values were not significantly affected by the flexibility coefficients.

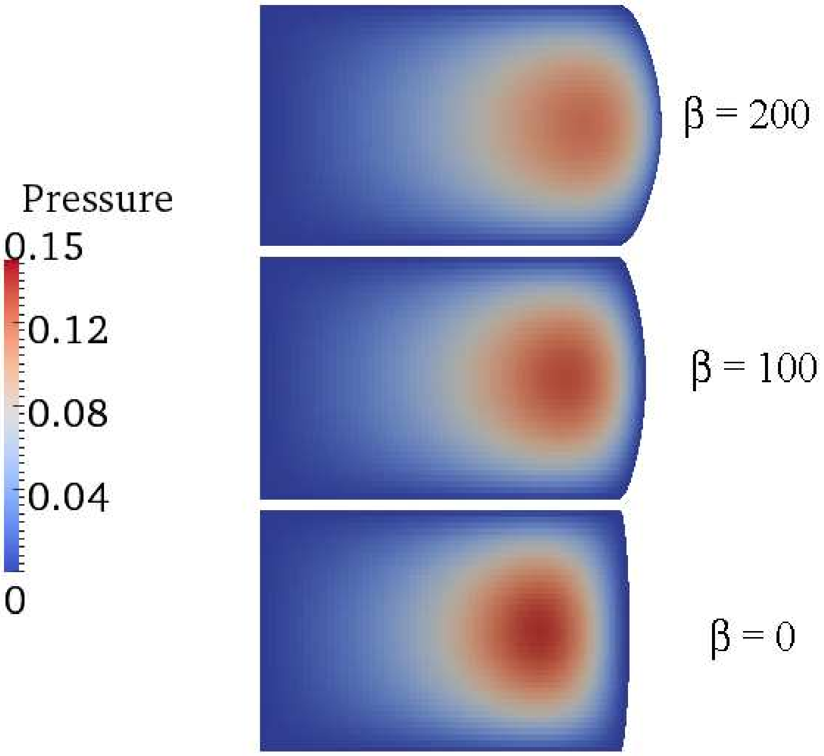

Figure 3 - Dimensionless pressure for D/L = 10 and $\varepsilon = 0.4$

5.2. Finite Bearing

Finite bearings are those which $1/4 < D/L < 4$ (Szeri, 2005). The results presented now have been obtained for $D/L = 1$.

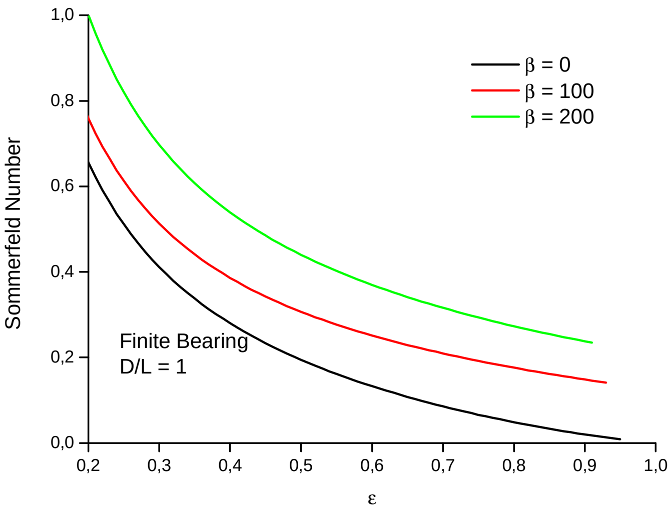

Figure 4 - Sommerfeld Number - $D/L = 1$

Significant differences were observed in the Sommerfeld number for different values of flexibility coefficient in Figure 4. The higher ε value, the higher this difference. Such differences were not observed for short bearing.

It is visible the influence of the flexibility coefficient on the boundary shape located at the cavitation region as well, as shown in Figure 5.

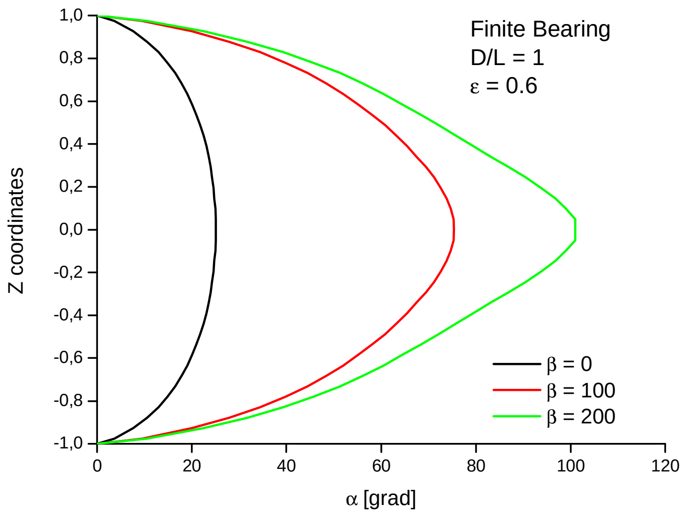

Figure 5 - Cavitation pattern - $D/L = 1$ and $\varepsilon = 0.6$

Figure 6 summarizes Figure 4 and Figure 5. Ones can observe that both pressure field and boundary shape were affected by flexibility coefficient in Eq. (12).

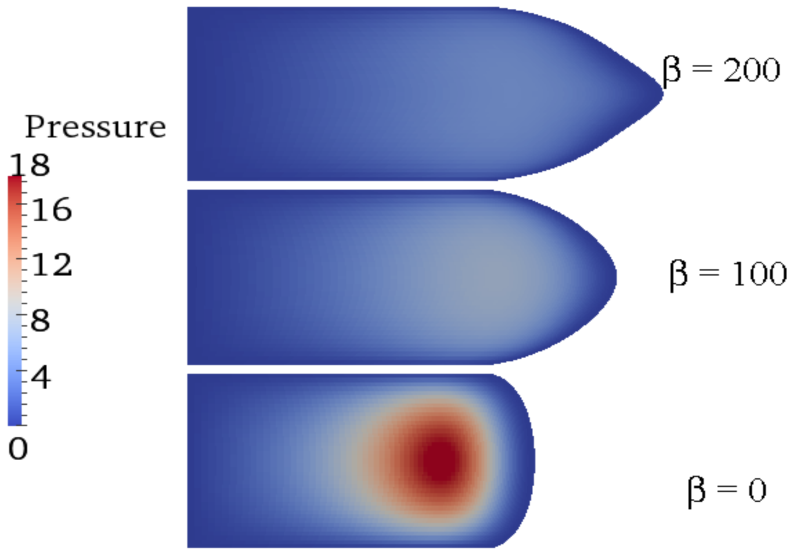

Figure 6 - Dimensionless pressure for $D/L = 1$ and $\varepsilon = 0.6$

5.1. Long Bearing

Long bearings are those which $D/L < 1/4$ (Szeri, 2005). The results presented now have been obtained for $D/L = 0.1$.

Significant differences were observed again in the Sommerfeld number for different values of flexibility coefficient in Figure **7**. It is clear that the load capacity is affected by any deformation of the pad.

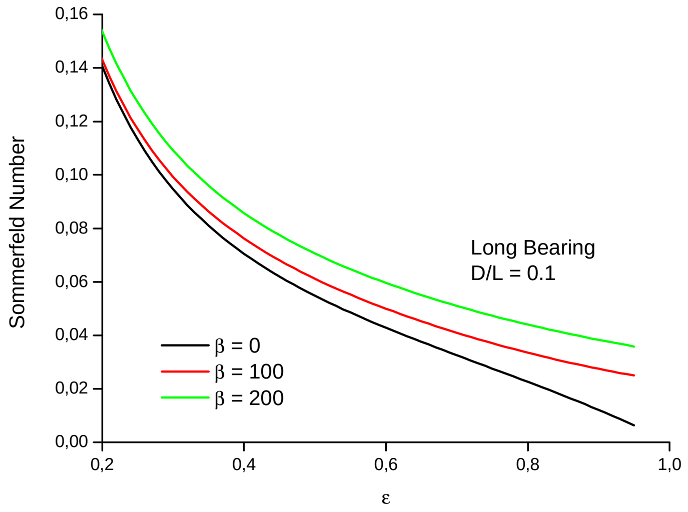

Figure 7 - Sommerfeld Number - $D/L = 0.1$

Figure 8 shows the cavitation boundary and how this boundary changes affected by the pad deformation.

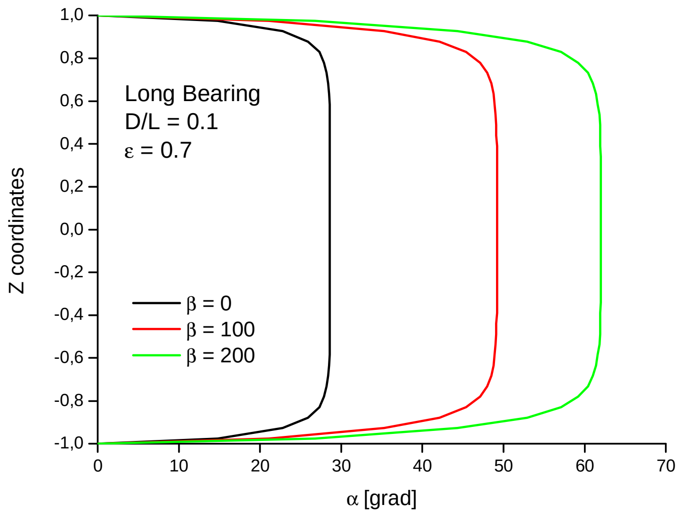

Figure 8 - Cavitation pattern - $D/L = 0.1$ and $\varepsilon = 0.7$

Figure 9 reviews Figure 7 and Figure 8. Pressure field and cavitation boundary has been affected by pad deformation as we observed in finite bearing.

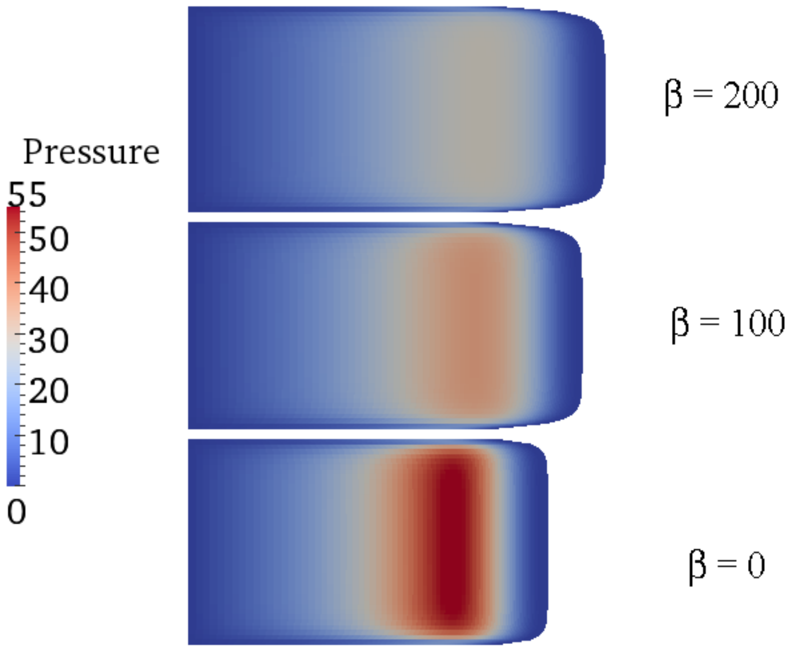

Figure 9 - Dimensionless pressure for $D/L = 0.1$ and $\varepsilon = 0.7$

6. CONCLUSIONS

In this work, the influence of pad deformation in the journal bearing performance has been investigated. A model where cavitation and pad deformation were taken under consideration has been analyzed numerically using finite volume method.

Results have shown that pad deformation caused by pressure field in the film gap affects the load capacity of the bearing and the pad deformation changes the cavitation boundary. It changes the position and the pattern as well.

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