ROBUST CONTROL SYSTEM DESIGN FOR LOW ALTITUDE PARACHUTE EXTRACTION OPERATION

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Abstract. *In military cargo transportation it is often observed the need for aircraft which are able to carry relatively small load with agility to conflicted zones or areas where access is difficult. In the case landing is either impossible or too dangerous, airdrop appears as a solution, although it implies a delicate situation for flight control once this type of operation means a fast variation in parameters related to stability such as center of gravity and moment of inertia. A solution based on robust design techniques is proposed in order to obtain an automatic control law which takes in consideration this variation, assuring aircraft stability during load extraction and airdrop. Performance requirements are represented explicitly by weighting functions adequately chosen to allow the design of a robust controller respecting the restrictions imposed by the project requirements. Stability and handling quality analysis are performed to evaluate the proposed solutions.*

Keywords: *lape operation, robust control, parameter identification, aircraft stability, cargo transportation*

1. INTRODUCTION

Low Altitude Parachute Extraction (LAPE) is a type of airdrop in which the aircraft flies near the ground, typically between $10\,m$ e $150\,m$. A parachute system is responsible for extracting the load and eventually used to slow it down after the cargo is dropped. A major technological challenge is to keep the flight stability during and immediately after extraction. This is due to the instant variation of some parameters such as center of gravity (CG), moment of inertia and mass. An accurate prediction for the aircraft behavior may be difficult to obtain in order to use classic control design tools. Hence, in order to keep its attitude stable the controller must take into account or be able to support such variation, which implies the use of robust design techniques. The objective of this paper is to present a method of robust control design for stabilization of a light cargo aircraft performing a hypothetical extraction operation at low altitude in the presence of weather disturbances in the form of wind gust.

2. MODELING

Main features of this light cargo aircraft are high wing, turboprop propulsion and presence of rear ramp and rear door. The airdrop maneuver is typically performed in straight and level flight. Although weather conditions may cause a lateral movement, only the longitudinal dynamics will be studied in this paper. Air speed, aircraft rotations around a coordinate system and their respective derivatives describe the motion. Aircraft's 3-view picture and the state-space variables and forces involved in the motion are shown in Fig. 1.

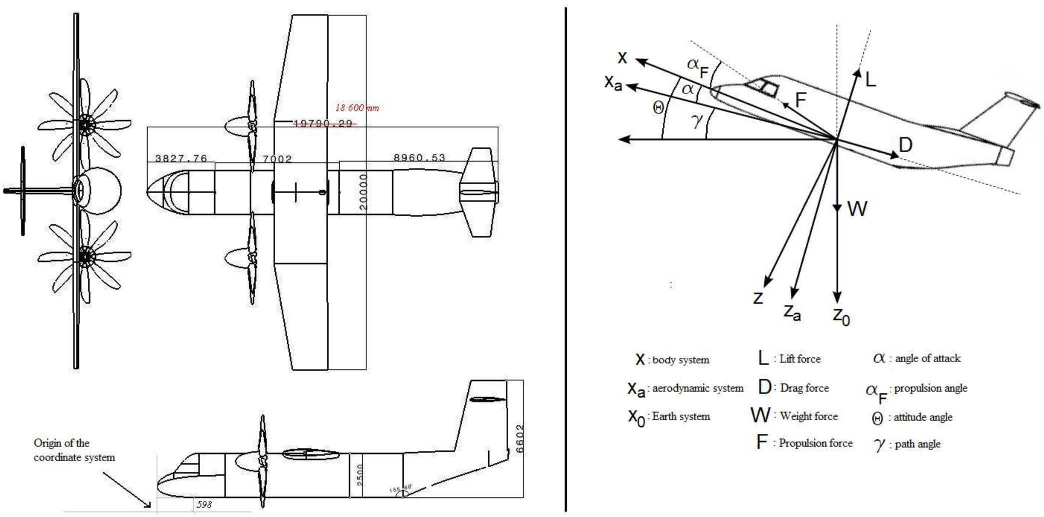

Figure 1. Aircraft's 3-view representation, coordinate systems and variables of longitudinal motion.

The maneuver is composed by the stages of cargo (mass) displacement along the aircraft interior and its eventual extraction, when a discontinuity of the total mass is observed. For simulation purposes, an initial mass of $8600\,kg$ is considered, and the center of gravity is placed at 18% of the mean aerodynamic chord (MAC). A $1000\,kg$ cargo moves toward the rear door. At this point, the CG position is 54%. The aircraft mass drops then to $7600\,kg$ and the final CG position is 22% (Mendonça, 2010). Other parameters such as aerodynamic derivatives and moment of inertia can be modeled as a function of the CG position. The output of interest in this work is the attitude angle θ as it describes the aircraft rotation in longitudinal motion. From Fig. 1 it can be seen that

$$\theta = \alpha + \gamma. \tag{1}$$

Because airdrop operation happens at low altitudes, the effects of the reduced distance between the wing and the ground over the aircraft movement must be considered. These effects are generally known as ground effect and can be described in simple terms by an increase in lift, decrease in drag and reinforcement of the pitch moment.

3. ROBUST CONTROLLER DESIGN

The design of a robust controller must start from a set of requirements to be fulfilled. It must assure stability in the presence of model uncertainties and disturbance rejection. The method is based on the choice of adequate weighting functions W_i covering the set of requirements and allows the representation of these requirements explicitly. *A posteriori* performance analysis for each requirement is also possible. This procedure makes use of the design structure presented in Fig. 2.

Weighting functions don't have physical implementation, serving exclusively as a design tool. Performance outputs Z_i are employed to obtain the transfer functions to the input R in function of the controller K and the plant G. The mathematical problem consists in deducing K from these functions. It can be written in a standard form as shown in Fig. 3.

Outputs Z and Y are related to inputs R and U by Eq. (2).

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Y \end{pmatrix} = \underbrace{\begin{pmatrix} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ 1 & -G \end{pmatrix}}_{P} \begin{pmatrix} R \\ U \end{pmatrix}. \tag{2}$$

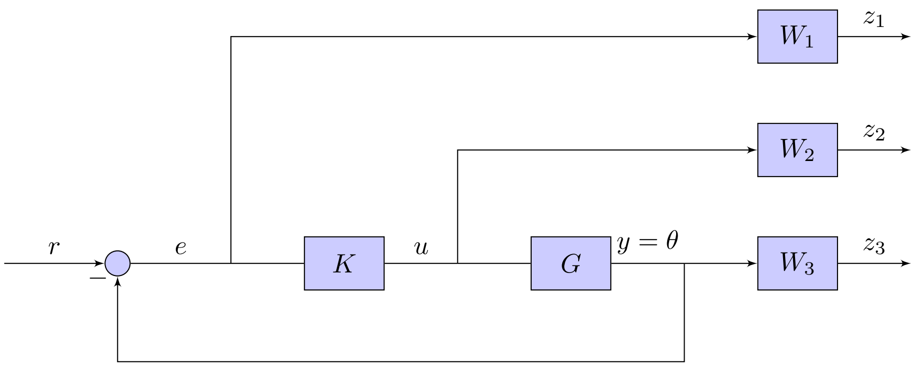

Figure 2. Weighting functions.

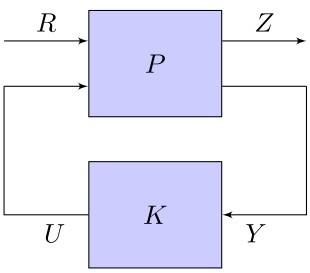

Figure 3. Block diagram - standard form.

Defining the components of the transfer function P as

$$P_{11} = \begin{pmatrix} W_1 \\ 0 \\ 0 \end{pmatrix} \quad P_{12} = \begin{pmatrix} -W_1 G \\ W_2 \\ W_3 G \end{pmatrix} \qquad P_{21} = 1 \quad P_{22} = -G. \tag{3}$$

and considering an output feedback $u = Ky$, the Fractionary Linear Transfer (FLT) is obtained, giving the relationship (4) between R and Z needed to design the controller.

$$F(P,K) = \frac{Z}{R} = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}. \tag{4}$$

Replacing the terms, this FLT can be written as

$$F(P,K) = \begin{pmatrix} W_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -W_1 G \\ W_2 \\ W_3 G \end{pmatrix} K \underbrace{(1 + GK)^{-1}}_{S} = \begin{pmatrix} -W_1 S \\ W_2 K S \\ W_3 T \end{pmatrix} \tag{5}$$

where S is the sensitivity function and T is the complementary sensitivity function. A H_∞ design searches K in order to assure internal stability of the closed loop respecting the restrictions

$$\|F(P,K)\|_\infty = \left\| \begin{matrix} W_1 S \\ W_2 K S \\ W_3 T \end{matrix} \right\|_\infty < \gamma, \tag{6}$$

where γ is the performance parameter. The problem turns into choosing weighting functions that represent the performance given by the set of requirements. Ideally, a value of 1 for γ means that all the stability criterions have been accomplished optimally. H_∞-norm properties allow the study of each component of $F(P,K)$ independently. Therefore the weighting functions must obey

$$\begin{aligned} \|W_1 S\|_\infty &\leq 1 \\ \|W_2 K S\|_\infty &\leq 1 \\ \|W_3 T\|_\infty &\leq 1. \end{aligned} \tag{7}$$

It is possible to associate the closed-loop performance, given by the poles of S, to the weighting function W_1 (Franklin, Powell and Emami-Naeini., 2006). The sensitivity function S describes the system response to an output disturbance. Attenuating its value means that this disturbance has small influence in the expected output of the system. Typically, this kind of disturbance is dominant at low frequencies and its spectrum defines the frequency band for which the design must result in sensitivity attenuation. An adequate weighting can make the longitudinal dynamics insensible to output disturbance, typically a sudden change in speed caused by wind gust. Therefore, a desired sensitivity S_d response must be small at the frequency range in which those disturbances are critical.

According to (UNITED STATES AIR FORCE, 1997), atmospherical disturbances modeled as stochastic wind have a more relevant effect up to $100\,rad/s$ $(15\,Hz)$ and may present different degrees of severity. The desired sensitivity S_d is then modeled as a high-pass filter as shown in Fig. 4.

From Eq. (7), a successful design should provide

$$\|S\|_\infty < \frac{\gamma}{\|W_1\|_\infty}. \tag{8}$$

The next step is to assure closed-loop stability considering the variation of the flight parameters during the operation. The initial flight condition is considered as the nominal plant. As mentioned above, at this condition the mass value is $8600\,kg$ and the center of gravity position is 18% of the MAC. Critical cases are the airdrop imminence, as CG reaches its maximum value, and the instant right after extraction, when a discontinuity in mass is observed. Linearization around these conditions results in nominal plant G_n and extreme plants G_{e1} and G_{e2}. Difference between extreme and nominal plants can be seen as input multiplicative uncertainty around the nominal model, given by

$$\Delta_i = \tfrac{G_{ei} - G_n}{G_n}, \tag{9}$$

leading to the expressions of Δ_1 and Δ_2. Figure 5 shows the diagram block including this uncertainty.

The resulting closed-loop transfer function is then

$$\frac{Y}{R} = \Delta \cdot \frac{KG}{(I + KG)} = \Delta \cdot T, \tag{10}$$

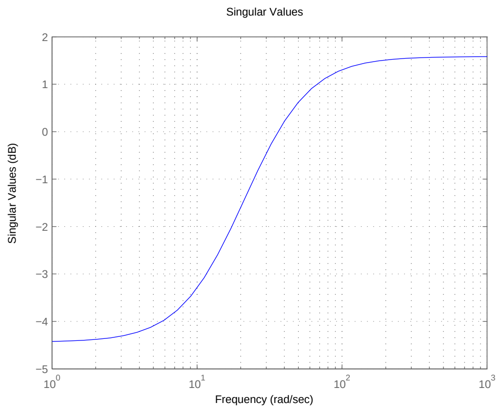

Figure 4. Desired sensitivity, modeled as a high-pass filter with cutoff frequency of $50\ Hz$.

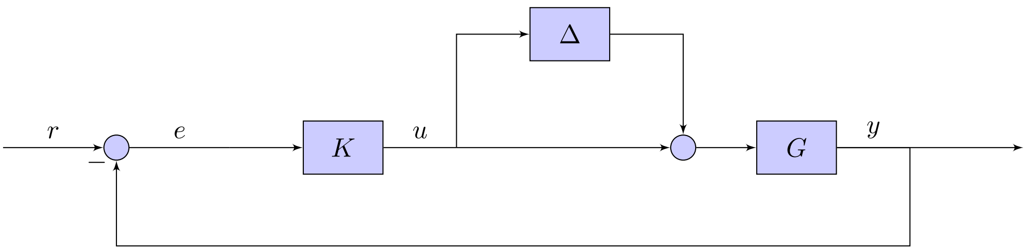

Figure 5. Feedback system with the input multiplicative uncertainty

where T is the complementary sensitivity function and also represents the initial closed-loop transfer function. The Small Gain Theorem states that a closed-loop system T remains stable in the presence of a multiplicative uncertainty Δ if (11) is satisfied (Siqueira, Paglione and Moreira, 2008).

$$\|\Delta \cdot T\|_{\infty} < 1. \tag{11}$$

In order to assure stability for all uncertainties, a comprehensive function Δ_{max} must be found. It is given by (12).

$$\Delta_{max}(j\omega) > sup[\sigma(\Delta_1(j\omega)), \sigma(\Delta_2(j\omega))],\ \forall\omega, \tag{12}$$

where $\sigma(\Delta_i(j\omega))$ represents the singular values of $\Delta_i(j\omega)$, shown in Fig. 6.

Combining (7) and (11), the weighting function that assures stability is found to be

$$W_3 \geq \Delta_{max}. \tag{13}$$

This function is built from the singular values of Δ_{max}. Figure 6 shows that W_3 can be represented by a low-pass filter according to (14), whose parameters must be adjusted to fit the supremum value of the variations.

$$W_3 = K_3 \frac{s + z_3}{s + p_3}. \tag{14}$$

The controller is found using the robust control package in *Matlab* (THE MATHWORKS INC, 2010). An augmented plant P is built from the nominal plant G_n and the weighting functions W_i executing the command *augw*. In the case any of the weighting functions is not used (W_2 in this case, which refers to the control effort), the command to be executed is

```
>>P=augw(Gn,W1,[],W3);
```

The augmented plant is built using the synthesis command *hinfsyn* if a H_{∞} problem is considered. An example is

```
>>[K,CL,GAM]=hinfsyn(P,ny,nu);
```

with ny and nu the number of outputs and inputs to the controller (in this case $ny = nu = 1$). As a result, besides the controller K, the closed-loop system formed by K and G_n and the performance parameter γ are provided. This last parameter should be ideally close to the unity.

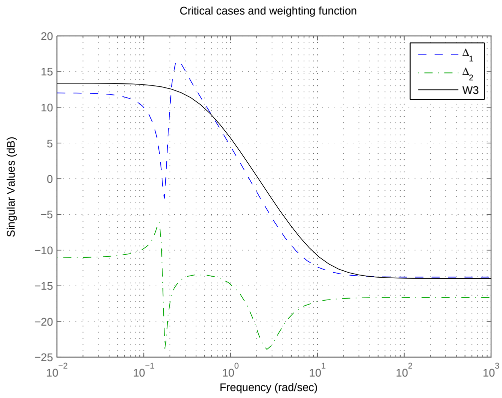

Figure 6. Weighting function adjusted to the supremum value of the variations.

4. RESULTS

The design method proposed previously has generated a 6th-order controller, whose frequency response is shown in Fig. 7(left). A performance parameter γ of $1,5$ is obtained, indicating a reasonable fulfilment of the stability requirements as shown in Fig. 7(right). In order to check the controller performance, the natural aircraft's behavior is presented in Fig. 8. The aircraft performs a straight and level flight $10\ m$ from the ground at $60\ m/s$ before a $1000\ kg$ cargo is dropped. The extraction operation takes $3\ s$ to be executed. Simulation result comes from the integration of the nonlinear equations of motion and can be compared to the closed-loop response at the same flight conditions. Stability, handling quality and disturbance rejection analysis are presented further.

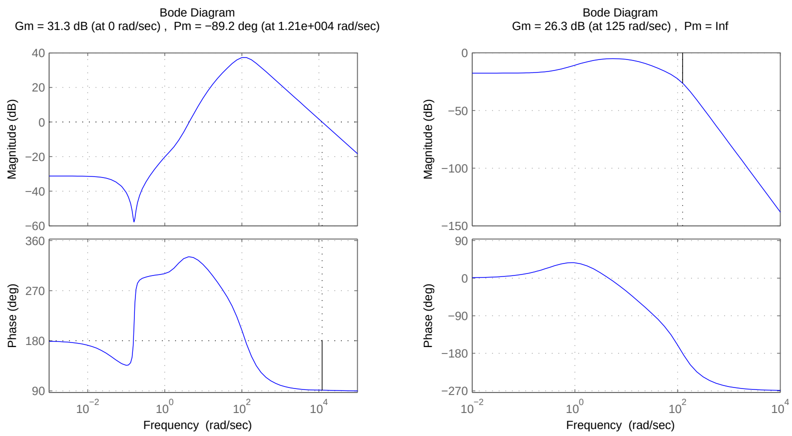

Figure 7. Frequency response of the 6ht-order controller K (left) and stability margins of the closed loop (right).

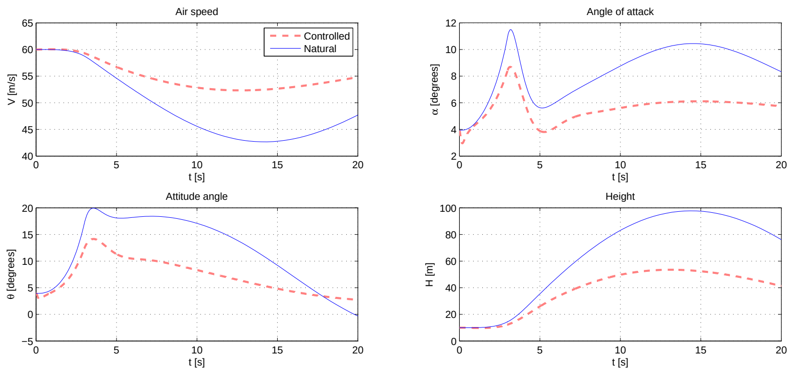

Figure 8. Time response of natural and controlled aircraft behavior during extraction.

4.1 Stability

One of the modern techniques employed in the analysis of robust stability is the use of exclusion zones representing stability margins in a Nichols diagram (Bates and Kureemun, 2003). This procedure consists in determining the stability margins for the open-loop frequency response of each inner loop in the system. This response can then be plotted in a diagram, in which the stability margins are represented by exclusion zones to be avoided in the frequency range of interest. Those "forbidden" regions may be represented in an elliptical form, centered at the critical point $(-180, 0)$ of the Nichols plan. The exclusion zones A and B considered in this paper refer to margins of $6dB$ $/36, 87°$ and $4, 5dB$ $/28, 44°$ respectively.

The resulting system presents a stable closed-loop behavior for all the flight conditions, including the critical CG at 54%, naturally instable, as shown in Fig. 9.

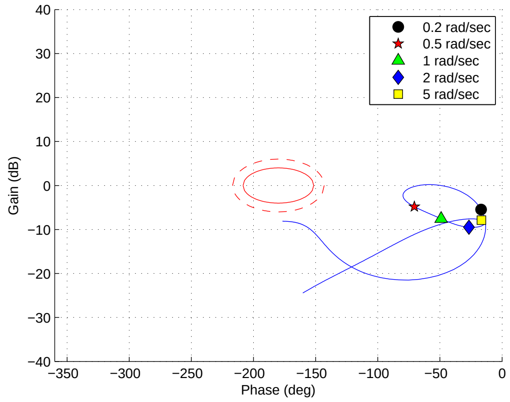

Figure 9. Stability margins satisfied in the Nichols plan for the critical case.

4.2 Handling quality

In the 80s, a handling quality criterium was presented aiming to evaluate an aircraft's tendency to develop Pilot Induced Oscillation (PIO) in the frequency domain. Known as Gibson/Dornier criterium, this tool is efficient from the flight safety point of view, being able to predict tendency to PIO in a large number of tested aircraft (NATO RESEARCH AND TECHNOLOGY ORGANIZATION, 2000). It compares the open-loop attitude angle response, plotted in a Nichols Diagram, to frontiers established from a database of aircraft whose behavior is considered good (Ferreira, 2008). Those frontiers represent regions of acceptable behavior, PIO and "pitch bobble" tendencies.

The Gibson/Dornier criterium fits well the case studied here as it deals with a behavior that can be critical in airdrop and founds its analysis on the attitude response. Figure 10 shows the handling quality analysis of the airdrop operation resulting from this method. It can be seen that for most of the frequency range the aircraft shows a satisfying behavior, even if it tends to oscillate at low frequencies (bobble zone).

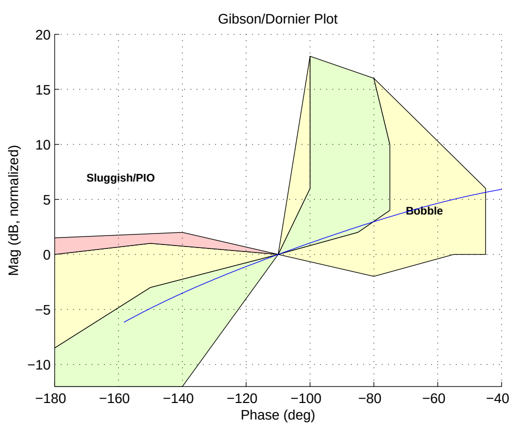

Figure 10. Handling quality analysis according Gibson/Dornier criterium for the controlled aircraft.

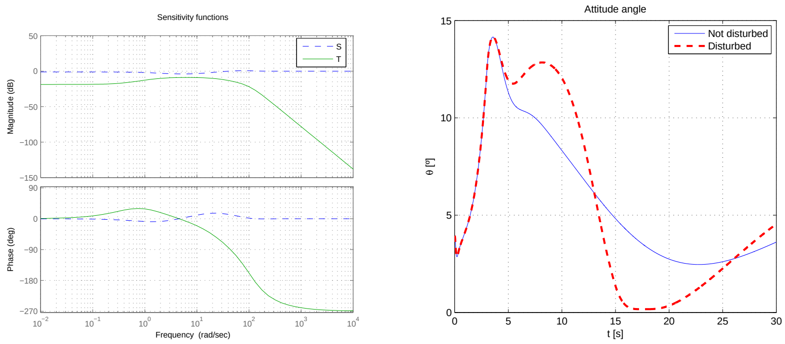

Figure 11. Sensitivy function and its impact on disturbance rejection.

4.3 Disturbance rejection

Although the resulting closed-loop system is stable, its sensitivity function does not have the desired behavior for low frequencies, as shown in Fig. 11. Disturbance is not adequately attenuated, indicating an inappropriate choice W_1 regarding W_3.

5. CONCLUSION

This paper presents a method for the design of a controller for a light cargo aircraft executing low altitude extraction operation. Given the instant change in critical parameters such as center of gravity and mass, a closed-loop robust controller is proposed. Analysis of sensitivity functions are carried out and weighting functions are used to explicitly represent design requirements regarding stability and disturbance rejection in the presence of parameter variation. Results show a trade-off to be found between performance and robustness, once the solution that best satisfies stability has very poor results in terms of disturbance rejection in a given frequency range of interest, indicating that the balance between the weighting functions initially proposed should be modified.

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