# SIMULATION OF THE QUADROTOR CONTROLLED WITH LQR WITH INTEGRAL EFFECT

**Mauricio Vladimir Peña G., mauriciovladimirp@gmail.com**
Universidad Libre Kr. 66A # 53-40, Bogotá, Colombia.

**Edilberto Carlos Vivas G., ecvivasg@unal.edu.co**
Universidad Pedagógica Nacional, Bogotá, Colombia.

**Carol Ivonn Rodríguez F., cirodriguezf@unal.edu.co**
Universidad Nacional de Colombia, Bogotá, Colombia.

***Abstract.*** *This work presents a dynamics study for a vehicle of the type quad-rotor helicopter. It consists of a central body and four beams joined to it. Each beam has a motor with two rotating wings at the far extreme. The body is formed by a box with batteries and on-board computers for control and avionic functions. The wings give sustentation to the vehicle and give the possibility of controlling the orientation and translation of the system. The dynamic model of the vehicle takes account of the dynamics of the rotating wings and results in a non-linear system. It is made a simplification by linearization of the nonlinear and unstable model for the attitude of the quad-rotor and it is designed a LQR (Linear Quadratic Regulator) control with integral effect to track reference paths for the roll, pitch and yaw angles. After that, it is validated the designed control running with the nonlinear model of the Quadrotor. It is compared both types of behavior, linear and non-linear, following the reference.*

***Keywords:*** *Model, LQR control, Integral effect, Quadrotor, attitude*

## 1. INTRODUCTION

This work presents a study of the dynamics for a vehicle type Quadrotor helicopter. This one consists of a central body and four beams joined it. Each beam has a motor with two rotating wings at the far extreme. The body is formed by a box with batteries and on-board computers for control and avionic functions. The wings give sustentation to the vehicle and the possibility of controlling the orientation and translation of the system. The dynamic model of the vehicle takes into account the dynamics of the rotating wings and results in a non-linear system. It is made a simplification by linearization of the nonlinear and unstable model for the Quadrotor attitude and it is designed a LQR (Linear Quadratic Regulator) control with integral effect to track reference paths for the roll, pitch and yaw angles. After that, the designed control is validated by running the Quadrotor non-linear model. Both types of behaviors are compared, linear and non-linear, following the reference.
In diverse articles models and controls of diverse forms have been implemented to make controls of movement to a Quadrotor, being the doctorate thesis (Bouabdallah, 2004), the more illustrative and complete document in this subject. Other authors we have left from its investigation to develop new control applications for devices with these characteristics, trying to improve the performance of the vehicle. In order to see other authors, the section two can be reviewed which part of the premise of a revision of the raised models.
The second section describes the dynamic model for a Quadrotor, and then this one is simplified and linealized to design the control. The thirst section shows basics aspects of the LQR concept and its added integral effect in this article. The next section presents the resultant control and simulations around a point of reference (altitude and actitude) and the conclusions of this study are mentioned in the final section.

## 2. MODEL

Figure 1 shows the Quadrotor and its reference frame which is represented with the rotational transformation of the Roll-Pitch-Yaw Euler angles.

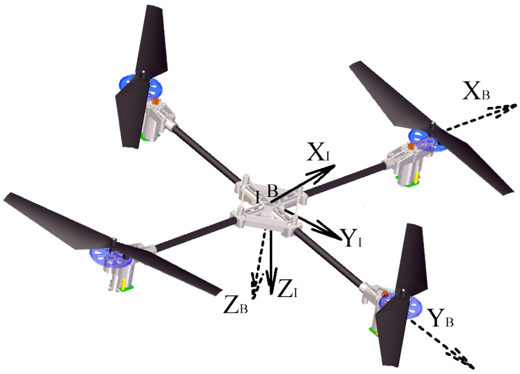



Figure 1. Reference frame in Quadrotor

Euler angles - Roll ($\phi$), Pitch ($\theta$) and Yaw ($\psi$) - are defined as reference frame $x$, $y$, $z$ rotations around $Z$, $y'$ and $x''$ in the amounts $\psi$, $\theta$ and $\phi$ respectively, as can be see in Fig. 2.

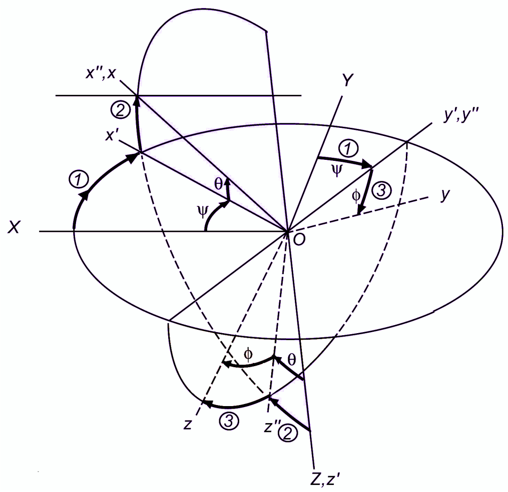



Figure 2. Definition of Roll, Pitch and Yaw angles

According with Newton-Euler equation the translational and rotational accelerations for the Quadrotor aerial vehicle can be written as in Eq. (1):

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\left[(\sin\phi\sin\psi + \cos\psi\sin\theta\cos\phi)\sum_{i=1}^{i=4} T_i - \sum_{i=1}^{i=4} H_{xi} - C_x \frac{1}{2} A_c \rho \dot{x}\,|\dot{x}|\right] \\ \frac{1}{m}\left[(-\cos\psi\sin\phi + \sin\psi\cos\phi\sin\theta)\sum_{i=1}^{i=4} T_i - \sum_{i=1}^{i=4} H_{yi} - C_y \frac{1}{2} A_c \rho \dot{y}\,|\dot{y}|\right] \\ \frac{1}{m}\left[mg - \cos\phi\cos\psi\sum_{i=1}^{i=4} T_i - C_z \frac{1}{2} A_c \rho \dot{z}\,|\dot{z}|\right] \\ \frac{1}{I_{xx}}\left[\dot{\theta}\dot{\psi}\,(I_{yy} - I_{zz}) + l_1\,(T_4 - T_2) + \sum_{i=1}^{i=4}(-1)^{i+1} R_{mxi} - l_2 \sum_{i=1}^{i=4} H_{yi} + J_r \dot{\theta}\Omega_r\right] \\ \frac{1}{I_{yy}}\left[\dot{\phi}\dot{\psi}\,(I_{zz} - I_{xx}) + l_1\,(T_1 - T_3) + \sum_{i=1}^{i=4}(-1)^{i+1} R_{myi} + l_2 \sum_{i=1}^{i=4} H_{xi} - J_r \dot{\phi}\Omega_r\right] \\ \frac{1}{I_{zz}}\left[\dot{\phi}\dot{\theta}\,(I_{xx} - I_{yy}) + l_1\left[(H_{x2} - H_{x4}) + (H_{y3} - H_{y1})\right] + \sum_{i=1}^{i=4}(-1)^{i} Q_i + J_r \dot{\Omega}_r\right] \end{bmatrix} \tag{1}$$

For other models see also (Bouabdallah, 2004a; Bouabdallah *et al.*, 2004b,c, 2005a,b; Castillo *et al.*, 2004; Holger, 2007; Dunfied *et al.*, 2004; Lai *et al.*, 2006; Pounds *et al.*, 2002,b; Altug *et al.*, 2003, 2002; Horn *et al.*, 2005; Wendel *et al.*, 2006; McKerrow, 2004).

Equation (1) can be rewritten in state variables as in the Eq. (2):

$$\begin{bmatrix} \dot{v}_x \\ \dot{x} \\ \dot{v}_y \\ \dot{y} \\ \dot{v}_z \\ \dot{z} \\ \dot{\omega}_\phi \\ \dot{\phi} \\ \dot{\omega}_\theta \\ \dot{\theta} \\ \dot{\omega}_\psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\left[\,(\sin\phi\sin\psi+\cos\psi\sin\theta\cos\phi)C_taU(1)+C_{Hx}aU(1)-C_xbv_x\,|v_x|\,\right] \\ v_x \\ \frac{1}{m}\left[\,(-\cos\psi\sin\phi+\sin\psi\cos\phi\sin\theta)C_taU(1)-C_{Hy}aU(1)-C_ybv_y\,|v_y|\,\right] \\ v_y \\ \frac{1}{m}\left[\,mg-\cos\phi\cos\psi C_taU(1)-C_zbv_z\,|v_z|\,\right] \\ v_z \\ \frac{1}{I_{xx}}\left[\,\omega_\theta\omega_\psi(I_{yy}-I_{zz})+C_tlaU(3)-C_{Hy}haU(1)+C_{Rx}RaU(2)+J_r\omega_\theta U(5)\,\right] \\ \omega_\phi \\ \frac{1}{I_{yy}}\left[\,\omega_\phi\omega_\psi(I_{zz}-I_{xx})+C_tlaU(4)+C_{Hx}haU(1)+C_{Rx}RaU(2)-J_r\omega_\phi U(3)\,\right] \\ \omega_\theta \\ \frac{1}{I_{zz}}\left[\,\omega_\phi\omega_\theta(I_{xx}-I_{yy})+C_qRaU(2)+C_{Hx}laU(3)+C_{Hy}laU(4)+J_r\dot{U}(5)\,\right] \\ \omega_\psi \end{bmatrix} \tag{2}$$

with $a=\rho AR^2$, $b=\frac{1}{2}A_c\rho$, $U(1)={\Omega_1}^2+{\Omega_2}^2+{\Omega_3}^2+{\Omega_4}^2$, $U(2)={\Omega_1}^2+{\Omega_3}^2-{\Omega_2}^2-{\Omega_4}^2$, $U(3)={\Omega_4}^2-{\Omega_2}^2$, $U(4)={\Omega_1}^2-{\Omega_3}^2$ y $U(5)=\Omega_1+\Omega_3-\Omega_2-\Omega_4$, where $\Omega_i$ are the angular velocities of the $i$-th propeller. The resistance, the thrust coefficients and others are showed by more detail in (Seddon, 1990; Bramwell, 2001; Peña, 2009a; Peña *et al.*, 2009b, 2010).

In Eq. (1) and Eq. (2) the model is non-linear and complex to design controls, thus it is valid to take some assumptions in order to simplify the mathematical model without big changes in vehicle dynamics. The first one is to suposse that the friction forces are system perturbations and extract them from the plant equations; the second one is to say that the roll and pitch angles are small enough, then the velocities are small too and the coefficients $C_{Hx}$, $C_{Hy}$, $C_{Rx}$, $C_{Ry}$ have near to 0 values.

$$\begin{bmatrix} \dot{v}_x \\ \dot{x} \\ \dot{v}_y \\ \dot{y} \\ \dot{v}_z \\ \dot{z} \\ \dot{\omega}_\phi \\ \dot{\phi} \\ \dot{\omega}_\theta \\ \dot{\theta} \\ \dot{\omega}_\psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\left(\sin\phi\sin\psi+\cos\psi\sin\theta\cos\phi\right)C_taU(1) \\ v_x \\ \frac{1}{m}\left(-\cos\psi\sin\phi+\sin\psi\cos\phi\sin\theta\right)C_taU(1) \\ v_y \\ \frac{1}{m}\left[\,mg-(\cos\phi\cos\psi)C_taU(1)\,\right] \\ v_z \\ \frac{1}{I_{xx}}\left[\,\omega_\theta\omega_\psi(I_{yy}-I_{zz})+C_tlaU(3)+J_r\omega_\theta U(5)\,\right] \\ \omega_\phi \\ \frac{1}{I_{yy}}\left[\,\omega_\phi\omega_\psi(I_{zz}-I_{xx})+C_tlaU(4)-J_r\omega_\phi U(3)\,\right] \\ \omega_\theta \\ \frac{1}{I_{zz}}\left[\omega_\phi\omega_\theta(I_{xx}-I_{yy})+C_qaRU(2)+J_r\dot{U}(5)\right] \\ \omega_\psi \end{bmatrix} \tag{3}$$

Equation (3) suggests that the rotation equations are independent from the translation equations, though the translational movement depends on the rotational movement, so the system can be divided in two singles models: the rotational model (attitude) and the translational model as is shown in Eq. (4), Eq. (5) and Fig. 3. In the matricial Eq. (3) the last six equations are dependent on the states $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$ and the inputs $U(2)$, $U(3)$, $U(4)$, $U(5)$, but they are independent from the translational positions, the translational velocities and the input $U(1)$. In the other side, the first six equations are dependent on $\phi$, $\theta$, $\psi$ and the input $U(1)$. This can be put in block diagrams like:

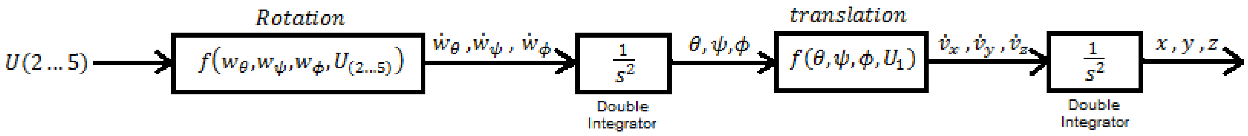



Figure 3. System divided in translation and rotation tasks

and put in state variables equations like:

$$\begin{bmatrix} \dot{v}_x \\ \dot{x} \\ \dot{v}_y \\ \dot{y} \\ \dot{v}_z \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\left(\sin\phi\sin\psi + \cos\psi\sin\theta\cos\phi\right) C_t a U(1) \\ v_x \\ \frac{1}{m}\left(-\cos\psi\sin\phi + \sin\psi\cos\phi\sin\theta\right) C_t a U(1) \\ v_y \\ \frac{1}{m}\left[ mg - (\cos\phi\cos\psi) C_t a U(1) \right] \\ v_z \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} \dot{\omega}_\phi \\ \dot{\phi} \\ \dot{\omega}_\theta \\ \dot{\theta} \\ \dot{\omega}_\psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{I_{xx}}\left[ \omega_\theta\omega_\psi(I_{yy} - I_{zz}) + C_t l a U(3) + J_r \omega_\theta U(5) \right] \\ \omega_\phi \\ \frac{1}{I_{yy}}\left[ \omega_\phi\omega_\psi(I_{zz} - I_{xx}) + C_t l a U(4) - J_r \omega_\phi U(3) \right] \\ \omega_\theta \\ \frac{1}{I_{zz}}\left[ \omega_\phi\omega_\theta(I_{xx} - I_{yy}) + C_q a R U(2) + J_r \dot{U}(5) \right] \\ \omega_\psi \end{bmatrix} \tag{5}$$

To control the translation is possible to control first the airship attitude. The linearization of the attitude will be realised in the equilibrium points as in Eq. (6). The equilibrium points are obtained by equaling the system to zero, thus:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{I_{xx}}\left[ \omega_\theta\omega_\psi(I_{yy} - I_{zz}) + C_t l a U(3) + J_r \omega_\theta U(5) \right] \\ \omega_\phi \\ \frac{1}{I_{yy}}\left[ \omega_\phi\omega_\psi(I_{zz} - I_{xx}) + C_t l a U(4) - J_r \omega_\phi U(3) \right] \\ \omega_\theta \\ \frac{1}{I_{zz}}\left[ \omega_\phi\omega_\theta(I_{xx} - I_{yy}) + C_q a R U(2) + J_r \dot{U}(5) \right] \\ \omega_\psi \end{bmatrix} \tag{6}$$

From previous system there is deduced that the attitude equilibrium points are independent from rotation angles $\phi$, $\theta$ and $\psi$. It is also observed that in order that this equality should be fulfilled, the angular velocities and the entrances must be a zero. Then when equaling this system to zero the equilibrium points are obtained, thus:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\left(\sin\phi\sin\psi + \cos\psi\sin\theta\cos\phi\right) C_t a U(1) \\ v_x \\ \frac{1}{m}\left(-\cos\psi\sin\phi + \sin\psi\cos\phi\sin\theta\right) C_t a U(1) \\ v_y \\ \frac{1}{m}\left[ mg - (\cos\phi\cos\psi) C_t a U(1) \right] \\ v_z \end{bmatrix} \tag{7}$$

In Eq. (7), $v_x$, $v_y$ and $v_z$ must be zero and in order that this is fulfilled, roll and pitch angles must be zero and the vehicle lift force must have the same magnitude and direction but opposed sense to that of the Quadrotor weight. Soon it is possible to be deduced that the multiple equilibrium points of the system are given by:

$$X_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi \end{bmatrix}^T \tag{8}$$

Any point of balance would mean "hover". The linear system in the equilibrium point is obtained with the help of symbolic Matlab:

$$\begin{bmatrix} \dot{v}_x \\ \dot{x} \\ \dot{v}_y \\ \dot{y} \\ \dot{v}_z \\ \dot{z} \\ \dot{\omega}_\phi \\ \dot{\phi} \\ \dot{\omega}_\theta \\ \dot{\theta} \\ \dot{\omega}_\psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{m} C_t a U(1) \cdot \theta \\ v_x \\ -\frac{1}{m} C_t a U(1) \cdot \phi \\ v_y \\ -\frac{1}{m} C_t a U(1) \\ v_z \\ \frac{1}{I_{xx}} C_t l a U(3) \\ \omega_\phi \\ \frac{1}{I_{yy}} C_t l a U(4) \\ \omega_\theta \\ \frac{1}{I_{zz}} C_q a R U(2) + J_r \dot{U}(5) \\ \omega_\psi \end{bmatrix} \tag{9}$$

## 3. LQR CONTROL WITH INTEGRAL EFFECT

### 3.1 LQR (Linear Quadratic Regulator)

LQR control (see (Ljung, 1994; Ogata, 1994; Astrom *et al.*, 2006)) is a method to find the optimum solution for a problem of minimization that assures the system stability in close-loop, in addition its calculation is easy. The most general problem that this method can solve is given by the equation of the dynamic system:

$$\dot{x}(t) = Ax(t) + Bu(t); \quad x(t_0) = x_0 \tag{10}$$

with $x(t) \in R^n$ and $u(t) \in R^m$,

$$z(t) = Cx(t) \tag{11}$$

with $z(t) \in R^p$. The Eq. (12) represent the quadratic cost function to minimize:

$$J = \frac{1}{2} \int_{t_0}^{t_f} \left[ z^T(t) R_{zz} z(t) + u^T(t) R_{uu} u(t) \right] dt + \frac{1}{2} x^T(t_f) P_{t_f} x(t_f) \tag{12}$$

$R_{zz} > 0$, $R_{uu} > 0$, $P_{t_f} \geq 0$, $A(t)$ is a continuous-time function, and $B(t)$, $C(t)$, $R_{zz}$ and $R_{uu}$ are continuous-time and bounded functions. Thus the general problem in LQR method is to find an input $u(t)$ in time domain between the initial and final given times. The optimum input is defined by the Eq. (13):

$$u_{op}(t) \;=\; -R_{uu}^{-1} B^T P(t) x(t) \;=\; -K(t) x(t) \tag{13}$$

where $P(t)$ in Eq. (14) is the solution to the Ricatti differential equation:

$$-\dot{P}(t) \;=\; AP(t) + P(t)A + C^T R_{zz} C - P(t) B R_{uu}^{-1} B^T P(t) \tag{14}$$

For linear time invariant systems, Eq. (14) reaches a value in stable state that is reduced to the Eq. (15):

$$A^T P + PA + R_{xx} - PBR^{-1} B^T P \;=\; 0 \tag{15}$$

named the Control Algebraic Ricatti Equation(CARE) and it is find the optimum value of $P$. The optimum input is defined in Eq. (16):

$$u(t) = -K_{ss} x(t) \tag{16}$$

The value $K_{ss}$ is easyly found by the Matlab control toolbox using the sintax: $K_{ss} = lqr(A, B, C^T R_{zz} C, R_{uu})$ (see (Ljung, 2006)). The diagram that describe the stabilizing control is shown in the Fig. 4. Here $A$, $B$ and $C$ are the matrix associated to the linearized system and $R_{zz}$ and $R_{uu}$ are weight matrix in order respectively to increase or to diminish the effect of the states and the entrances of individual form and are select for the designer in agreement with the required performance.

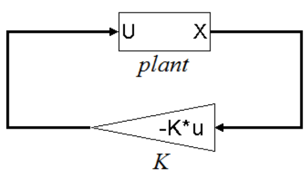



Figure 4. LQR control to stabilize the system

### 3.2 LQR with integral effect

Using the concept of the control above and some aditional mathematical treatment the airship can follow the references angles $\phi$,$\theta$ and $\psi$, thus with $y(t) = Cx(t)$ the necesary outputs and $r(t)$ the reference which is the aditional new states $x_I(t)$ herefore the equations states $\dot{x}_I(t)$ with its respectives costs in the cost function. The new system in state variables is defined by Eq. (17):

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(t) \tag{17}$$

With $\overline{x}(t) = [x(t)\ x_I(t)]^T$ the new cost function is defined in Eq. (18):

$$J = \int_0^\infty \left[ \overline{x}^T(t) R_{zz} \overline{x}(t) + u^T(t) R_{uu} u(t) \right] dt \tag{18}$$

therefore:

$$u(t) = -\begin{bmatrix} K & K_I \end{bmatrix} \begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix} = -\overline{K}\overline{x}(t) \tag{19}$$

Once the control has been designed, it must take the shape in the close-loop architecture shown in the diagram in Fig. 5:

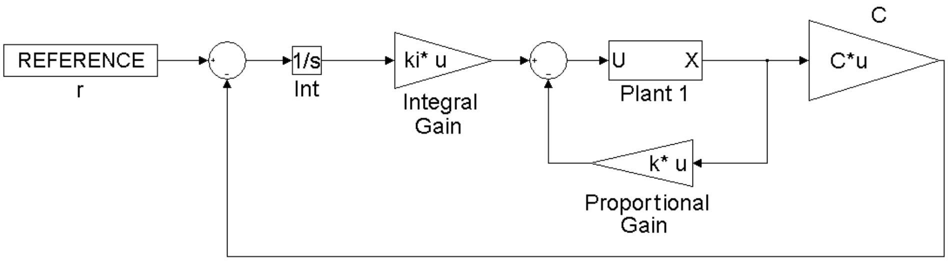



Figure 5. LQR control with integral effect following the reference

## 4. DESIGN OF LQR CONTROL AND SIMULATION

In this section, the values of the parameters are found for a real Quadrotor. A graphic model and the real model are placed in the linear model of the form $\dot{X}(t) = A_{LN}X(t) + B_{LN}U(t)$, with $Y(t) = C_{LN}X(t) + D_{LN}U(t)$, whose values for $A_{LN}$, $B_{LN}$, $C_{LN}$ and $D_{LN}$ are these:

$$A_{LN} = \begin{bmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix} \quad B_{LN} = \begin{bmatrix} 0 & 127.4725 & 0 & 0 \\ 0 & 0 & 127.4725 & 0 \\ 0 & 0 & 0 & 278.0868 \\ -7.5188 & 0 & 0 & 0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$$

$$C_{LN} = \begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix} \quad D_{LN} = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$$

With the previous matrices, the weight matrix $Q$ (values of weight for the most important states) and the matrix $R$ (values of weight for the amplitud signal control), the LQR problem find the solution with the $lqr$ command in control toolbox of Matlab, thus:

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And the constant matrix of control $k$ is calculated. Its value is:

$$k = \begin{bmatrix} 0.0000 & -0.0000 & 0.0000 & -10.2295 & 0.0000 & -0.0000 & 0.0000 & -17.4525 \\ 10.0136 & 0.0000 & 0.0000 & -0.0000 & 17.3283 & -0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 10.0136 & 0.0000 & 0.0000 & 0.0000 & 17.3283 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 10.0062 & -0.0000 & 0.0000 & -0.0000 & 17.3241 & -0.0000 \end{bmatrix}$$

$$k_i = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & -10.0000 \\ 10.0000 & -0.0000 & -0.0000 & -0.0000 \\ 0.0000 & 10.0000 & 0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 10.0000 & -0.0000 \end{bmatrix}$$

The control of the linear system is optimum for these matrices weights $Q$ and $R$, so the performance objective for the system is modified by changing these weights to check that the signal control is between the limits of the actuator. In the linear system, the dynamic of the four motors (actuators) and the friction forces are despised, while in the non-linear system this dinamics were not despised. For that it is necessary to compare both systems and to verify that the designed control works appropriately.

The complete architecture of the system including the control is shown in the diagram blocks of Fig. 6 which include the linear and non-linear systems:

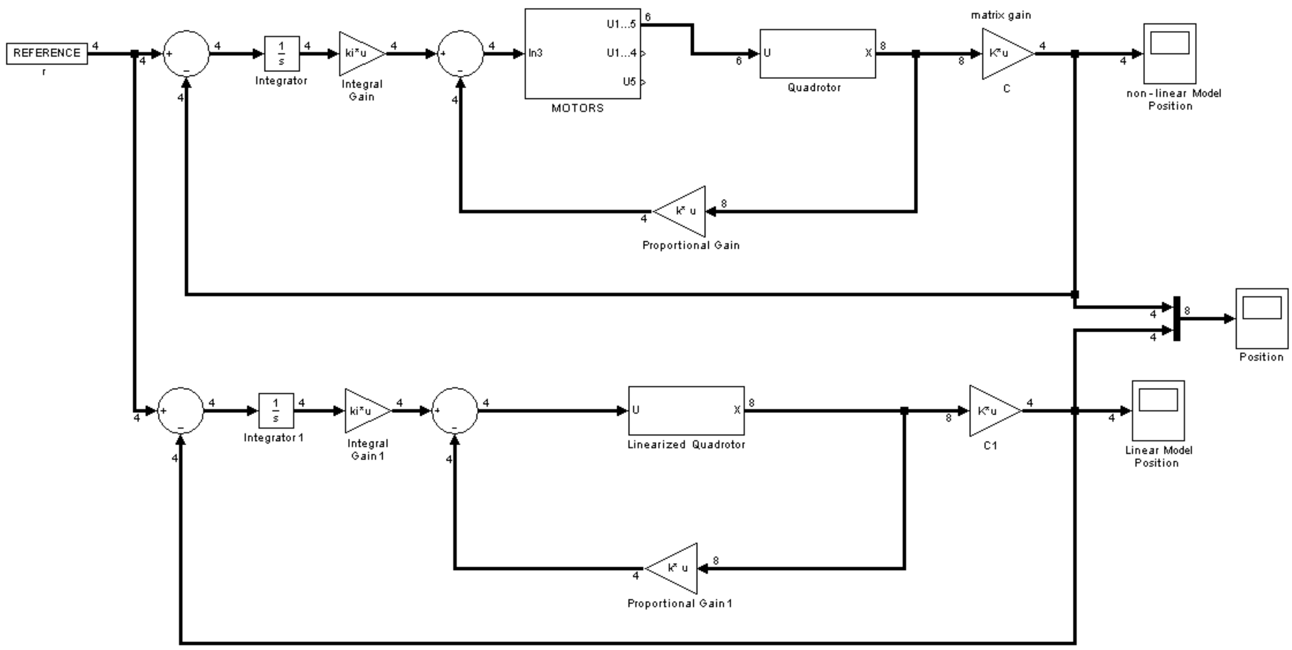
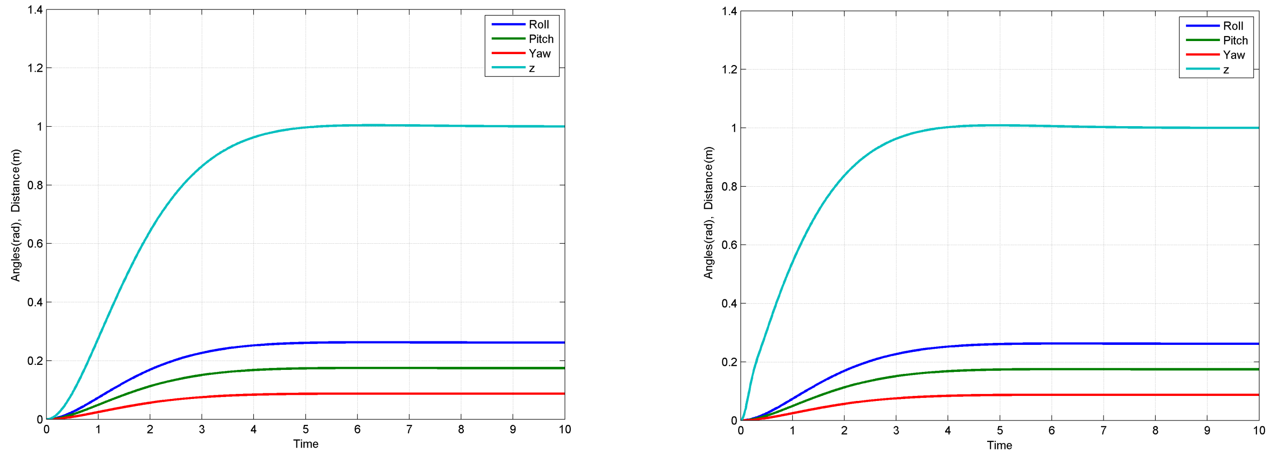



Figure 6. Block Diagram of the LQR control with integral effect

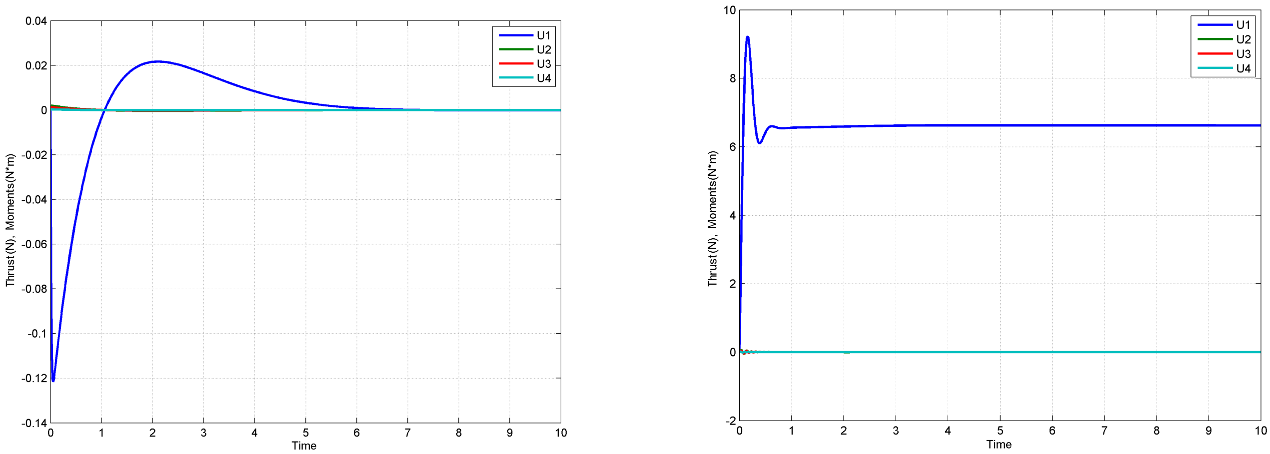
The following figures show the results of the simulations of the linearized and nonlinear systems for inputs of the step type. Figure 7 shows no significant differences between both systems when these are excited with step functions like signal reference, but this figure shows large times of stabilization in the references value because the control was designed for this especification.




(a) Linear system (b) Non-linear system

Figure 7. Systems responses to reference steps in 0.2618, 0.1745 and 0.087 radians (15, 10 and 5 degrees) in roll, pitch, yaw and 1 meter of altitude respectively

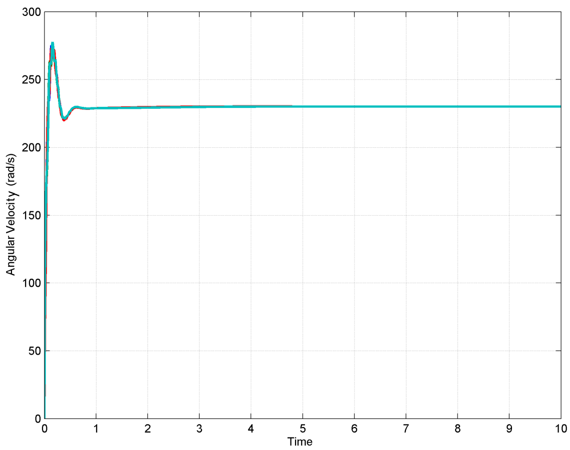
Figure 8 shows the system in the linear system accepting negative thrusts, but this is not realistic. The non linear system does not accept negative velocities and is that the differences between graphics. When the non-linear model is linearized, the term due to the gravity (Quadrotor's weight) disappears, then $U1$ only takes values while it reaches the reference height. In the non-linear model the entry $U1$ must offset the weight of the vehicle and take this one at the height of reference which means that exists a transitory response and a stationary non-null condition.



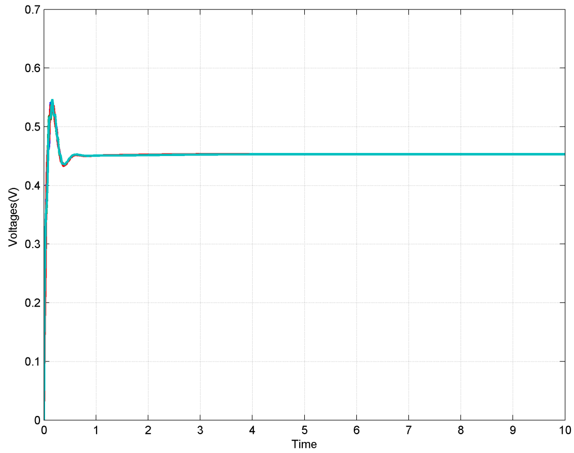

(a) Linear system (b) Non-linear system

Figure 8. Control signals (thrusts and moments)

Finally, Fig. 9 guarantees that it is a good design because the saturation value is respected.




(a) Angular velocity of the propellers




(b) Voltage to the power circuit

Figure 9. Control signal to the actuators

## 5. CONCLUSIONS

The modeling of any aircraft can approach a Newton-Euler model based on a frame of inertial reference, a second frame fixed to the aircraft and the third reference frame describes the rotation of the same one referred to the mobile frame previous; Suppositions that manage to diminish the complexity of the model and help to increase the understanding of the behavior of the object in flight. The forces and external moments depend on every aircraft and on multiple variables both on the aircraft itself and on the environment; nevertheless there are phenomena that affect more that others in certain conditions and in this case the sustentation forces and drags moments generated by the propellers are the most important since that the conditions are assumed to hover (flotation). The dynamics of the engines is faster that the dynamics of the aircraft by the which the dynamics can be assumed despicable.

If only there is had interest in controlling the attitude of the quadrotor, the system has infinite equilibrium points; but when the rotation and translation are controlled and the equilibrium point always is hover.

The system is unstable in opened loop and its linear approximate resultant model is an double integrator in the diagonal.

An important characteristic of the model is that the inputs of the system are functions of the square of the speeds of the engines; then, to include the dynamics of the engines and to keep in mind an real signal of control given in terms of the voltage, it is necessary to find the value of the angular speeds from a system of equations that relates the variables $U$ and the angular speeds to the square.

There was designed the control LQR with integral effect to obtain a small time of establishment and low overshoot without saturating the actuators. The maximum value of the signals of control was obtained of the specifications of the engines provided by the manufacturer.

The LQR control ensures stability and good performance at all operating points.

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## 7. RESPONSABILITY NOTICE

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