STRESS ANALYSIS AND FAILURE CRITERIA OF ADHESIVE BONDED SINGLE LAP JOINTS

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Abstract. *The demand for adhesive applications in structural joints has increased considerably over recent years. This growth is due to the benefits provided by adhesives, when compared to conventional joining methods, like rivets, bolts or welding. Therefore, there is a specific need for analysis and design tools that can provide physical insight and accurate results for bonded joints applications. Preliminary approach requires the stress distribution and a suitable failure criterion. Several analytical methods for the calculation of stress distributions in adhesively bonded joints are available in literature. In this paper was done a revision of the main analytical methods known in the literature, such as Volkersen, Goland & Reissner, Hart-Smith and Ojalvo & Eidinoff. Due to the complexity of including non-linearity of the materials, both adherends and adhesives are considered to be linear elastic in most of the analyses. Nevertheless, as the degree of complexity increases, the initial analytical problem must be solved numerically. For this reason, a study using the finite element method (FEM) was performed. Comparisons between the FEM analysis and closed-form models implemented were achieved showing great approximation. Finally, a revision of the main failure criteria found in literature was done, giving special emphasis on three special groups: (a) Maximum stress or strain criteria, (b) Critical stress or strain at a distance or over a zone and (c) Limit state criteria.*

Keywords: *Bonded joints, adhesives, analytical methods, FEM analysis, Single Lap Joints*

1. INTRODUCTION

The application of adhesives in bonded joints has been greatly increased over recent years. This growth is due to the benefits provided by adhesives, when compared to conventional joining methods, such as: more efficient load transfer, possibility to conform light weight structures, enhanced fatigue properties, improved corrosion resistance, smoother surfaces, among others. Several analytical methods for the calculation of stress distributions in adhesively bonded joints are available in literature. In this paper was done a revision of the main analytical methods and failure criteria known in the literature, such as Volkersen, Goland & Reissner, Hart-Smith and Ojalvo & Eidinoff. This paper also shows a comparison between the FEM analysis and closed-form models implemented. For the experimental validation, predicted strengths (by using the failure criterion previously analyzed) were compared with test data obtained by several tests performed according to the ASTM D1002 standard.

2. ANALYTICAL MODELS

2.1 VOLKERSEN

The first analytical method known in literature for the stress analysis of bonded joints was developed by (Volkersen, 1938). Volkersen method, also known as the shear-lag model, introduced the concept of differential shear. The bending effect caused by the eccentric load path is not considered. The adhesive shear stress distribution τ is given by:

$$\tau = \frac{P\omega}{2b} \cdot \frac{\cosh(\omega x)}{\sinh\left(\frac{\omega l}{2}\right)} + \left(\frac{t_t - t_b}{t_t + t_b}\right) \cdot \left(\frac{\omega l}{2}\right) \cdot \frac{\sinh(\omega x)}{\cosh\left(\frac{\omega l}{2}\right)} \tag{1}$$

where,

$$\omega = \sqrt{\frac{G_a}{E t_t t_a}\left(1 + \frac{t_t}{t_b}\right)}$$

The reciprocal of ω has units of length and is the characteristic shear-lag distance, a measure of how quickly the load is transferred from one adherend to the other. t_t is the top adherend thickness, t_b is the bottom adherend thickness, t_a is the adhesive thickness, b is the bonded area width, l is the bonded area length, E is the adherend modulus, G_a is the adhesive shear modulus and P is the force applied to the inner adherend. The origin of x is the middle of the overlap and is shown in Fig. 1.

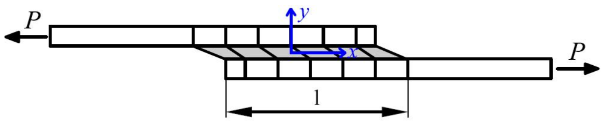

Figure 1. Volkersen model

Eq. (1) shows that for a joint with different adherends the adhesive stress is maximum (and thus failure most likely) at the overlap end where the loaded adherend is thinnest. This principle is shown in Fig. 2, for $t_t >> t_b$

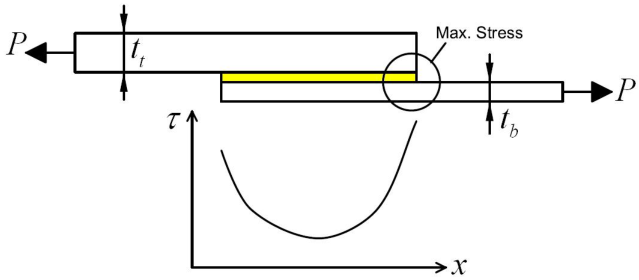

Figure 2. Principle of maximum stress for joints with different adherends

Further, the lowest adhesive stresses are obtained when the adherends are identical, $t_t = t_b = t$. Assuming that the joint is sufficiently long such that $\sinh(\omega l) = \cosh(\omega l)$, Eq. (1) will result in:

$$\tau = \frac{\omega P}{2} = \sqrt{\frac{G_a}{E t_t t_a}\left(1 + \frac{t_t}{t_b}\right)} \cdot \frac{P}{2} \tag{2}$$

This is an extremely useful formula which shows a number of very important features about the size of the peak adhesive stress in a single overlap joint:

- For long joints it is independent of the joint length
- It increases with increasing adhesive shear modulus
- It increases with decreasing adherend modulus and thickness and adhesive thickness

2.2 GOLAND & REISSNER

(Goland and Reissner, 1944) were the first to consider the effects due to rotation of the adherends, Fig. 3. They divided the problem into two parts: (a) determination of the loads at the edges of the joints, using the finite deflection theory of cylindrically bent plates and (b) determination of joints stresses due to the applied loads.

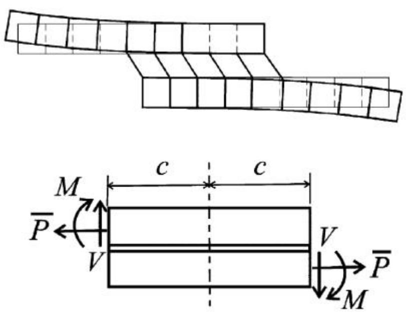

Figure 3. Goland & Reissner model

The adhesive shear stress distribution τ found by Goland & Reissner is given by:

$$\tau = -\frac{1}{8}\frac{\bar{P}}{c}\left\{\frac{\beta c}{t}(1+3k)\frac{\cosh\left(\frac{\beta c}{t}\frac{x}{c}\right)}{\sinh(\beta c/t)} + 3(1-k)\right\} \tag{3}$$

where, $\bar{P}$ is the applied tensile load per unit width, c is half of the overlap length, t is the adherend thickness, ν is Poisson's ratio and k is the bending moment factor:

$$k = \frac{\cosh(u_2 c)}{\cosh(u_2 c) + 2\sqrt{2}\sinh(u_2 c)},\ u_2 = \sqrt{\frac{3(1-\nu^2)}{2}}\frac{1}{t}\sqrt{\frac{\bar{P}}{tE}},\ \beta^2 = 8\frac{G_a}{E}\frac{t}{t_a}$$

The adhesive peel stress distribution σ is given by:

$$\sigma = \frac{1}{\Delta}\frac{\bar{P}t}{c^2}\left[A + B\right] \tag{4}$$

where:

$$A = \left(R_2\lambda^2\frac{k}{2} + \lambda k' \cosh(\lambda)\cos(\lambda)\right)\cosh\left(\frac{\lambda x}{c}\right)\cos\left(\frac{\lambda x}{c}\right),\ B = \left(R_1\lambda^2\frac{k}{2} + \lambda k' \sinh(\lambda)\sin(\lambda)\right)\sinh\left(\frac{\lambda x}{c}\right)\sin\left(\frac{\lambda x}{c}\right)$$

and $k' = \frac{kc}{t}\sqrt{3\left(1-\nu^2\right)\frac{\bar{P}}{tE}}$ is the transverse force factor, also:

$$\lambda = \gamma\frac{c}{t},\ \gamma^4 = 6\frac{E_a}{E}\frac{t}{t_a},\ \Delta = \frac{1}{2}\left(\sin(2\lambda) + \sinh(2\lambda)\right)$$

$$R_1 = \cosh(\lambda)\sin(\lambda) + \sinh(\lambda)\cos(\lambda),\ R_2 = -\cosh(\lambda)\sin(\lambda) + \sinh(\lambda)\cos(\lambda)$$

2.3 HART-SMITH

In contrast with (Volkersen, 1938) or (Goland and Reissner, 1944), (Hart-Smith, 1973) considered adhesive plasticity. In the report presented for the NASA they analyzed both, the single lap joint (SLJ) and the double lap joint (DLJ). For both analyses they combined elastic peel stress with plastic shear stresses. According to (Hart-Smith, 1973), the adhesive elastic shear stress distribution $\tau_(x)$ is given by:

$$\tau_{(x)} = A_2\cosh(2\lambda' x) + C_2 \tag{5}$$

where:

$$\lambda' = \sqrt{\left[\frac{1+3(1-\nu^2)}{4}\right]\frac{2G_a}{t_a E t}},\ A_2 = \frac{G_a}{t_a E t}\left[\bar{P} + \frac{6\left(1-\nu^2\right)M}{t}\right]\frac{1}{2\lambda'\sinh(2\lambda' c)},$$

$$C_2 = \frac{1}{2c}\left[\bar{P} - \frac{A_2}{\lambda'}\sinh(2\lambda' c)\right],\ M = \bar{P}\left(\frac{t+t_a}{2}\right)\frac{1}{1+\xi c+\left(\frac{\xi^2 c^2}{6}\right)},\ \xi^2 = \frac{\bar{P}}{D}$$

and D is the adherend bending stiffness given by: $D = \frac{Et^3}{12(1-\nu^2)}$.

Variables $\bar{P}$, G_a, t_a, E, E_a, ν, t, c has the same meaning as presented by (Volkersen, 1938) and (Goland and Reissner, 1944) models. The adhesive peel stress distribution $\sigma_{(x)}$ is given by:

$$\sigma_{(x)} = A\cosh(\chi x)\cos(\chi x) + B\sinh(\chi x)\sin(\chi x) \tag{6}$$

The shear plastic stress was modeled using a bi-linear elastic-perfectly plastic approximation. The overlap is divided into three regions, a central elastic region of length l and two outer plastic regions. Coordinates x and x' are defined as shown in Fig. 4.

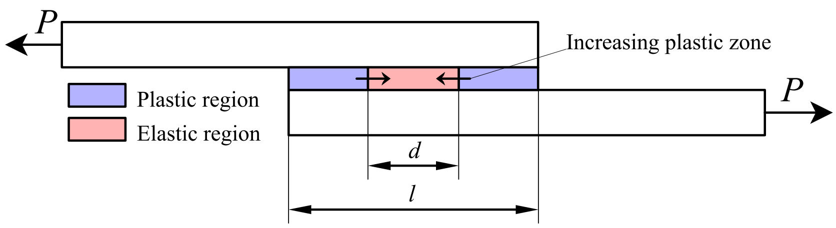

Figure 4. Regions considered by Hart-Smith

The problem is solved in the elastic region in terms of the shear stress according to:

$$\tau_{(x)} = A_2\cosh(2\lambda' x) + \tau_p(1-K) \tag{7}$$

And the shear strain in the plastic region according to:

$$\gamma_{(x')} = \gamma_e\left\{1 + 2K\left[\left(\lambda' x'\right)^2 + \lambda' x' \tanh(\lambda' d)\right]\right\} \tag{8}$$

where τ_p is the plastic adhesive shear stress and $A_2 = \frac{K\tau_p}{\cosh(\lambda' d)}$. K and d are solved by an iterative approach using the following equations:

$$\frac{\bar{P}}{l\tau_p}\left(\lambda' l\right) = 2\lambda'\left(\frac{l-d}{2}\right) + (1-K)\left(\lambda' d\right) + K\tanh\left(\lambda' d\right) \tag{9}$$

$$\left[1+3k\left(1-\nu^2\right)\left(1+\frac{t_a}{t}\right)\right]\frac{\bar{P}}{\tau_p}\lambda^2\left(\frac{l-d}{2}\right) = 2\left(\frac{\gamma_p}{\gamma_e}\right) + K\left[2\lambda'\left(\frac{l-d}{2}\right)\right]^2 \tag{10}$$

$$2\left(\frac{\gamma_p}{\gamma_e}\right) = K\left\{\left[2\lambda'\left(\frac{l-d}{2}\right) + \tanh\left(\lambda' d\right)\right]^2 - \tanh^2\left(\lambda' d\right)\right\} \tag{11}$$

where, γ_e and γ_p are the elastic and plastic adhesive shear strain respectively.

2.4 OJALVO & EIDINOFF

(Ojalvo and Eidinoff, 1978) model is based (Goland and Reissner, 1944) model. They modified some coefficients in the shear stress equations by adding new terms in the differential equation and considering new boundary conditions for bond peel stress calculation. Their leading work was the first in predicting the variation of shear stress through the bond thickness. The adhesive nondimensional shear stress τ^* distribution found by (Ojalvo and Eidinoff, 1978) is given by:

$$\tau^* = Acosh\left(\lambda\sqrt{2+6(1+\beta)^2}x^*\right) + B \tag{12}$$

where:

$$A = \frac{2\lambda\left(1+3(1+\beta)^2 k\right)}{\sqrt{2+6(1+\beta)^2}\sinh\left(\lambda\sqrt{2+6(1+\beta)^2}x^*\right)},\ B = 1 - \frac{A\sinh\left(\lambda\sqrt{2+6(1+\beta)^2}\right)}{\lambda\sqrt{2+6(1+\beta)^2}},\ \lambda^2 = \frac{G_a c^2}{E^* th} \text{ and } \beta = \frac{h}{t}$$

$E^* = E$ for adherends in plane stress and $\frac{E}{1-\nu^2}$ for adherends in plane strain. G_a, c, E, t are the same variables as defined previously and h is the adhesive thickness. k is the bending moment factor as seen in Hart-Smith model. The maximum nondimensional stress at the bond/adherend interfaces is given by:

$$\tau^{**} = \tau^* \pm \Delta\tau^* \tag{13}$$

where:

$$\Delta\tau^* = \frac{G_a h}{2E_a}\sigma^{*'}$$

The solution for the nondimensional peel stress $\sigma^{(*')}$ is given by:

$$\sigma^* = C\sinh\left(\alpha_1 x^*\right)\sin\left(\alpha_2 x^*\right) + D\cosh\left(\alpha_1 x^*\right)\cos\left(\alpha_2 x^*\right) \tag{14}$$

where:

$$\alpha_1{}^2 = \frac{3\beta\lambda^2}{2} + \frac{\rho}{2};\ \alpha_2{}^2 = -\frac{3\beta\lambda^2}{2} + \frac{\rho}{2} \text{ and } \rho^2 = \frac{24E_a c^4}{E^* ht^3}$$

Constants C and D are obtained upon substitution of the derivatives of Eq. (14) into Eq. (15) and Eq. (16):

$$\sigma^{*'''}\left(\pm 1\right) - 6\beta\lambda^2\sigma^{*'}\left(\pm 1\right) = \mp k\gamma\rho^2\left(1+\beta\right) \tag{15}$$

$$\sigma^{*''}\left(\pm 1\right) = k\gamma\rho^2\left(1+\beta\right) \tag{16}$$

where $\gamma = \frac{t}{2c}$.

All analysis was done in a nondimensional basis. Equivalence is given by: $\tau^* = \frac{\tau}{\bar{\tau}}$, $\sigma^* = \frac{\sigma}{\bar{\tau}}$, $x^* = \frac{x}{c}$, where $\bar{\tau} = \frac{\bar{P}}{2c}$.

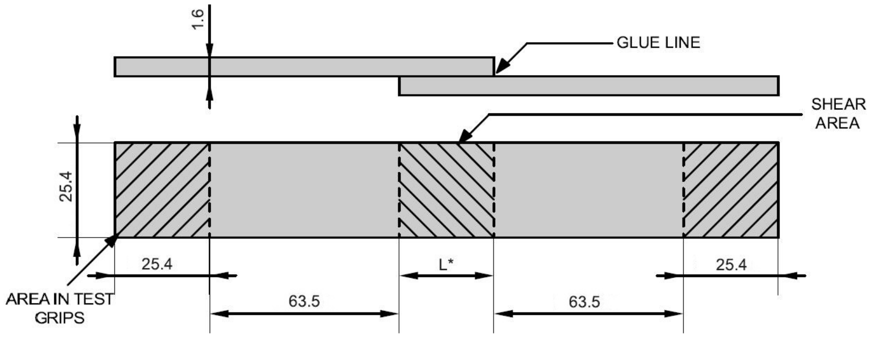

Figure 5. Form and dimensions of the test specimens

3. NUMERICAL COMPARISONS

Most of the analytical methods exposed above considered both adherends and adhesives as been perfectly elastic. Including the non-linearity of the material turns the problem complex, and as the degree of complexity increases, the initial analytical problem must be solved numerically. This paper shows a comparison between analytical models and numerical results for single lap joints. For the numerical validation was used the commercial software ABAQUS®. Geometry was in accordance with the ASTM D1002 standard (ASTMD1002, 2001), as shown in Fig. 5. For the analytical comparison was considered an applied load of 5000N. For the analysis was used the 2D Abaqus element CPS8R for the adhesive and the adherends. The CPS8R is an eight - node biquadratic plane stress quadrilateral element. In sum were used 400 elements in the adhesive and 960 elements in the adherend. Results of the FEM analysis for both shear and peel are shown in Figures 6 and 7. Results of the comparison between FEM and analytical methods are shown in Figures 8 and 9.

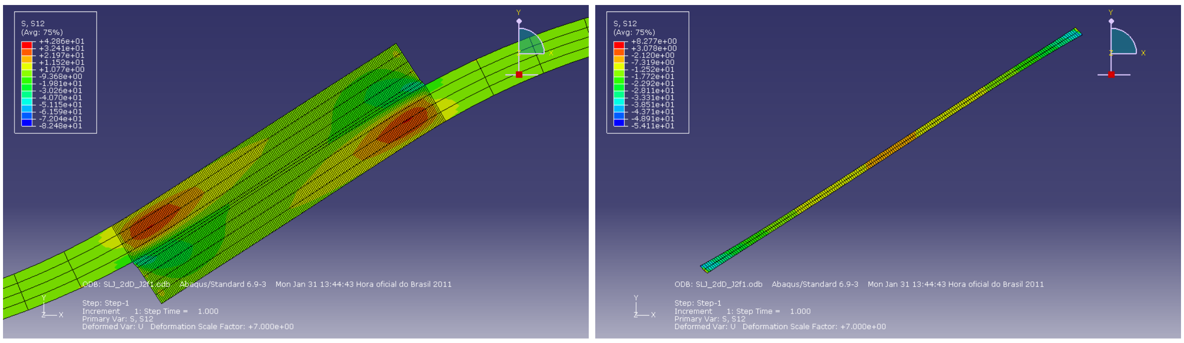

Figure 6. Shear abaqus FEM analysis

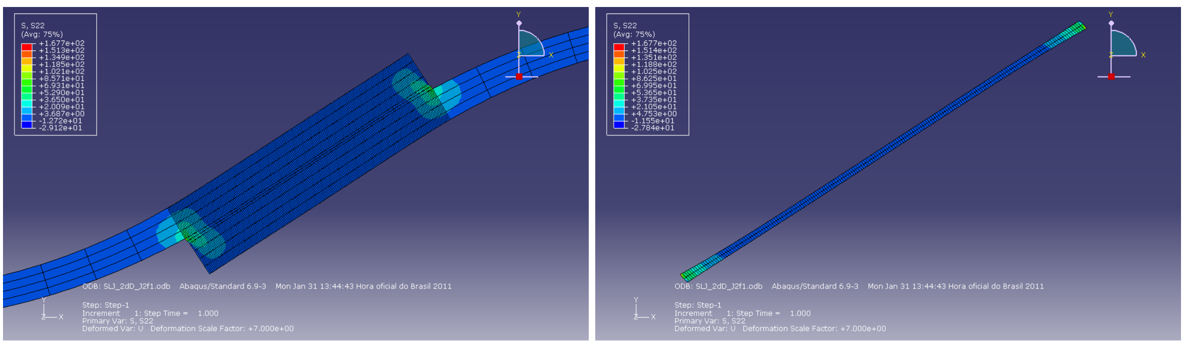

Figure 7. Peel abaqus FEM analysis

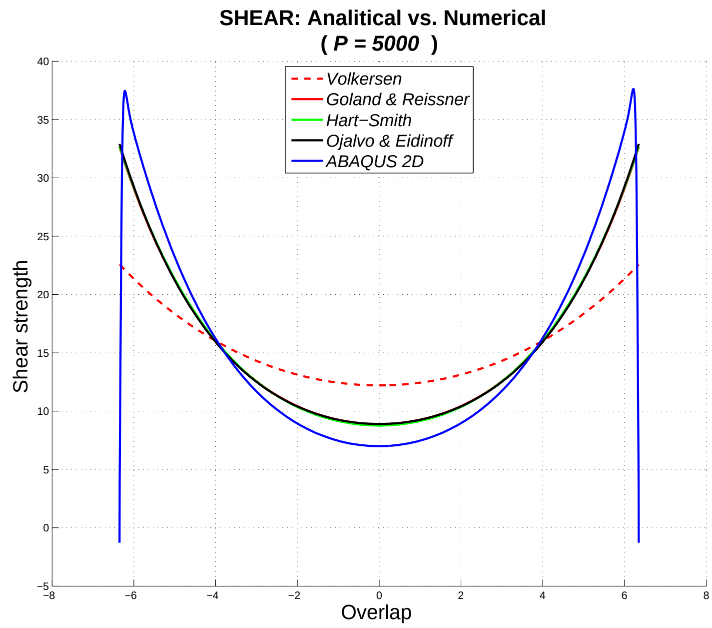

Figure 8. Shear comparison, analytical methods vs. abaqus FEM analysis

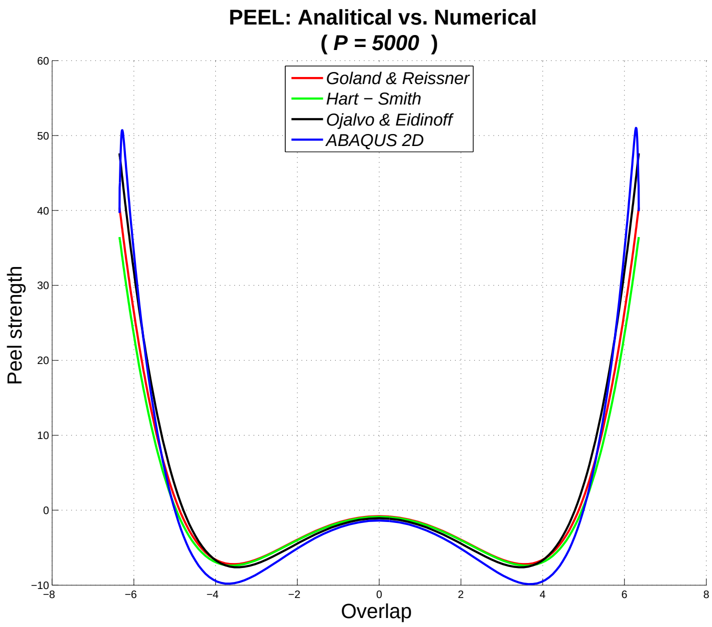

Figure 9. Shear comparison, analytical methods vs. abaqus FEM analysis

4. FAILURE CRITERIA

Alike analytical methods, literature for failure criteria is extensive. In this paper is shown a small review of the main failure criteria known in literature for the case of static loads. Generally, failure criteria can be grouped into the following categories: Maximum stress or strain criteria, critical stress or strain at a distance or over a zone, limit state criteria, fracture mechanics criteria and damage mechanics criteria, (Randolph and Clifford, 2004). As our analysis is static, just the first three criteria will be analyzed.

4.1 Maximum stress or strain criteria (Maximum value criteria)

This type of failure criteria is considered to be the biggest and the most intuitive category for bonded joints. Failure criteria have evolved naturally as analytical methods evolved too. For example, in the (Volkersen, 1938) model, the adhesive was assumed to deform just in shear. It becomes natural to consider the maximum shear stress as the failure criteria. This approach was also used by (Greenwood *et al.*, 1969). They used a (Goland and Reissner, 1944) analysis and found that the maximum shear stress occur at about 45° across the adhesive layer.

Likewise, various quantities have been used to predict bonded joint strength:

- Maximum peel stresses (Hart-Smith, 1973; Crocombe and Tatarek, 1985; Adams and Panes, 1994). Peel stresses should be minimized by design rather than used as a design limit on the strength of bonded joints (Hart-Smith, 1983).
- The maximum principal tensile stress and strain criteria were used by (Harris and Adams, 1984), predicting the strength of Single Lap Joints (SLJ's) to about 10% accuracy using elastoplastic finite element analysis. This same criteria was used on cleavage and compressive shear test by (Crocombe *et al.*, 1990).
- The maximum von Mises stress was used by (Ikegami *et al.*, 1989) as a failure criterion for bonded scarf joints. This criterion was found ineffective on double lap joints as described by (Charalambides *et al.*, 1997). This criteria is ineffective because the von Mises criteria does not considered the hydrostatic stress, which significantly affects the yield and deformation behavior of polymers.
- The maximum shear strain criteria were used by (Lee and Lee, 1992) and the effective uniaxial plastic strain were used by (Crocombe and Adams, 1982).

4.2 Critical stress or strain at a distance over a zone (Finite zone criteria)

This kind of criteria has been naturally adopted due to the high dependency of the mesh with the previous category (as a result of the inevitable requirement of dealing with singularities). (Zhao, 1991) used a weighted averaged maximum stress criteria, where the adhesive thickness is used as the distance over which the stresses are averaged and then compared to the adhesive yield strength. (Charalambides *et al.*, 1997) more lately showed that for double-lap joints, the location of the maximum stress occurs further down the fillet edge, outside the averaged zone.

(Clarke and Mcgregor, 1993) stated that for failure to occur, the maximum principal stress must exceed the ultimate tensile stress of the adhesive over a finite zone. They used three kind of geometry (single-lap, double strap and T-peel joints) to demonstrate that such zone was independent of the joint geometry. Lately (Charalambides *et al.*, 1997) found that predictions for Double Lap Joints (DLJ's) overestimated experimental data by approximately 68% for long overlaps.

Critical strain at a distance were used by (Towse *et al.*, 1997a) on DLJ's. They used a nonlinear analysis which included the effect of residual thermal stresses. The joint was demonstrated to fail when the strain near the singularity reached the adhesive ultimate strain. Same criteria were used by (Towse *et al.*, 1997b) in their study of a novel combjoint. For both studies, the applied distance had to be determined experimentally. Other authors (Trantina, 1972; Chow and Lu, 1992) also used experimental data to determine that characteristic distance. The problem with this approach is that becomes unviable to predict the strength of any joint that does not use same adherends and adhesives characterized previously.

4.3 Limit state criteria

This type of criteria was firstly introduced by (Crocombe, 1989). Based on a concept termed global yielding, which applies when a path of adhesive along the overlap region reaches the state in which it can sustain no further significant increase in applied load. For demonstrate the applicability of this concept three different geometries were tested experimentally, SLJ, DLJ and compressive shear. Non-linear finite element analyses were carried out and the load at which the adhesive layer was completely yielded was determined. (Zhao, 1991) used this approach in a CTBN (carboxyl terminated butadiene nitrile) toughened adhesive lap joint. (Schmit and Fraisse, 1992) include global yielding as one of the possible failure mechanisms in their analysis of stepped double lap joints. This criteria is only applicable to a limited range of adhesive joints. The majority of structural epoxy adhesives do not have sufficient ductility for the entire layer to yield prior to joint failure, (Crocombe and Kinloch, 1994).

4.4 Failure criteria considered for the analytical methods implemented

In this paper was done a revision of the main analytical methods known in the literature, such as (Volkersen, 1938; Goland and Reissner, 1944; Hart-Smith, 1973; Ojalvo and Eidinoff, 1978). Now, it becomes imperative to define a failure criteria for each method. This failure criteria will define a strength for each analytical method, then the comparison with experimental data will be finally possible. Failure criteria considered for each method is shown in Tab. 1.

Table 1. Failure criterion for each analytical method

Analytical method	Analysis type	Failure criterion
Volkersen	Elastic	$\tau > \tau_a$
Goland & Reissner	Elastic	$\sigma > \sigma_a$
Hart-Smith	Elastic	$\tau > \tau_a$ or $\sigma > \sigma_a$
	Elastic-perfectly plastic	GY
Ojalvo & Eidinoff	Elastic	$\tau > \tau_a$ or $\sigma > \sigma_a$

Where, τ_a is the shear strength of the adhesive, σ_a is the peel strength of the adhesive and GY is the global yielding failure criteria as described by (Crocombe, 1989).

5. EXPERIMENTAL RESULTS

Analytical methods implemented in the software were validated with experimental results. Failure criteria can be seen in Tab. 11. For each implemented model was obtained a failure load, then this value was compared with the ultimate load found experimentally, Fig. 10.

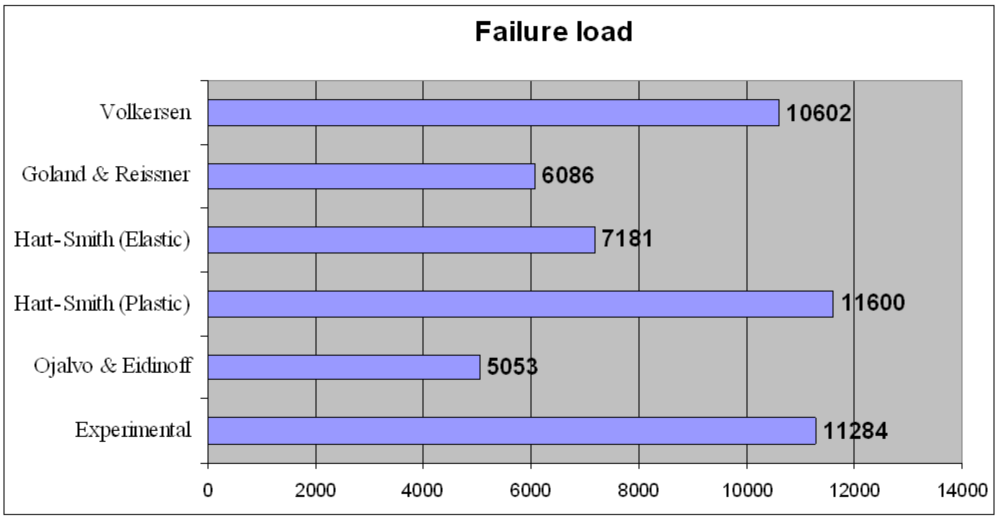

Figure 10. Failure load of each implemented analytical method

6. CONCLUSIONS

There exist many analytical methods and failure criteria available in literature for bonded joint analysis. However, in this paper were implemented only four analytical methods (Volkersen, 1938; Goland and Reissner, 1944; Hart-Smith, 1973; Ojalvo and Eidinoff, 1978). There was done a review of the main failure criteria available in literature for the analysis of static loads. Then, each analytical method was associated with the best failure criteria. Methods implemented were considered sufficient to achieve a consistent result, which would be useful for preliminary design purposes and as a consequence would reduce costly tests. For the validation of the analytical methods implemented were used experimental data in accordance with the ASTM D1002 standard and numerical results obtained by ABAQUS®. A small review and numerical validation for the stress distribution of Single Lap Joints was done also by (Fanton *et al.*, 2011). Their paper could be considered as complementary to the present work, even though, the main objective of such paper was the analysis of doublers structures.

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8. Responsibility notice

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