ESTIMATION OF WAITING TIME IN SWITCHABLE BATTERY STATION MODEL

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Abstract. *Electric vehicles use just energy of electrons and open the era of zero emission mobility. They use the batteries as sources of energy for driving. But the principal drawback of the battery is its big time for recharging. To resolve this problem the model of switchable battery station was suggested. Switchable battery station consists of m fast chargers (30 minutes for full battery recharging) and k full batteries at the beginning. Electric cars arrive to the station randomly according to Poisson flow with the known rate (for example 20 cars per hour). Each car gives its battery for recharging in one of the free chargers. If all the chargers are occupied, then the battery goes to the waiting queue for recharging. If there are full batteries at the station, then the car may take the full battery and does not wait. If there is not ready full battery at the moment of the arrival, then the car is forced to wait. The aim of the paper is to try to choose the parameters of the station (m - number of chargers and k - number of batteries at the beginning) to provide short average waiting time for the car. It turns out that the number of chargers may be chosen a little bit more than some critical number (which is equal to 10 for the rate of 20 cars per hour) and the number of batteries at the beginning may be chosen dependently on the requirements to the waiting time (more strict requirement implies more batteries at the beginning of the process). The method does a computer simulation of random car arriving and also the arbitrary distribution of the charging time with the condition that the average time of recharging is 30 minutes. A pseudo random number generator is used for this purpose. A time interval of 10 hours is chosen for simulation and 200 cars arrive to the station during this time. The algorithm for calculating the waiting time was proposed. The use of this algorithm permits to calculate the waiting time of each of 200 cars in the sequence. This gives one random experiment with the cars. But these results are not very stable. Therefore the 1000 tests are repeated and stable results are obtained for various number of chargers (from 11 to 29) and for various number of full batteries at the beginning (from 0 to 29). Average waiting times and their standard deviations are obtained for each car (total 200). It turns out that these waiting times vary a little bit at the beginning, but turn to be more stable with greater number of arriving cars. Because of this the average of these waiting times for the last 100 cars is chosen for obtaining the table which gives the waiting times values for various numbers of chargers m and various numbers of full batteries at the beginning k. This table permits to give recommendation on how to choose the number of chargers (a little bit more than critical, 13 or 14 for example) and the number of full batteries at the beginning (dependent on the waiting time requirement). The results of the paper can be used to optimize these parameters and to save financial resources.*

Keywords*: electric vehicles, switching batteries, waiting timeMonte Carlo Simulation, queue formation.*

1. INTRODUCTION

Our time is characterized by massive use of fossil fuels as a source of energy. This leads to depletion of the natural resources and of the oil in particular, and to the climate change, air and environment pollution in general. Oil is a unique fuel resource for our transport. Most of transport based on fuel, about 99%, has its origin from petroleum. Big countries that were in the past completely independent became dependent on several relatively small countries that have big fields of petroleum. Those are a good reason to look for alternative transportation means which are not based on petroleum. There are different alternatives to solve this problem (Electrification Roadmap, 2009). The most economically viable alternative technologies employ biofuels, natural gas, hydrogen and electrification.

One of the first alternatives is to use biofuels, ethanol may serve as a famous example. They represent already a significant amount of the fuel market of some countries in the world and most of them are produced domestically. Advanced biofuels are used in car, ship and aircraft industries. However, biofuel prices tend to track oil prices closely. This is because the market price is determined by the marginal price of the crude oil barrel in the global oil market. The world economy is still dependent on the oil price, and so does the chain of biofuel production and distribution. Therefore, when gasoline price rises or falls, the same happens proportionally to ethanol. Consequently, when the price of gasoline falls below the marginal price of producing ethanol, also the production of ethanol declines.

A second alternative is the use of natural gas. Consumption of natural gas emits about 30% less CO_2, than oil, and 45% less than coal on an energy equivalent basis. However, using natural gas in transportation would require a significant expansion of distribution and refueling infrastructure. Electrification would also require infrastructural upgrades, but of a very different and significantly less substantial nature. If the electricity produced from 28 cubic meters of natural gas burned in a current power generation plant were used to fuel an electric vehicle, it would provide enough energy to travel 735 kilometers. The same 28 cubic meters burned in a natural gas vehicle would only provide enough energy to move 360 kilometers.

A third alternative is the hydrogen fuel cell. At some point in the future the progress in fuel cell technology and fall of the cost of fuel cells will allow for hydrogen vehicles to be a successor or supplement to battery-powered electric vehicles (Kelly *et al.*, 2010). Commercialization of hydrogen-fueled vehicles, however, faces several obstacles that are far more significant than those facing battery-powered vehicles. First, the cost of hydrogen fuel cells is currently in excess of the cost of a comparable battery cell. Second, reliance on hydrogen would require the construction of an entirely new infrastructure to distribute it to consumers. At the same time there is no clear ability to manufacture sufficient quantities of hydrogen to fuel the automotive fleet. And perhaps the largest obstacle to the development of the hydrogen-fueled light-duty fleet is the fact the hydrogen itself is much more expansive than electricity, and likely always will be.

Electric vehicles get attention as most perspective alternative for transportation. Instead of Internal Combustion Engine they have electric motor and use batteries as a source of energy. But the batteries have one essential shortage because they need a relatively long time to recharge, so other than fast recharging battery technology, efficient recharging systems are also required (Solero, 2001). The most recent electric car designs have their own charger that allow for recharging the batteries from 0 to 100% in approximately 18 hours. Home chargers allow that in 8 hours. Fast chargers do this in half an hour, however they are also more expensive (Wang *et al.*, 2004 and Wang *et al.*, 2005). Conventional recharging systems coupled to the vehicle are not as fast as the refueling process in a gasoline station for regular cars. Therefore, a new business model in which the battery and the vehicle are separated has to be established. Switch Battery Stations in which the car owners leave their batteries for recharging and take full batteries in exchange are suggested to increase competitiveness of the electric car technology (Nansai*et al.*, 2001).

Several works in the literature deals with different aspects of the electric car technology (Solero, 2001; Wang *et al.*, 2004; Wang *et al.*, 2005; Egan *et al.*, 2007 and Pellegrino *et al.*, 2010) but few have explored the development of methodologies to design the |charging infrastructure (Nansai *et al.*, 2001). Design variables such as the number of batteries, chargers, flux of cars and waiting time are important characteristics to define electric cars as a competitive alternative to other commonly employed technologies. Therefore, the main goal of this work is to develop a consistent and reliable mathematical methodology to design switch battery stations taking into account flux of cars and waiting time as basic characteristics. This work intends to design switch battery stations with waiting time similar or even smaller to that of other fuel supply stations to show the potential of use of electric cars.

The mathematical methodology, here introduced, was firstly studied by Yudai and Osamu (2009) under the assumption that recharge times are exponentially distributed, which permitted to apply the queuing theory (Adan and Resing, 2002). The work of Yudai and Osamu (2009) described the probability distribution of system states and found the functional dependence of the loss probability when there is no possibility to exchange the batteries because the stock of full batteries is empty. They explored characteristics such as the stock size of full batteries for a fixed number of chargers and the number of chargers for a fixed battery stock. They also determined the functional dependence between a recharge time and a reasonable stock size of batteries.

The work is subdivided in different items, the first one refers to this text of introduction where many aspects of the system of propulsion of cars and in particular the electric cars are discussed. The second item presents the concepts behind the switch battery station model, and the third item introduces a mathematical algorithm based on the waiting time characteristic. The fourth item discuss the possibility to account for random vehicles arrivals and charging time distribution, and the fifth item shows the results of some test cases used to illustrate the methodology developed. The sixth and last item presents the main conclusions and some suggestions for future work.

2.SWITCH BATTERY STATION MODEL

Suppose that the arrival of vehicles to the station occurs according to the Poisson flow with parameter λ. It means that for each moment t the probability that the vehicle arrives in the time interval $(t,t+\Delta t)$ is equal to $\lambda\Delta t+o(\Delta t)$ and does not depend on t chosen from the real axis. Suppose also that the time for recharging of the battery from 0 to 100% of its capacity is 30 minutes or half an hour. Suppose that at the beginning of the process there are k full batteries. The vehicle arrives and gives its battery for recharging. If there is a free charger, the battery goes directly for recharging. If there is not free charger, then it goes to the queue to wait. If there are full batteries at this moment, then the vehicle takes the full battery and leaves the station. If there are not full batteries at this moment, then the vehicle has to wait in queue for a swap.

Consider one example of service when the vehicles arrive at specific moments. Suppose that there are 10 chargers and 10 batteries at the beginning. Suppose also that the vehicles arrive each 3 minutes. The first vehicle arrives, gives its battery for recharging, takes a full battery and leaves the station. The second vehicle arrives and also swaps its battery. This situation continues until the arrival of the vehicle number 11. Now, there are not full batteries, but the first battery is finishing, at this moment, its recharging and can be used by the vehicle number 11 for swap. Further vehicle number 12 arrives and uses the battery of the second vehicle which will be ready at this moment for the swap. Thus, this process continues without stopping and without any wait.

3. WAITING TIME CALCULATION ALGORITHM

Suppose that the first vehicle arrives at the moment t_1, the second vehicle arrives at the moment t_2, and so on, the vehicle number$n$ arrives at the moment t_n. Suppose also that the first battery has recharging time τ_1, the second battery has recharging time τ_2, and so on, the battery number n has recharging time τ_n. At the beginning, all the chargers are not occupied. Therefore, the first battery can get recharged at the first charger, the second battery can get recharged at the second charger, and so on, the battery number m can get recharged at the last charger number m. The first battery arrives at the moment t_1 and can start its recharging and finish it at the moment $g_1^{(1)} = t_1 + \tau_1$, the second battery arrives at the moment t_2 and can start its recharging and finish it at the moment $g_2^{(1)} = t_2 + \tau_2$,…, the battery number m arrives at the moment t_m and can start its recharging and finish it at the moment $g_m^{(1)} = t_m + \tau_m$. Now, we can find index i_1 such that

$$g_{i_1}^{(1)} = \min_{i=1,\dots,m} \left(g_i^{(1)}\right). \quad (1)$$

This defines the moment of the first battery at the exit

$$f_1 = g_{i_1}^{(1)}. \quad (2)$$

Now, the vehicle number $m+1$ arrives at the moment t_{m+1} and brings its battery for recharging. This battery can get recharged at the charger number i_1, which is liberated at the moment f_1. Therefore, the initial moment of this new recharging is

$$y^{(1)} = \max(t_{m+1}, f_1). \quad (3)$$

The final moment of the recharge of this battery is

$$z^{(1)} = y^{(1)} + \tau_{m+1}. \quad (4)$$

Now, we have to change the final moment of recharging for the charger i_1.

$$g_{i_1}^{(2)} = z^{(1)} \text{and } g_i^{(2)} = g_i^{(1)} \text{if } i \neq i_1. \quad (5)$$

We can find index i_2, that leads to the minimum

$$g_{i_2}^{(2)} = \min_{i=1,\dots,m} \left(g_i^{(2)}\right). \quad (6)$$

This defines the moment of the second battery at the exit

$$f_2 = g_{i_2}^{(2)}. \quad (7)$$

Now,the vehicle number $m+2$ arrives at the moment t_{m+2} and brings its battery for recharging. This battery can get recharged at the charger number i_2, which is liberated at the moment f_2. Therefore, the initial moment for a new recharge is

$$y^{(2)} = \max(t_{m+2}, f_2). \quad (8)$$

Therefore, the final moment for recharging of this battery is

$$z^{(2)} = y^{(2)} + \tau_{m+2}. \quad (9)$$

Now, we have a change in the recharging final moment for the charger number i_2.

$$g_{i_2}^{(3)} = z^{(2)} \text{and } g_i^{(3)} = g_i^{(2)} \text{if } i \neq i_2. \quad (10)$$

We can find index i_3 such that

$$g_{i_3}^{(3)} = \min_{i=1,\dots,m} \left(g_i^{(3)}\right). \quad (11)$$

This defines the moment of the third battery at the exit

$$f_3 = g_{i_3}^{(3)}. \quad (12)$$

Continuing at this manner, we can obtain all these moments. They form an increasing sequence

$$f_1 \le f_2 \le \dots \le f_n. \quad (13)$$

Now, we are going to calculate the waiting times for a given number of batteries at the beginning. Let k be the number of batteries at the beginning of the process. Define waiting times of the vehicles that arriving as

$$w_1, \dots, w_n. \quad (14)$$

The first k vehicles do not need to wait, because they can swap their batteries with the full batteries at the beginning

$$w_1 = w_2 = \dots = w_k = 0. \quad (15)$$

Now, the vehicle number $k+1$ arrives at the moment t_{k+1}. If it arrives before the first battery at the exit ($t_{k+1} \leq f_1$), then it should wait until moment $f_1$in order to swap this battery. In opposite case it does not wait. Therefore, waiting time here is

$$w_{k+1} = \max(0, f_1 - t_{k+1}). \quad (16)$$

Now, the vehicle of general n arrives at the moment t_n. If it arrives before the $(n-k)$ batteries at the exit ($t_n \leq f_{n-k}$), then it should wait until the moment f_{n-k} in order to swap its battery. In the opposite case it does not wait. Therefore, waiting time here is

$$w_n = \max(0, f_{n-k} - t_n). \quad (17)$$

4. GENERALIZATION OF THE ALGORITHM FOR TWO DIFFERENT BATTERY TYPES

Let again $t_1 \leq t_2 \leq \cdots \leq t_n$ be the arrival moments and $\tau_1, \tau_2, \dots, \tau_n$be the recharging times of the corresponding batteries. Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be the types of batteries ($\alpha_i = 1{,}2$ - two types). The first m batteries go to the free chargers with termination times $g_1^{(1)} = t_1 + \tau_1, g_2^{(1)} = t_2 + \tau_2, \dots, g_m^{(1)} = t_m + \tau_m$. These batteries have types $\beta_1^{(1)} = \alpha_1, \beta_2^{(1)} = \alpha_2, \dots, \beta_m^{(1)} = \alpha_m$, respectively. Now, denote by $f_1 \leq f_2 \leq \cdots \leq f_n$ for the first, the second and then^{th} battery at the exit. This order can be different from the order of arrivals. Denote, also, by $f_{1,1} \leq f_{2,1} \leq \cdots$ and $f_{1,2} \leq f_{2,2} \leq \cdots$ corresponding moments for the batteries of type 1and 2, respectively. Now, we can subsequently find all these numbers. The vehicle $m+1$ arrives at the moment t_{m+1} and gives its battery for recharging. It will use the charger that finishes its recharge first. To find this charger we define index i_1, such that $g_{i_1}^{(1)} = \min_i\left(g_i^{(1)}\right)$. If $\beta_{i_1}^{(1)} = 1$, then the type of the battery that finishes first is 1 andwe have $f_1 = f_{1,1} = g_{i_1}^{(1)}$. If $\beta_{i_1}^{(1)} = 2$, then the type of the battery that finishes first is 2 and we have $f_1 = f_{1,2} = g_{i_1}^{(1)}$. Now, the battery of the vehicle $m+1$can start its recharge with charger i_1 at the moment $y^{(1)} = \max\ (t_{m+1}, f_1)$ and finish it at the moment $z^{(1)} = y^{(1)} + \tau_{m+1}$. We put now $g_{i_1}^{(2)} = z^{(1)}, \beta_{i_1}^{(2)} = \alpha_{m+1}$ and $g_i^{(2)} = g_i^{(1)}, \beta_i^{(2)} = \beta_i^{(1)}$if $i \neq i_1$. Now, the vehicle $m+2$ arrives at the moment t_{m+2}. To define the number of the next released charger, we can find indexi_2 such that $g_{i_2}^{(2)} = \min_i\left(g_i^{(2)}\right)$. If $\beta_{i_2}^{(2)} = 1$, then the type of battery that finishes second is 1 and we have $f_2 = g_{i_2}^{(2)}$. If this battery is also the second battery of the type 1, then $f_{2,1} = f_2$. If it is only the first battery of the type 1, then $f_{1,1} = f_2$. If $\beta_{i_2}^{(2)} = 2$, then the type of the battery that finishes second is 2 and we have $f_2 = g_{i_2}^{(2)}$. If this battery is also the second battery of the type 2, then $f_{2,2} = f_2$. If it is only the first battery of the type 2, then $f_{1,2} = f_2$. Now, the battery of the vehicle $m+2$can start its recharging with charger i_2 at the moment $y^{(2)} = \max\ (t_{m+2}, f_2)$ and finish it at $z^{(2)} = y^{(2)} + \tau_{m+2}$. We define, now,$g_{i_2}^{(3)} = z^{(2)}, \beta_{i_2}^{(3)} = \alpha_{m+2}$ and $g_i^{(3)} = g_i^{(2)}, \beta_i^{(3)} = \beta_i^{(2)}$if $i \neq i_2$. By the same way we can find all the numbers $f_{1,1} \leq f_{2,1} \leq \cdots \leq f_{n,1}$and $f_{1,2} \leq f_{2,2} \leq \cdots \leq f_{n,2}$. Denote, now, by $r_{i,1}$ and $r_{i,2}$ the numbers of vehicles of the types 1 and 2 respectively after i^{th} arrival. Then, we can define them subsequently. If $\alpha_1 = 1$, then $r_{1,1} = 1, r_{1,2} = 0$. If $\alpha_1 = 2$, then $r_{1,1} = 0, r_{1,2} = 1$. Now,if $\alpha_2 = 1$, then $r_{2,1} = r_{1,1} + 1, r_{2,2} = r_{1,2}$. If $\alpha_2 = 2$, then $r_{2,1} = r_{1,1}, r_{2,2} = r_{1,2} + 1$. By the same way, we can obtain all $r_{i,1}$and $r_{i,2}$. Let, now,k_1 and k_2 be the numbers of batteries of type 1 and 2 respectively at the beginning of the process. Denote by $w_1, w_2, \dots, w_n$waiting times of the vehicles. Now we define them. Consider first case $\alpha_i = 1$. If $r_{i,1} \leq k_1$, then $w_i = 0$. If $r_{i,1} > k_1$, then $w_i = \max\ \left(0, f_{r_{i,1}-k_1,1} - t_i\right)$. Consider now the case $\alpha_i = 2$. If $r_{i,2} \leq k_2$, then $w_i = 0$. If $r_{i,2} > k_2$, then $w_i = \max\ \left(0, f_{r_{i,2}-k_2,2} - t_i\right)$.

5. SIMULATION OF RANDOM ARRIVALS OF THE VEHICLES AND CHARGING TIME DISTRIBUTION

We have used a pseudo random number generator to simulate 200 arrivals in a time interval of 10 hours. The ordering of these arrival times and corresponding vehicles leads us to the Poisson flow. There could be several different cases: the recharging time is a constant and equal to half an hour; probability of a given recharging time is proportional to the time itself; or an arbitrary recharging time distribution with an average of half an hour. As a result of 1000 simulations we have found the means of the waiting times and their standard deviations. After this, we took the second part of the series (last 100 vehicles of a total of 200) and found the average values (the means and standard deviations). The tables of the waiting times for different number of chargers and different number of batteries at the beginning were obtained. It was also done for different rates of arrival. These tables demonstrate explicitly how a waiting time changes with the number of chargers and the number of batteries. There exists some critical number of chargers for a given rate of recharging and a given rate of arrival such that taking a number of chargers less or equal to the critical number may lead to the growth of the queue. After some limit, which is a little bit greater than the critical number, an increase of the number of chargers does not affect significantly the waiting time. The increase of the number of batteries at the beginning affects essentially how rapid decrease the waiting times. It is not necessary to bring additional batteries during the process. The batteries at the beginning will supply the whole process, without the need for new batteries. The number of chargers may be chosen as a little bit greater than the critical number and the number of batteries may be chosen according to the tables and requirement for waiting time.

6. RESULTS

Several simulations of the same queue formation process were carried out in order to obtain statistically representative results. In this sense, our simulation is a Monte Carlo procedure (Kroese and Taimre, 2011). It was supposed that the unit of time is one hour and that the average time of charging is 0.5 hour (μ=2 batteries per hour). For simplicity we present graphics only for the constant recharging time.

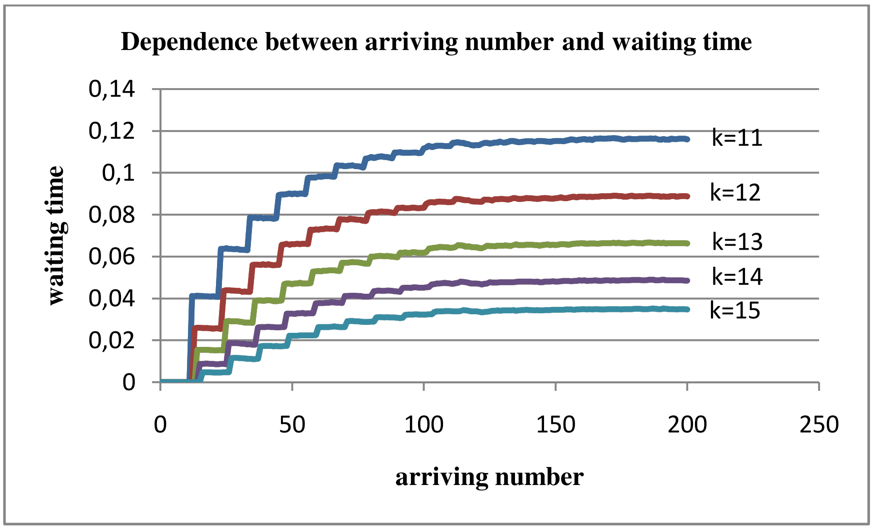

Figure 1.Test case for an arrival rate λ of 20 vehicles per hour, a number of chargers m=11 and a number of batteries at the beginning k=11, 12, 13, 14, 15.

The critical number of chargers (when the arrival rate is equal to the recharge rate) here is 10. The waiting time at the beginning of the process is zero. We can easily see that the mean waiting time (after 50000 tests) is jumping at some definite points. Each jump is followed by a horizontal level which is reaching the next jump. After a sufficiently large number of iterations (which leads to a stable mean) it is possible to explain the jumps of waiting time as a consequence of the initial condition of the system. At each curve of Fig. 1, a number of vehicles, equal to the number of batteries at the beginning of the process, does not need to wait, because there are full batteries available. The behavior, in a general case (arbitrary distribution of recharging time), is slightly different. In this case, the horizontal levels here are not expressed so clearly. When the arrival number exceeds a hundred of vehicles per unit of time, we observe the stabilization of the waiting time. Here also we can see clearly that the waiting time is decreasing with the number of batteries growth from 11 to 15.

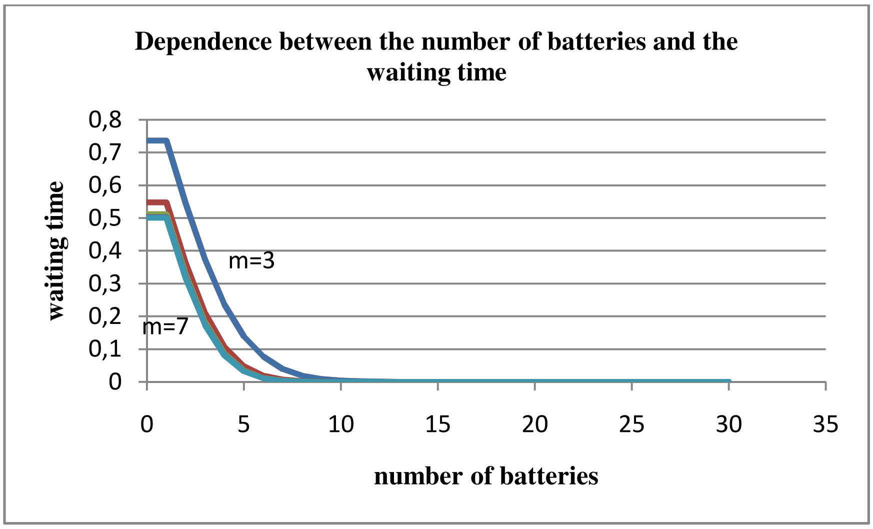

Figure 2.Test case for an arrival rate λ of 5 vehicles per hour and a number of chargers m = 3, 4, 5, 6, 7.

Figure 2 shows the behavior of the waiting time as a function of the number of batteries, for several different numbers of charges. We choose an arrival rate of 5 vehicles per hour. At this rate, the critical number of charges is 2.5 (not an integer). These 5 curves demonstrate how the waiting time is changing with the number of batteries at the beginning of the process for the different numbers of chargers m = 3, 4, 5, 6, 7. We can see here that the curves for m = 4, 5, 6, 7 are *quasi*-coincident, which demonstrates that the number of charges is not significantly affecting the waiting time. For 3 chargers (m = 3) and for the number of 5 batteries at the beginning, we have approximately 10 minutes (0.17 hour) average waiting time (after 50000 tests). For the number of chargers m = 4, 5, 6, 7 and for the same number of batteries we have less than 2.4 minutes (less than 0.04 hour) for the average waiting time.

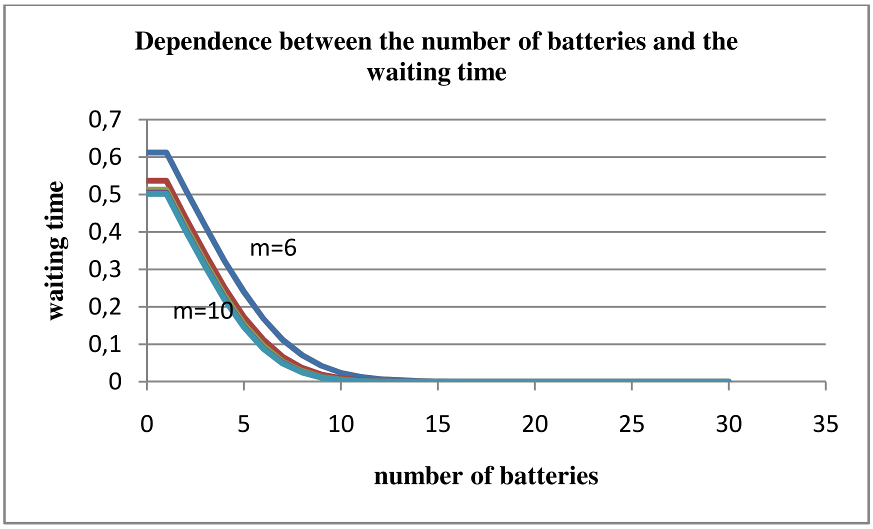

Figure 3.Test case for an arrival rate λ of 10 vehicles per hour and a number of chargers m = 6, 7, 8, 9, 10.

In the Fig. 3, the arrival rate is equal to 10 and the critical number of chargers is 5. These 5 curves demonstrate how the waiting time is changing with the number of batteries at the beginning of the process for the different numbers of chargers m = 6, 7, 8, 9, 10. We can see here that the graphs for m = 7, 8, 9, 10 practically coincide. For 6 chargers and for the number of 5 batteries we have approximately 15 minutes (0.25 hour) average waiting time. For the number of chargers m = 7, 8, 9, 10 and for the same number of batteries, we have approximately 10 minutes for the average waiting time. The shape of the curves in the Fig 2 and Fig. 3 is qualitatively the same. In contrast, we notice an important increase of the performance of the station. In the first case, Fig. 2, 6 batteries are required to provide a waiting time of 6 minutes, for 3 chargers, at an arrival rate of 5 vehicles per hour. If the arrival rate would be of 10 vehicles per hour, two stations should be necessary to keep the waiting time equal 6 minutes. In other words, 12 batteries and 6 chargers are necessary to maintain the waiting time at this level. In the case of Fig. 3, where the station is two times higher, only 7 batteries and 6 chargers are necessary to reach a waiting time of 6 minutes. This represents an economy of 5 batteries or a relative gain of 41%.

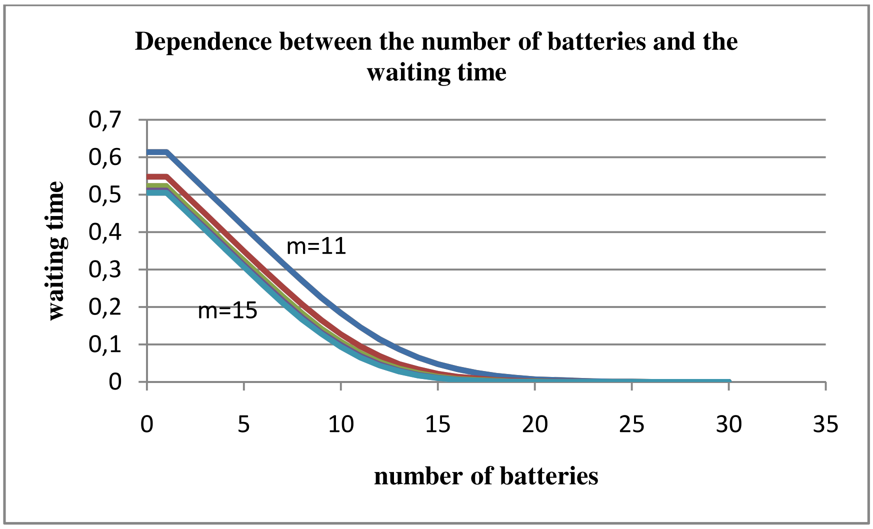

Figure 4.Test case for an arrival rate λ of 20 vehicles per hour and a number of chargers m = 11, 12, 13, 14, 15.

In the Fig. 4, the arrival rate is equal to 20 and the critical number of chargers here is 10. Comparing results of Fig. 4 with Fig. 3, similarly to what was done for Fig. 3 and Fig. 2, we found that to keep the waiting time equal to 6 minutes, at an arriving rate of 20 vehicles per hour, a single station with 12 chargers would need 10 batteries while two stations of 6 chargers would need 14 batteries. In this case, the gain is 28%, which is less than in the previous situation. Actually, for greater stations (in the sense of the number of chargers), the relative gain obtained with the integration of stations, progressively decrease.

To take into account the deviations in each test we give also the curves with standard deviation as a function of the number of batteries. Each one of the following figures may provide subsequently more reliable estimation compared to the above figures, for the individual waiting time. In the Fig. 5, we show curves for mean plus standard deviation waiting time, as a function of the number of batteries. It means that 84.14% of the vehicles that arrives in the station would wait less than the value defined by the curve. In the figures 6 and 7, the curves correspond to mean plus two (97.73% of confidence level) and three (99.87% of confidence level) standard deviation waiting time, respectively.

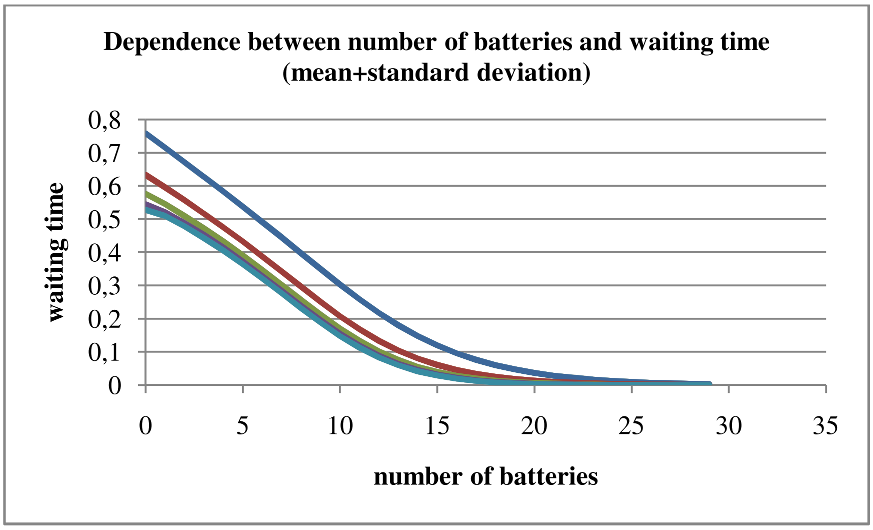

Figure 5.Test case for an arrival rate λ of 20 vehicles per hour and a number of chargers m = 11, 12, 13, 14, 15.

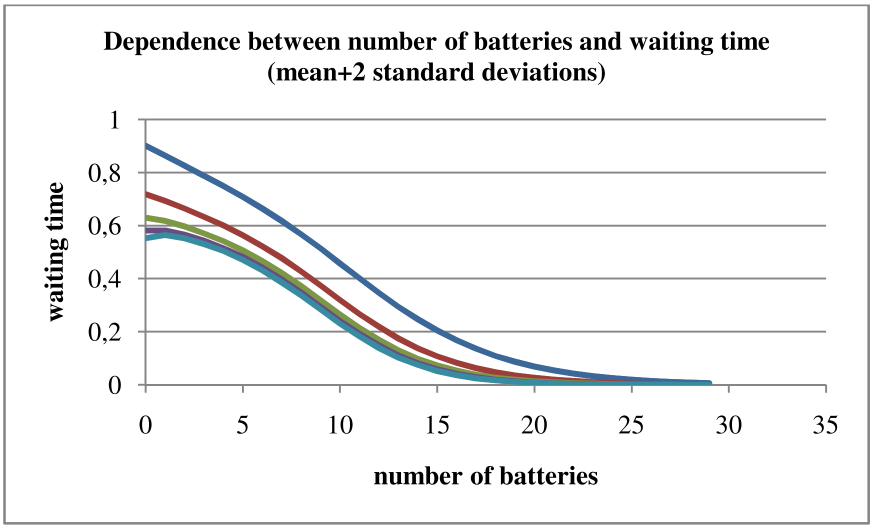

Figure 6.Test case for an arrival rate of 20 vehicles per hour and a number of chargers m = 11,12,13,14, 15.

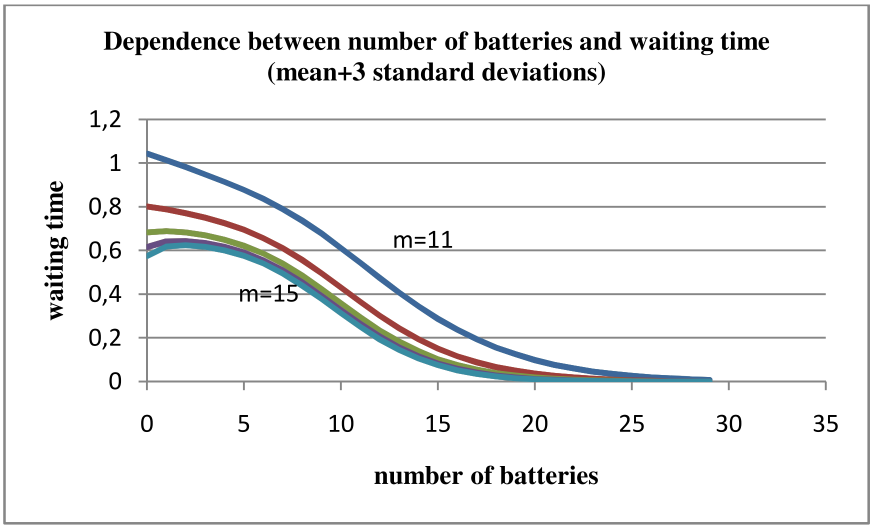

Figure 7.Test case for an arrival rate of 20 vehicles per hour and a number of chargers m = 11, 12, 13, 14, 15.

We have also studied the dependence of the waiting time as a function of the number of batteries for different service time rates. In the Fig. 8, we can see that the waiting time is inversely proportional to the service rate. To reach the waiting time of 0.1 hour (6 minutes) we need approximately 80 batteries at the beginning for the rate of 0.25 batteries per hour, 43 batteries for the rate of 0.5, 28 batteries for the rate of 0.75 and 25 batteries for the rate of 1.0. It means that the number of batteries at the beginning of the process decrease proportionally to the recharging rate of the charger. We can compare here two chargers: the regular charger with a rate of 0.125 batteries per hour and the fast charger with a rate of 2.0 batteries per hour. The cost of the installation of the first one is approximately US$2000 and of the second one is $45000. To serve the arrival rate of 20 vehicles per hour we need at least 160 regular chargers which cost US$320000. The corresponding number for the fast chargers is 10 which cost US$450000. So, the cost of the 10 fast chargers is greater than the cost of the 160 regular chargers. But to reach a good waiting time we need approximately 160 batteries at the beginning for the case of regular chargers and just 10 batteries for the case of fast chargers. The cost of the battery pack for the Nissan Leaf is approximately $9000. The cost of 160 batteries is $1440000 and the cost of 10 batteries is just $90000. The total cost of 160 regular chargers and 160 batteries is $1760000. The total cost of 10 fast chargers and 10 batteries is $540000. It means that the case of the regular chargers is more than three times costly than the case of fast chargers.

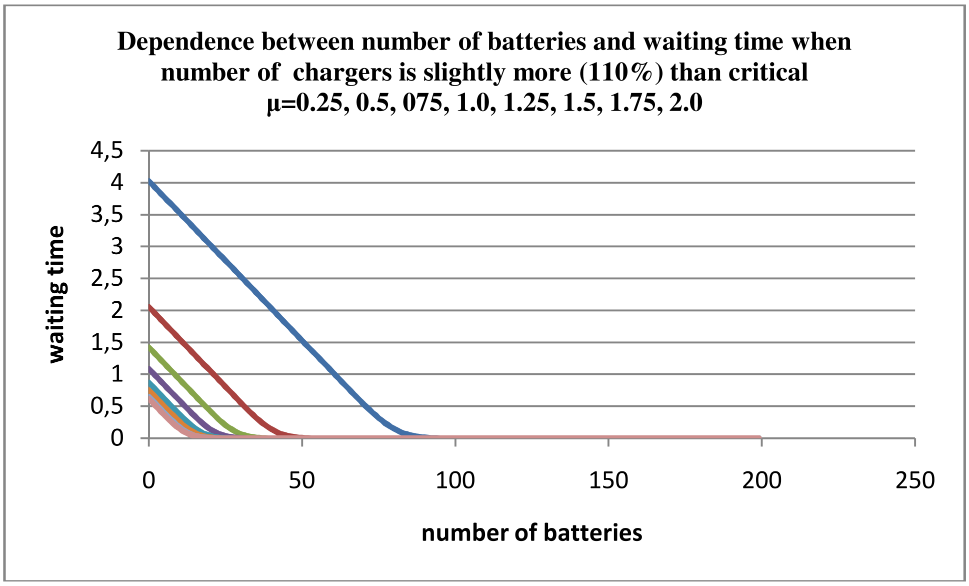

Figure 8.Test case for an arrival rate of 20 vehicles per hour and a recharge rate μ = 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0.

7. CONCLUSION

In the present work we have studied the queue formation process in switchable battery model for electric cars. A direct Monte Carlo methodology was developed where the main parameters are the number of chargers m, the number of batteries k, the arrival rate λ and service rate of each charger μ. We attempted to describe the behavior of the waiting time as the function of these parameters.

In each case there is a definite critical number of chargers. If we take the number of chargers less or equal to this critical value, than the queue will grow up and we cannot obtain a stable situation. But if we take the number of chargers a little bit more than critical, then the waiting time will be defined mostly by the number of batteries at the beginning. We obtained the tables with two entries: the number of chargers and the number of batteries. Each intersection of these entries gives the corresponding waiting time. These tables permit to find the number of chargers and the number of batteries in such a manner that the waiting time would be less than some small value. We do not suppose that the recharging time of the battery is exponentially distributed as in the paper of Yudai and Osamu, 2009. These authors use this assumption to apply the classical approach of the theory of queues. But we use just the computer simulation.

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