FREEZING TIME PREDICTION FOR REGULAR THREE-DIMENSIONAL SHAPED FOOD USING THE ENTHALPY MODEL

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Abstract. *In this study the original heat transfer equation was reformulated using an enthalpy model in order to obtain accurate numerical results and enhance the computational speed of the program. Thermo-physical properties used in the numerical model were evaluated by the equations suggested in literature, as well as the thermo-physical properties of the pure components. A three dimensional geometry with parallelepiped shape was used to describe the beet shape. The predicted temperature history of beets in freezing process is validated with a set of actual experimental data. The numerical predictions agreed with the experimental time-temperature curves during freezing of beets (maximum absolute error < 5ºC). The computational code was applied to determine the required processing times for different operating conditions with minimum computational efforts. So it could be used to give a guideline for freezing experiments of products with similar characteristics.*

Keywords*: freezing, beet, heat transfer, enthalpy method, difference finite method*

1. INTRODUCTION

General foods, for example, beets can be submitted either whole or sliced to several processes in order to obtain longer durability, for example low (refrigeration or freezing) or high (sterilization, pasteurization, dehydration, etc.) temperature thermal processes. The process of freezing is generally regarded as being superior to canning or dehydratation when judged on the basis of retention of sensory attributes and nutritive properties (Fennema, 1977).

Besides freezing is an important operation in food preservation, particularly, because it involves millions of tons of food per year (Pierce, 2002). It is known that the freezing of food effectively reduces the activity of microorganisms and enzymes, thus retarding deterioration. In addition, crystallization of water reduces the amount of liquid water in food items and inhibits microbial growth (Heldman, 1975). Thus freezing process has attracted the attention of many researchers with continuous interest in improving and simplifying prediction methods of food freezing and thawing times (Delgado and Sun, 2001).

In order for food freezing operations to be cost-effective, it is necessary to optimally design the refrigeration equipment to fit the specific requirements of the particular freezing application. The design of such refrigeration equipment requires estimation of the freezing times of foods, as well as the corresponding refrigeration loads.

Models which have been proposed in the past for the prediction of freezing times have been reviewed by numerous researchers (Ramaswamy and Tung, 1984; Hung, 1990; Delgado and Sun, 2001). Due to the difficulty of deriving analytical freezing time estimation methods, numerous semi-analytical/empirical food freezing time estimation methods have been developed which make use of various simplifying assumptions (Becker and Fricke, 1999a, 1999b, 1999c). The majority of empirical models are modifications and extensions of the Plank equation (1941). López-Leiva and Hallstrom (2003) published a thorough survey on the Plank equation, its modifications and extensions. The Plank equation only predicts the freezing time for a food at its freezing point, sub-cooling is not considered (Ilicali and Icier, 2010).

All of these semi-analytical/empirical freezing time estimation methods require knowledge of the surface heat transfer coefficient for the freezing process. The empirical nature of the surface heat transfer coefficient can introduce significant error in the freezing time calculation (Becker and Fricke, 2004; Fricke and Becker, 2002; 2004). For predicting of freezing times, the evolution and distribution temperatures in the whole dominium of the food during freezing must be known. To this end, in this study an enthalpy numerical model using the finite difference method (Smith, 1985; Allen *et al*., 1988), an established formulation for numerical predictions of the transient temperature in heat conduction problems such as freezing, is applied to predict freezing times of beets.

The purpose of the present study is to develop a finite difference code that simulates the freezing process of a parallelepiped shaped food using an enthalpy transformation method (helps to avoid inaccuracies and/or divergence of the numerical method, caused by the latent heat peak release at the phase transition), to apply the numerical model during freezing of beets considering a tri-dimensional geometry to validate the numerical solutions comparing the temperature predictions with experimental data during the freezing process of beets, to predict processing times for different operating industrial conditions; different external fluid temperature and surface heat transfer coefficients.

2. MATHEMATICAL MODEL

The heat transfer through product called the thermal conduction process is induced because of the temperature gradient in the frozen food. For the sake of simplicity, we assume that the food temperature distribution and physical property distribution are central symmetric in the freezing food, and there is no mass transfer and evaporation during the freezing process although there are moisture phase change and corresponding heat release.

Detailed modeling of heat transfer in freezing or thawing of foods leads to strongly nonlinear differential balances, due to the rapid variation of the thermal properties with temperature in the freezing range (Sanz *et al.*, 1987). Generally, the balance to be solved is (Eq. (1)):

$$\frac{\partial H}{\partial t} = \vec{\nabla}(k\vec{\nabla}T), \tag{1}$$

where the Laplacian ∇ can have one, two or three components, depending on whether the heat transfer is one-dimensional (plate, infinite cylinder or sphere) or multidimensional (finite cylinder, parallelepiped or irregular shapes), t (s) is the time, k (W/m^{o}C) is the thermal conductivity, and H (J/m^3) is the volumetric specific enthalpy obtained by performing a change in variables such as $H = \int_{T^*}^{T} \rho c_p dT$, where T^*(^{o}C) is the reference temperature which corresponds to the zero value of H, ρ (kg/m^3) is the apparent density, c_p (J/kg^{o}C) is the apparent specific heat capacity and T (^{o}C) is the temperature (Fikiin, 1996; Mannapperuma and Singh, 1988).

The present work considers three-dimensional geometry in rectangular coordinates, as shown in Figure 1. The initial condition refers to the distribution of variable to be calculated at the beginning of unsteady heat transfer process, which could be constant, or, a function of the spatial coordinates. In this paper the initial temperature is uniform at the beginning of food freezing, namely, everywhere the temperature is T_0, so the initial conditions are expressed as follows:

$$T(x, y, z, t) = T_0;\ (t = 0,\ 0 \le x \le \mathrm{A};\ 0 \le y \le \mathrm{B};\ 0 \le z \le \mathrm{C}) \tag{2}$$

where A, B and C is the face length in directions x, y, and z.

We assume that there is no radiation and evaporation and the convective condition is considered on the surface,

$$\left. -k\frac{\partial T}{\partial n}\right) = h(T_s - T_a)\ ;\ t \ge 0;\ x = 0,\ x = \mathrm{A},\ y = 0,\ y = \mathrm{B} \text{ and } z = \mathrm{C} \tag{3}$$

$$\left. -k\frac{\partial T}{\partial n}\right) = h_c(T_s - T_a)\ ;\ t \ge 0;\ z = 0 \tag{4}$$

where h and h_c (W/m^{2o}C) are coefficients of convective heat transfer nevertheless different, T_s is the surface temperature of food, and T_a is the temperature of cooling medium, namely, the freezing air.

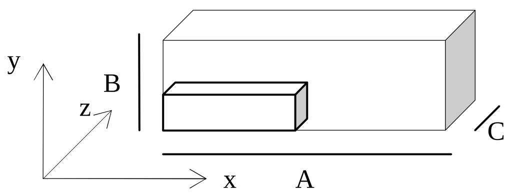


Figure 1. Computational domain.

3. MATERIALS AND METHODS

3.1 Finite Difference Simulation

Equation (1) together with boundary and initial conditions Eqs. (2)–(4) forms a system of coupled non-linear partial differential equations. This kind of system has no analytical solution. Therefore, it has to be solved numerically. The numerical solution to the partial difference equations is a set of numerical representations expressing the distribution of specific undetermined variables in the field of definition, and the discretization is to set up a group of algebraic equation for the variables to be determined in the field of definition.

The implicit method is used in this paper, whose accuracy is superior to that of the standard explicit schemes (Smith, 1985; Allen *et al.*, 1988). In order to solve the above equations, each partial derivative is replaced with the corresponding difference expression; and the range of variation of x, y and z is divided into A, B, and C points ($1 \le i \le$ A; $1 \le j \le$ B; $1 \le k \le$ C), respectively, with constant spacing Δx = a/(A-1), Δy = b/(B-1) and Δz = c/(C-1). Likewise, the time variation is divided into constant time intervals Δt. The time, therefore, is calculated as $t = n\Delta t$, ($0 \le n$), with no upper bounds (open interval). Thus, $T_{i,j,k,n}$ implies the temperature evaluated at the position $x = (i - 1)\Delta x$, $y = (j - 1)\Delta y$, $z = (k - 1)\Delta z$ and at the time $t = n\Delta t$.

The equation discretization by finite difference method is presented in this paper only to central volume, as illustrated in Fig. 2, where W, E, S, N, B and F are the center nodal points while w, e, s, n, b and f are placed in the faces of the control volume in x, y and z directions, respectively.

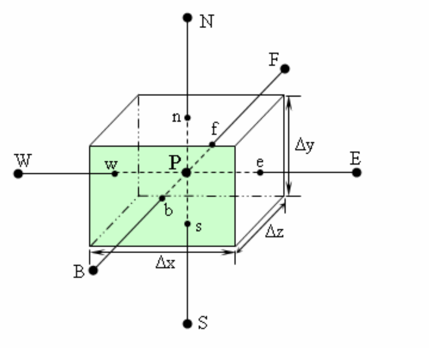


Figure 2. Three-dimensional control volume.

The volumetric enthalpy in the center of the control volume centered in P is calculated by:

$$H_P^{t+\Delta t} = \left[ke\left(\frac{T_E - T_P}{\Delta x^2}\right) - kw\left(\frac{T_P - T_W}{\Delta x^2}\right) + kn\left(\frac{T_N - T_P}{\Delta y^2}\right) - ks\left(\frac{T_P - T_S}{\Delta y^2}\right) + kf\left(\frac{T_F - T_P}{\Delta z^2}\right) - kb\left(\frac{T_P - T_B}{\Delta z^2}\right)\right]\Delta t + H_P^t \quad (5)$$

3.2 Thermo-physical Properties

The specific heat, thermal conductivity and density of the beet (between -40°C and 20ºC) were considered dependent with temperature. The typical composition of the *Beta vulgaris* beets considered to estimate the thermal properties were: 87.58% moisture content, 6.76% carbohydrates, 0.17% fat, 1.61% protein and 1.08% ash, as given by USDA Nutrient Database for Standard Reference (2001). The moisture contents (%) of the *Beta vulgaris* beets used in the present work were verified experimentally by drying triplicate samples in an oven at 80ºC until reaching constant weight.

The models proposed by Choi and Okos (1986) were implemented to estimate the thermal properties as a function of temperature and composition of the foodstuff. The thermal conductivity was obtained as mean between parallel and serial models, given respectively by:

$$k(T) = \sum X_i^v k_i(T) \quad (6)$$

$$k(T) = 1 \Big/ \sum \left(X_i^v / k_i(T) \right)_i \quad (7)$$

where k is the global conductivity, k_i is the thermal conductivity of the component i (see Table 1) i corresponds to water, ice (if the temperature is lower than the initial freezing temperature T_{cc}), carbohydrate, fat, ash, etc, X_i^v corresponds to the volumetric fraction of each component obtained as:

$$X_i^v = \frac{X_i/\rho_i}{\sum (X_i/\rho_i)_i} \quad (8)$$

The density of the product was calculated as:

$$\rho(T) = \frac{1}{\sum X_i/\rho_i} \quad (9)$$

where ρ is the global density, ρ_i is the density of the component i (see Table 2) and X_i corresponds to the mass fraction of each component.

The specific heat of the beet was calculated as

$$c(T) = \sum c_i X_i^v \tag{10}$$

where c is the global specific heat, c_i is the specific heat of the component i (see Table 3).

The empirical equation obtained by Pham (1996) was adopted to obtain the initial freezing temperature based on the composition of the beet

$$T_{cc} = -4,66\left(X_o / X_{wtot}\right) - 46,4\left(X_m / X_{wtot}\right) \tag{11}$$

where X is the mass fraction, with the indices m = minerals, o = other components, $wtot$ = water total; T_{cc} (°C). The initial freezing point of the beet was -1,18°C.

Table 1. Thermal conductivity of the components of the beet.

Components	Thermal conductivity ($Wm^{-1}°C^{-1}$)*
Protein	$k_p = 1.7881\times10^{-1} + 1.1958\times10^{-3}T - 2.7178\times10^{-6}T^2$
Fat	$k_f = 1.8071\times10^{-1} - 2.7604\times10^{-3}T - 1.7749\times10^{-7}T^2$
Carbohydrate	$k_c = 2.0141\times10^{-1} + 1.3874\times10^{-3}T - 4.3312\times10^{-6}T^2$
Fiber	$k_{fb} = 1.8331\times10^{-1} + 1.2497\times10^{-3}T - 3.1683\times10^{-6}T^2$
Ash	$k_a = 3.2962\times10^{-1} + 1.4011\times10^{-3}T - 2.9069\times10^{-6}T^2$
Water	$k_w = 5.7109\times10^{-1} + 1.7625\times10^{-3}T - 6.7036\times10^{-6}T^2$
Ice	$k_i = 2.2196 - 6.2489\times10^{-3}T + 1.0154\times10^{-4}T^2$

*As temperaturas são expressas em °C.

Table 2. Density of the components of the beet.

Components	Density ($kg.m^{-3}$)*
Protein	$\rho_p = 1.3299\times10^3 - 5.1840\times10^{-1}T$
Fat	$\rho_f = 9.2559\times10^2 - 4.1757\times10^{-1}T$
Carbohydrate	$\rho_c = 1.5991\times10^3 - 3.1046\times10^{-1}T$
Fiber	$\rho_{fb} = 1.3115\times10^3 - 3.6589\times10^{-1}T$
Ash	$\rho_a = 2.4238\times10^3 - 2.8063\times10^{-1}T$
Water	$\rho_w = 9.9718\times10^2 + 3.1439\times10^{-3}T - 3.7574\times10^{-3}T^2$
Ice	$\rho_i = 9.1689\times10^2 - 1.3071\times10^{-1}T$

Table 3. Specific heat of the components of the beet.

Components	Specific heat ($kJ.kg^{-1}.°C^{-1}$)*
Protein	$c_{p,p} = 2.0082 + 1.2089\times10^{-3}T - 1.3129\times10^{-6}T^2$
Fat	$c_{p,f} = 1.9842 + 1.4733\times10^{-3}T - 4.808\times10^{-6}T^2$
Carbohydrate	$c_{p,c} = 1.5488 + 1.9625\times10^{-3}T - 5.9399\times10^{-6}T^2$
Fiber	$c_{p,fb} = 1.8459 + 1.8306\times10^{-3}T - 4.6509\times10^{-6}T^2$
Ash	$c_{p,a} = 1.0926 + 1.8896\times10^{-3}T - 3.6817\times10^{-6}T^2$
Water	$c_{p,w} = 4.1762 - 9.0864\times10^{-5}T + 5.4731\times10^{-6}T^2$
Ice	$c_{p,i} = 2.0623 + 6.0769\times10^{-3}T$

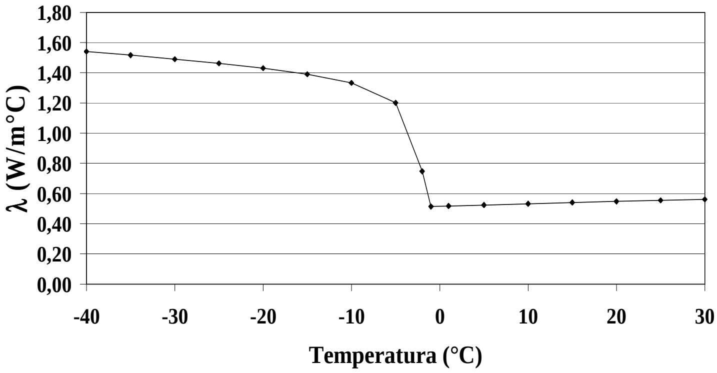
During the freezing process, which involves the phase change of water into ice in the food product, the thermo-physical properties such as specific heat, thermal conductivity, and density undergo abrupt changes due to the latent heat release. The system is then established as a highly non-linear mathematical problem. Several techniques were applied to deal with the large latent heat release when using the numerical method. One of the traditional methods is the use of the apparent specific heat, where the sensible heat is merged with the latent heat to produce a specific heat curve with a large peak around the freezing point, that can be considered a quasi delta-Dirac function with temperature (depending on the amount of water in the food product). The abrupt change in the apparent specific heat curve requires several iterations for each time step and usually destabilizes the numerical solution.

Another method applied is the enthalpy obtained from the integration with the temperature of the values of volumetric specific heat multiplied by density. Figure 3 shows the behavior of the thermal conductivity and specific heat, respectively, while Fig. 4 illustrates the enthalpy function. The equations for thermal conductivity, enthalpy function dependents of temperature, and temperature dependents of enthalpy are described, respectively, as follows:

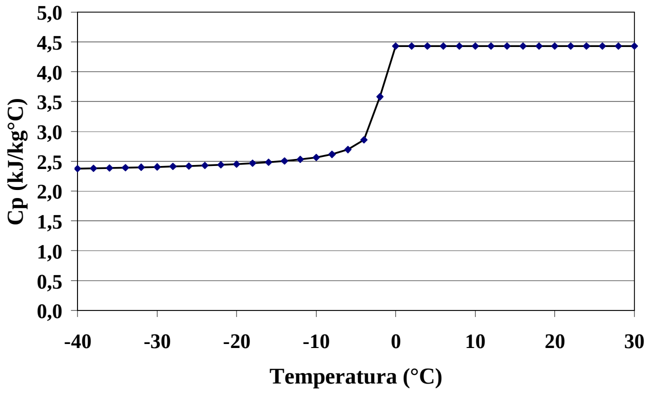
$$k = \begin{cases} \left(\dfrac{1}{0{,}62 - 1{,}27/T}\right); -40 \le T < -1 \\ (0{,}52 + 0{,}0016T; -1 < T < 30 \end{cases} \tag{12}$$

$$H(T) = \begin{cases} 46.3 + 1.47T - 585.34/T; -40 \le T < -1 \\ 349.07 + 3.856T; -1 \le T < 30 \end{cases} \tag{13}$$

$$T = \begin{cases} \dfrac{1}{-0{,}025 - 7{,}71 \cdot 10^{-5} H^{1{,}5}}; 0 \le H < 341{,}37 \\ -90{,}52 + 0{,}26H; 341{,}37 \le H < 464{,}76 \end{cases} \tag{14}$$



(a)



(b)

Figure 3. Thermo physical properties of the beet used for the freezing process as a function of temperature (a) thermal conductivity and (b) specific heat.

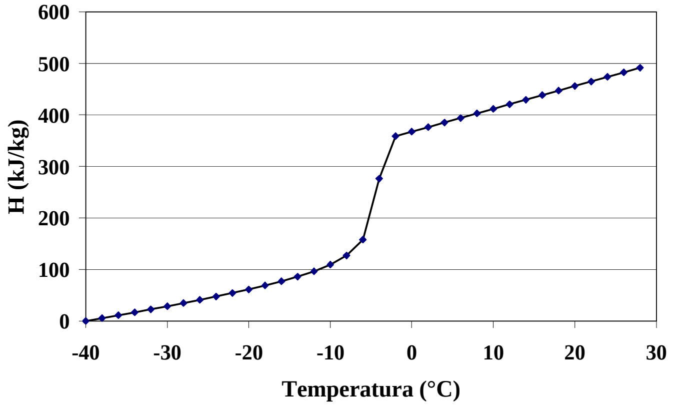


Figure 4. Functional relationships used in the combined formulation of the freezing process: enthalpy vs. temperature.

3.3 Experimental Procedure

All experimental procedures described below were performed by Strapasson (2006). For the experimental procedure were used 20 kg of beet in nature, the main equipments are shown in Table 4.

Table 4. Equipments and instruments.

Equipments	Quantity
PC containing software FORTRAN, Mattlab version 5.3 and Office	1
Electrolux freezer of 300 liters (-20°C)	1
Central data acquisition HP Agilent 3792[a]	1
Termocouples type T	8
Fluxmeters RdF 27032-3	2
Fan Arno	1

The horizontal freezer required internal modifications to be used under conditions of planned experiments. It was necessary to divide the internal volume into two chambers, having controlled conditions of ventilation and heat transfer coefficients in each of these chambers. Figure 5 shows the internal arrangement of the freezer, placing the fan for air movement at different speeds and with a sample of beet instrumented and ready for the freezing experiment, ie, illustrates the arrangement used, showing the surface of the parallelepiped ventilated beets. The convection heat transfer coefficient was determined using fluxmeter and thermocouple type T on the surface of the beet.

(a)

(b)

Figure 5. (a) Internal arrangement of the freezer with a three-dimensional sample of beets ready for the freezing. (b) Sample parallelepiped-shaped bulbs beets.

4. RESULTS AND DISCUSSION

The heat transfer coefficient by convection was determined by fluxmeter placed on the product surface that evaluated the heat flux on the surface and the implantation of a thermocouple for measuring the surface and environment temperatures. Figure 6 shows experimental results in which the beet sample is cooling by forced convection. In Fig. 6a began freezing the beets sample using speed 1 (high) then change to speed 2 (medium) and finally it is used the speed 3 (low) of the fan. In Figure 6 are shown temperature measured by thermocouples, surface temperature and heat flux on the sample surface.

There were always two thermocouples used to measure surface temperatures, since this measure is subject to experimental errors, since the location of the junction of the thermocouple measure exactly on the sample surface is difficult. It is noticed that the signal from the fluxmeter oscillates with reasonable amplitude, around an average value. This follows the fan pulse at high frequency and also because of unavailability of any signal filter. One possible treatment that could be done is to apply an algorithm for smoothing the signal, for example, a moving average algorithm. This was not done and will be subject to future research with the sign of fluxmeters under conditions of convection.

The signals obtained from the experiments illustrated in Figure 6a were used in Newton equation obtaining the heat transfer coefficient for different fan speeds. Figure 6b shows the convection coefficients obtained for the three tests shown in Fig. 6a. Both figures clearly show that, at the opening of the freezer to make the change of fan speed, there is a great disturbance in flow and heat transfer. For the calculation of the average coefficient of heat transfer, only took points from 2.5 minutes of opening the freezer, not to take into account the noise caused by the doors opening. The isolated area is considered in all surfaces that do not get the air by forced convection.

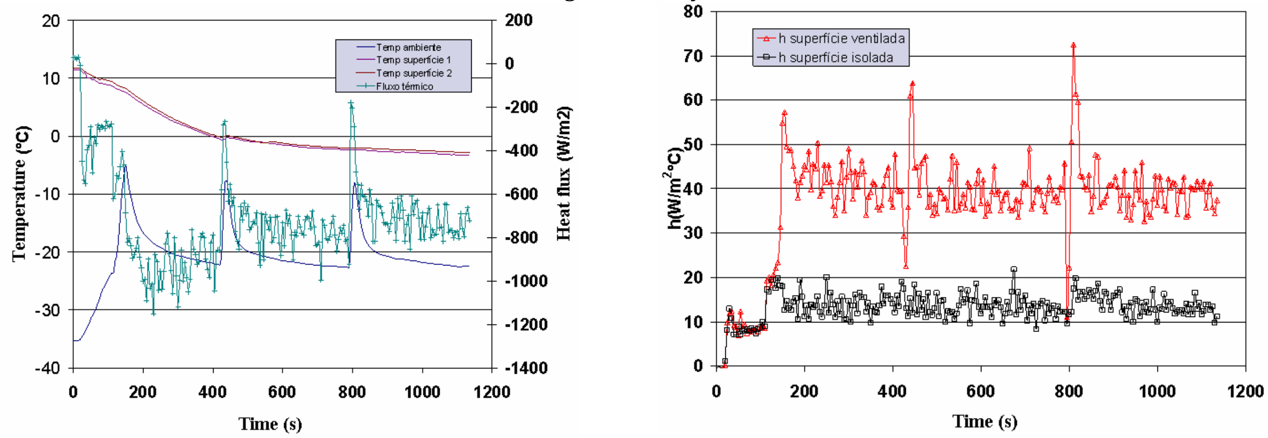


Figure 6. (a) Surface temperature, ambient temperature, surface heat flux on the beet sample.
(b) Heat transfer coefficient calculated on the ventilated surface (velocities 1, 2, and 3) and isolated surface.

The parameters used in the experiment and the mathematical model are shown in Table 5, and the numerical simulations obtained with the computational code, were contrasted with experimental time-temperature freezing curves of beets in Fig. 7. It can be seen from the results shown in Fig. 7a that a good agreement was obtained between the experimental measurements and the numerical predictions in the center of the product, the biggest discrepancies between numerical and measurement results were found in the surface, as indicated in Fig. 7b. The finite difference method produces a satisfactory result, and closely models the physical characteristic of the freezing food material. Therefore, the model's prediction has a good accuracy. The heat transfer coefficients used to computational simulations were h = 20 W/m^{2o}C and h_c = 50 W/m^{2o}C like described in Eqs. (3) and (4), while the temperature of cooling medium was -14.66^{o}C, such parameters are described in Table 5.

Table 5. Parameters about beets.

T_a(^{o}C)	h(W/m^{2o}C)	h_c(W/m^{2o}C)	T_0(^{o}C)	A (cm)	B (cm)	C (cm)
-14.66	20	50	11.66	5	3	2

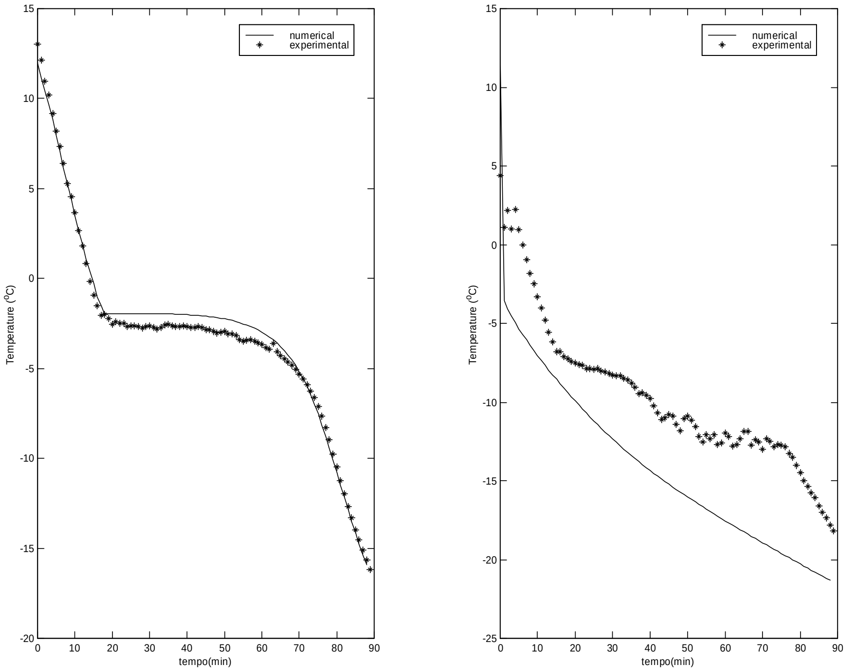


Figure 7. Experimental and predicted temperature values using a 3D model at
(a) product centre and (b) product surface.

4.1 Application of the numerical model to predict freezing times

Since the temperature distribution within a product varies considerably during freezing process, freezing time must be defined with respect to a position. The thermal center is generally taken as reference, which is the location where the temperature changes most slowly. The freezing time is usually defined as the time to reach a particular temperature at the slowest cooling point (the thermal center) (Hossain *et al*., 1992). For freezing, a number of final center temperatures have been used -5ºC, -10ºC, -15ºC and -18ºC.

In this way, simulations were carried out in order to obtain the freezing times for different external fluid temperatures and surface heat transfer coefficients values, considering an initial temperature of 11.66 ºC. The numerical freezing times for final center (in the warmest point) temperature of -10ºC and initial temperature of T_0 = 11.66ºC are shown in Table 6 for T_a = -20ºC and -30ºC, respectively. Table 6 concerns the same results used to build Fig. 8, where can see that for final center temperature of -10ºC the finite difference method predicted lower freezing times to the biggest heat transfer coefficient and the lower temperature of cooling medium.

Table 6. Freezing time considering different operating conditions for final center temperature of -10ºC.

	Temperature of cooling medium (ºC)	
h (W/m^{2}ºC)	-20	-30
5	18581.20	11489.61
10	10089.00	6592.18
15	7383.96	4700.34
20	5909.62	3650.00

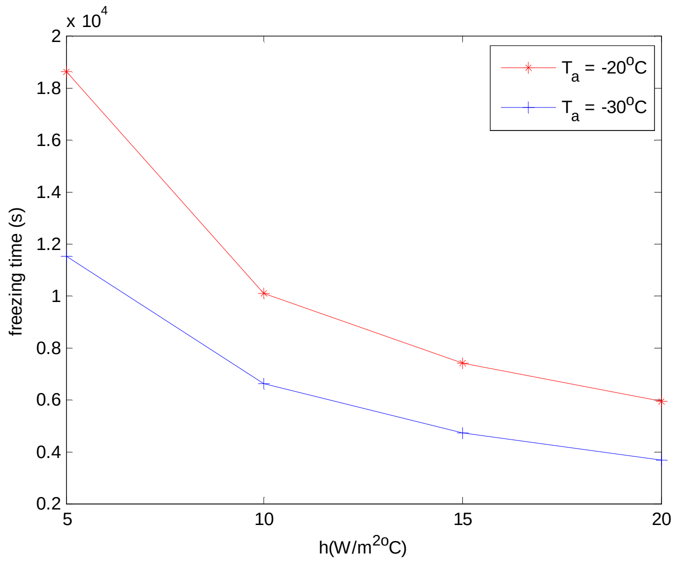


Figure 8. Heat transfer coefficient versus freezing time.

5. CONCLUSIONS

Difference finite computational code was developed to simulate the phase change transition during freezing of an irregular foodstuff. An enthalpy formulation of the heat transfer equation was applied to deal with the highly non-linear mathematical problem and to enhance the computational speed of the numerical runs. In order the computational code was found to be proper, no oscillations of the temperature predictions were encountered in the numerical solutions. The numerical model was compared with experimental freezing curves of beet cut in a three-dimensional shape. Numerical runs agreed well with experimental time-temperature curves. The computational code was applied to obtain freezing times of beets considering different operating conditions, such as different surface heat transfer and temperature of cooling medium. That information can be useful for food processors to obtain valuable information to design freezing equipment and optimize processes.

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8. RESPONSIBILITY NOTICE

The authors, Vagner Neckel and Viviana Cocco Mariani are the only responsible for the printed material included in this paper.