FUZZY PATTERNS FOR OF A HEART RATE RHYTHM MODEL

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Abstract. *Parameter tuning is a complicated and laborious task. Many techniques have been employed to solve this problem, most of them based on optimization, with a deterministic or frequently heuristic approach. Anyway, a reference pattern for comparison is necessary, in order to determine how good a set of parameters is; therefore, experimental data is often used as reference, what is called model training. In case of a deterministic model of heart rhythm dynamics, the experimental data is available from electrocardiograms(ECG); however, for models which do not consider the heart rate variability, distortions in signal period will appear if the model is trained directly from the ECG. To find the appropriate set of parameters to simulate specific situations as cardiopathies, a pattern generation by fuzzy models is proposed in this work. The fuzzy model works as a knowledge base, which retains the main characteristics of each cardiac situation, enabling to state a metric for comparison.*

Keywords*: fuzzy logic, ECG, pattern generation*

1. INTRODUCTION

When working with biological models, very often the determination of parameter values is so complicated as, or even more than, to determine the biological mechanism (Babloyantz, 1998). In the case of the electrocardiogram, is not very different; however the real signal is well known and presents a relative good behavior. That makes possible to think of a pattern to compare the results of different models, or different sets of parameter in the same model. In that sense, such a pattern would be very useful to train models, playing the role of the goal to be achieved.

The purpose is not to state a model which reproduces the functional mechanism of a biological system (Savi, 2005), instead is to develop a fuzzy model able to generate a signal which follows the same pattern as the real signal. The point is: this fuzzy system is not necessarily able to have the same response as the real system or its analytical mathematical model if subjected to a different situation.

A fuzzy system needs to be trained too, then, a question arises: why not to train directly the system? In the case of ECG signal, it is very easy to answer. First of all, as the real signal is well know, with a characteristic shape, it is easy to identify relevant parts of the signal to state a first prototype of the fuzzy system.

A second, but important issue, is that by means of fuzzy logic it is possible to deal with noisy signal without losing its real shape. For the particular case of the electrocardiogram, where we have a frequency variation (Carvajal et al, 1998) called heart rate variability (HRV) the ability of fuzzy systems to deal with non precise information is very valuable. Actually, considering that the HRV is chaotic and very small, the fuzzy system will face it just like a white noise on the signal amplitude.

2. THE ECG SIGNAL

The electrocardiogram is a measure of the cardiac electrical behavior. There are groups of cells in the heart able to generate electric signal (Guyton & Hall, 1997,), which will stimulate the cardiac muscle to contract, taking the heart to execute its function. Such groups of cells are called natural pacemakers and are a total of three: the sino-atrial node (SA node), the atrio-ventricular node (AV node) and the His-Purkinje complex (HP node). The synchronized work of those nodes stimulates successively the different regions of the cardiac muscle, coordinating the heart work and dictating its rate.

The combination of the electric fields generated by the pacemakers creates the heart electric signal, which is not stationary, instead, it can be considered as a rotating dipole (Koelliker & Mueller, 1855,). The dipole is represented in Fig. 1 as an arrow of magnitude P and direction angle α, where both are variable during heart activity.

But the electrocardiogram is actually the measured signal and, as many different measurement systems are available, different signals are obtained (Malmivuo & Plonsey, 1995). Basically, leads are attached to the patient body and the electric potential difference between them is measured. The arrangement in focus here is the 3-leads system (Einthoven, 1902), where a lead is attached to each wrist and the third one is on the left leg, as shown in Fig. 1. Then, the electric potential difference between leads is measured, what is the same as to say that the magnitude of the projection of the rotating dipole onto the line defined by two leads is measured. Each of those projections is called a derivative, and in this work, it will be considered the second derivative, that is, the projection along the line from the right wrist to the left leg. That derivative was chosen because it is the most representative for a wide variety of cardiopathies.

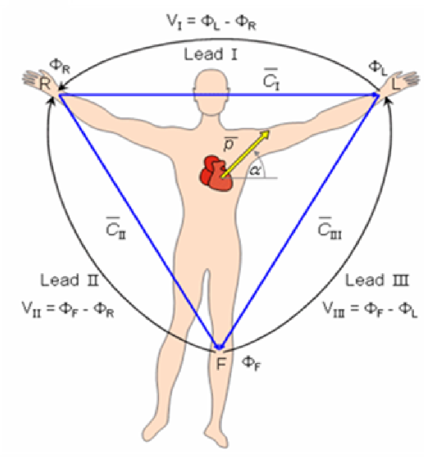

Figure 1 – Three Leads System

The measured signal has some characteristic peaks and valleys which are very important in determining malfunctioning of the heart. Each of them is related to a specific step in heart beat cycle. Therefore, taking a period of the ECG, the peaks are P, R and S, as shown in Fig.2, and the valleys are Q and S. As already mentioned, the HRV leads to a variation in the ECG period, then this term is not strictly correct, but, what the fuzzy system will consider is an average period.

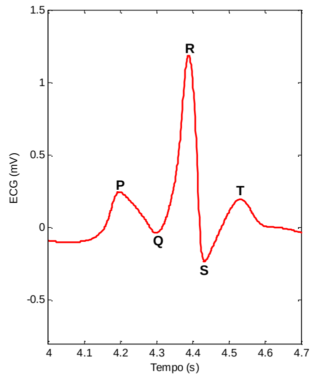

Figure 2 – Normal ECG period

There is a standard representation of the ECG, in order to make its interpretation easier, and universal (Dubin, 1996). The curve is plotted on a grid, where the cells sides are 1mm long. In the vertical scale, 5mm corresponds to 0.5mV, as in the horizontal scale, 5mm corresponds to 0.2s. In Fig. 3, a sketch of a normal ECG is shown, detailing each time interval which is of interest for the medical analysis.

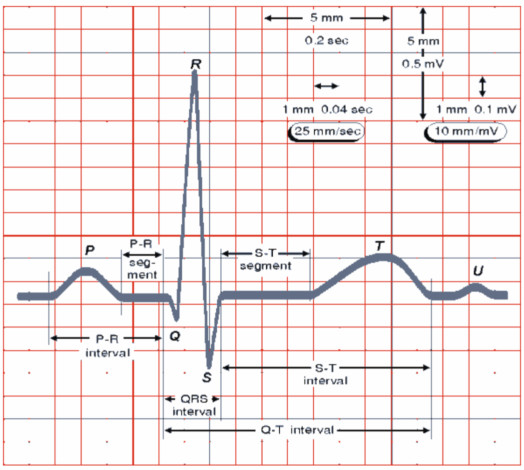

Figure 3 – Standard ECG plot

3. THE FUZZY SYSTEM

Now that the main characteristics to be captured from the ECG are known, it is possible to think of stating a fuzzy inference system to reproduce the signal pattern. Considering the extremely nonlinear nature of the signal, what seems more appropriate is to construct the curve in parts, taking the representative value of each one to reconstruct it (Jang, Sun & Mizutani, 1997).

It suggests that the appropriated structure for the fuzzy inference system (FIS) is the Takagi-Sugeno-Kang (TSK), where the consequent parts of each rule i is an explicit function f_i of the input, called output function. Many different output functions could be used, but in order to simplify the problem, they will be taken as zero order polynomials, that is, constant values, with the function input as the instant t considered.

$$f_i(t) = \theta_i \tag{1}$$

There is no general rule to choose the order of the output function, but in general, a first order function tends to generate smoother surfaces than a zero order does; however, this last one can give a more precise representation of edges and environments with abrupt changes, which is the case of the ECG.

The first consideration to state the curve to be reproduced is to consider the ECG as periodic; therefore, the desired curve will be taken as a period of the electrocardiogram. Periods are defined as intervals limited by reference points on the signal, for example, the middle point between two successive R peaks. In order to eliminate the HRV, an average period is calculated, then, each interval is linearly scaled to the average period, by stretching or folding it. Now, the input data for the TSK FIS is ready, which has its the general structure shown in Figure 3.

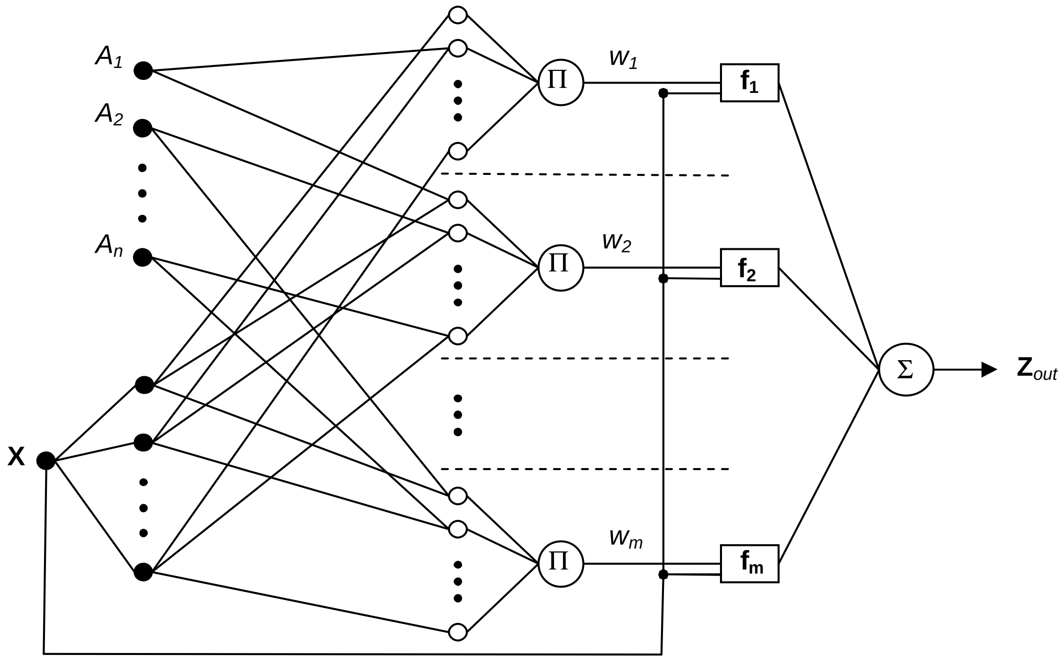

Figure 3 – General TSK fuzzy inference system

As the ECG is a time series, the input variable is time (in seconds), and the output will be the amplitude (in mV). In this case the input is scalar, and each rule will have an only membership function in its antecedent part. Then, the output of the fuzzy system may be written as:

$$\hat{z} = \frac{\sum_i w_i \cdot f_i(t)}{\sum_j w_j} \tag{2}$$

Where z is the real signal value and therefore, $\hat{z}$ is an estimation of it. The w_i are the firing strength of each rule i. The firing strengths are calculated as follows:

$$w_i = \sup(\mu_{i1}(t), \ldots, \mu_{in}(t)) \tag{3}$$

But in the case of a single input, with a single fuzzy set in each rule antecedent, it is clear that the firing strength of the rule becomes directly the membership function μ_i of the corresponding fuzzy set. If zero-order output functions are employed, then the output may be simplified to:

$$\hat{z} = \frac{\sum_i \mu_i(t) \cdot \theta_i}{\sum_j \mu_j(t)} = \sum_i W_i \cdot \theta \tag{4}$$

where W_i is the normalized firing strength of the corresponding rule *i*.

4. PARAMETER FITTING

Once the system structure is stated, now it is possible to adjust the parameters of the system. It is important to stress that, in spite of being a well behaved signal, the ECG is plenty of noise, what is problematic to most of fitting methods because they are based on the derivative of the signal, what makes necessary first to apply filtering to the signal.

Optimization is employed, through the minimization of the error between the estimated signal $\hat{z}$ and the measured one *z*. The optimization process is performed by application of the *Least Squares Estimation* method (Morgado de Gois, 2005). Consider a group of *N* samples (measurements) to start the optimization and define a cost function aiming at the minimization of the error, where $\mathbf{z} = [z_1\ z_2\ \ldots\ z_N]^T$, and the index refers just to the order in which the measurements are taken.

$$J = \frac{1}{N} \cdot \sum_{l=1\ldots N} (z_l - \hat{z}_l)^2 = \frac{1}{N} \cdot [\mathbf{z} - \hat{\mathbf{z}}]^T \cdot [\mathbf{z} - \hat{\mathbf{z}}] \tag{5}$$

Since a zero order function is used to find the estimates $\hat{z}$, applying Eq. 4 to Eq. 5, it can be rewritten in the matrix form from Eq. 6, where *P* is called the design matrix.

$$\hat{z} = P(t) \cdot \Theta \Rightarrow \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \vdots \\ \hat{z}_N \end{bmatrix} = \begin{bmatrix} W_1^1 & W_2^1 & \cdots & W_n^1 \\ W_1^2 & W_2^2 & & \\ \vdots & & \ddots & \vdots \\ W_1^N & W_2^N & \cdots & W_n^N \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \tag{6}$$

where the upper index in W refers to the sample (measurement) order in the signal, while $\Theta = [\theta_1, \ldots, \theta_n]$. Substituting it back in Eq. 5 and finding the minimum, it is obtained:

$$\begin{aligned} J &= \frac{1}{N} \cdot [\mathbf{z} - P(t) \cdot \Theta]^T \cdot [\mathbf{z} - P(t) \cdot \Theta] \\ \nabla_\Theta J &= 0 \Rightarrow \Theta = \left(P(t)^T \cdot P(t)\right)^{-1} \cdot P(t)^T \cdot \mathbf{z} \end{aligned} \tag{7}$$

Then, the set of optimal parameters Θ is stated, and the FIS defined. Now, from any normal ECG it is possible to train the system.

5. MODEL TUNING

Once the parameters of the fuzzy system are set after the training phase (based on the real signal), it is possible to use its output as the goal to be achieved by a more representative mathematical model of the physical phenomena behind the ECG signal, from here on called just as mathematical model.

Since the signal generated by the fuzzy system does contain neither noise nor random frequency variation, it is possible to use this FIS to create an objective function and then apply optimization methods to carry on the parameter tuning of the mathematical model. If the output of the mathematical model is $\bar{z}$, then a appropriate objective function is:

$$\bar{z}(q,t) - \hat{z}(\Theta,t) = 0 \tag{8}$$

where q are the parameters of the mathematical model to be tuned. With this objective function, any optimization methodology could be applied to the problem, even those which use derivatives of the objective, since it is explicitly available from Eq. 2.

6. RESULTS

A real ECG signal, obtained from the data bank PhysioNet was employed. A normal ECG was taken because of its very characteristic pattern, what makes easier to verify the result of the FIS training. Besides, abnormal ECG signals have specific details which may be determinant for a correct diagnose, what makes the evaluation of the training much more complicated, depending on the analysis of specific cardiopathies (Moffa & Sanches, 2001). Since that is not in the scope of this work, the real ECG from Fig. 4 was used.

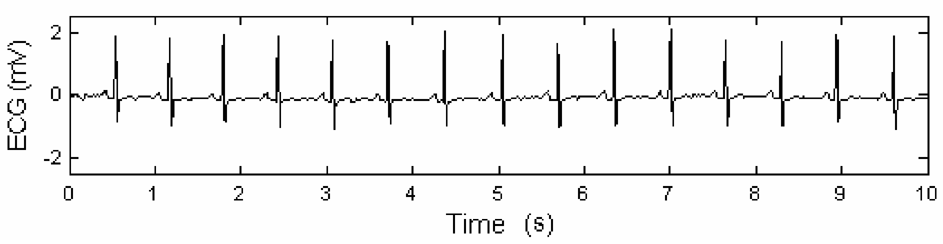

Figure 4 – Real ECG signal

Next, the data from the real ECG is separated in two sets, a training set and a test set. No especial criteria was used, the time span of the ECG it was separated in two intervals, one corresponding to each set. Then, a fuzzy inference system with TSK structure was trained. The FIS was composed of thirteen rules, with simple antecedents, i.e., an only fuzzy set in each antecedent, as shown by the representation of the membership functions (*mf*) in Fig. 5. This number of rules corresponds to the number of the fuzzy sets in which the input crisp set was partitioned, according to the topological characteristics of the real ECG signal.

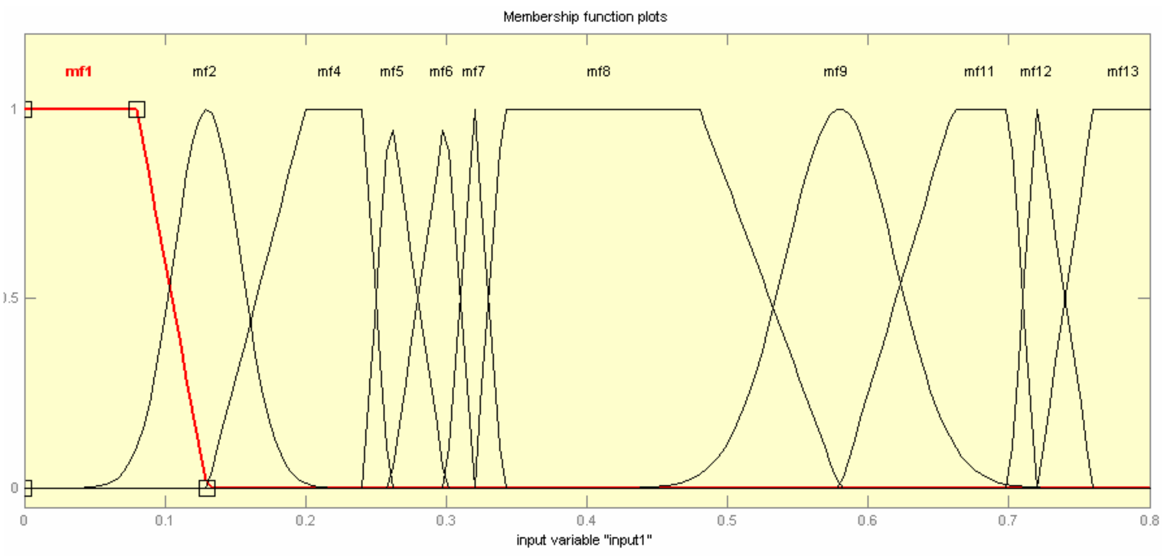

Figure 5- Membership functions

After the training step, the set of parameters obtained for the output functions of the fuzzy inference system was given by Θ = [0; 0.1; 0; -0.1; 1.1; -0.3; 0; 0.22; 0; 0.05; 0]. With this result it is possible to calculate the objective function from Eq. 8 and then, to apply the optimization.

The model to be tuned is shown in Eq. 9 and it was first presented in (Gois & Savi, 2009). It comprehends a total of third three parameters, therefore, the optimization it was separated in two steps: a first one executed by a direct search, and a second step for refinement, execute by genetic algorithm. In this model, three coupled oscillators are used; representing the natural pacemakers of the heart and *x* corresponds to their states. The ECG signal is generated by linear combination of them, representing the superposition of the electric fields.

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -a_{SA}x_2(x_1 - w_{SA_1})(x_1 - w_{SA_2}) - x_1(x_1 + d_{SA})(x_1 + e_{SA}) + \rho_{SA}\sin(\omega_{SA}t) + \\
&\quad + k_{SA-AV}(x_1 - x_3^{\tau_{SA-AV}}) + k_{SA-HP}(x_1 - x_5^{\tau_{SA-HP}}) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -a_{AV}x_4(x_3 - w_{AV_1})(x_3 - w_{AV_2}) - x_3(x_3 + d_{AV})(x_3 + e_{AV}) + \rho_{AV}\sin(\omega_{AV}t) + \\
&\quad + k_{AV-SA}(x_3 - x_1^{\tau_{AV-SA}}) + k_{AV-HP}(x_3 - x_5^{\tau_{AV-HP}}) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= -a_{HP}x_6(x_5 - w_{HP_1})(x_5 - w_{HP_2}) - x_5(x_5 + d_{HP})(x_5 + e_{HP}) + \rho_{HP}\sin(\omega_{HP}t) + \\
&\quad + k_{HP-SA}(x_5 - x_1^{\tau_{HP-SA}}) + k_{HP-AV}(x_5 - x_3^{\tau_{HP-AV}})
\end{aligned}
\tag{9}
$$

After the optimization, the tuned mathematical model generates the signal shown in Fig. 6, which is superimposed to the real ECG signal as shown in Fig. 7 for the sake of comparison.

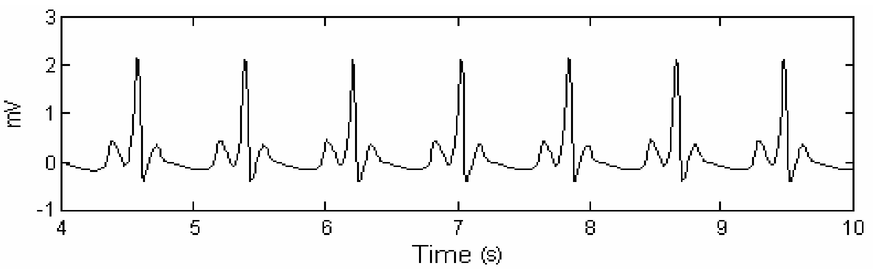

Figura 6 – ECG signal simulated by the mathematical model

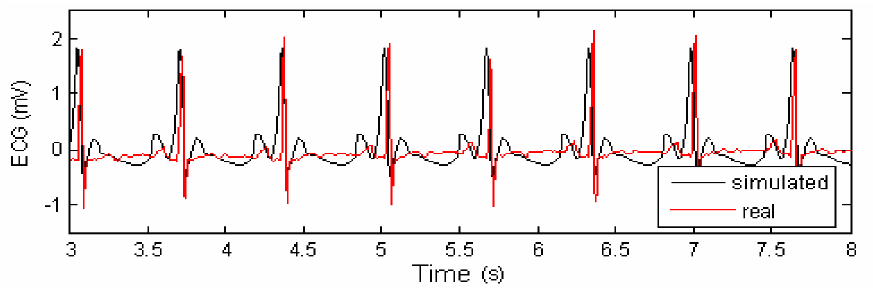

Figure 7 – Comparison between Real and Simulated ECG signal

7. CONCLUSION

The results obtained by the mathematical model with the optimal set of parameters show a good qualitative agreement to the real sign, what indicates that the procedure is promising. It makes possible to obtain in a systematic way the parameters of the system; what in a model with a total of third three parameters it is almost impossible to think of doing manually.

In a first glance, the definition of the input membership functions may seem a little complicated to do, requiring specific knowledge of the signal or of the system. Actually it is not so; the presented membership functions were taken based only on the shape of the signal, identifying topological characteristics, nothing else. Anyway, simple triangular functions could be employed, in the same number of peaks and valleys of the average period shown in Fig. 2.

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