# EFFECTIVE FLUID COMPRESSIBILITY MODULUS IN HYDRAULIC SYSTEMS 

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Abstract. The compressibility of hydraulic fluids is the main factor in the determination of the hydraulic frequency of hydraulic systems. Under conditions of rapid changes and high pressure, the fluid behaves like an hydraulic spring. In most of the cases this leads to a limitation in the response speed of a component or system to a given entrance signal, which means, the limitation in the dynamical behaviour. The spring effect of the hydraulic fluid (hydraulic spring) is characterized by the value of the compressibility modulus. The presence of air affects the bulk modulus value substantially, as well as the elastic deformation of the elements that contain the fluid. The ISO 6073 standard stipulates a procedure for predicting the bulk modulus of petroleum derivative fluids, free from bubbles, including also means for the determination of medium and instant values. Thus in order to proceed the systems analysis, we should consider a compressibility modulus that incorporates the volumetric variation effect of the parts. The effective compressibility modulus of the fluid considerably oscillates with the operation pressure of the hydraulic systems, although in most of the published papers its value is considered as constant. In practice the effective compressibility modulus of the fluid is difficult to be determined. The elasticity modulus of a mineral oil ranges from orders $14 \times 10^{8} \mathrm{~Pa}$ to $18 \times 10^{8} \mathrm{~Pa}$. The water, that nowadays is being used in hydraulic circuits again, has from order $22 \times 10^{8} \mathrm{~Pa}$. The objective of this file is to show a simple experimental technique that measures the effective compressibility modulus range of oil with the pressure in an hydraulic system.

Keywords: compressibility modulus; hydraulic systems; fluid properties; experimental methods.

## 1. INTRODUCTION

The compressibility of hydraulic fluids is the predominant factor in the determination of the hydraulic frequency of hydraulic systems. In most of the cases this leads to a limitation in the response speed of a component or system to a given entrance signal, which means, a limitation in the dynamical behavior. Under conditions of rapid changes and high pressure, the fluid behaves like a hydraulic spring. This effect is characterized by the value of the compressibility modulus, according to Jinghong (1994).

The analysis of the compressibility effects is usually made by using the so called compressibility modulus ( $\beta_{l}$ ), as shown:

$$
\begin{equation*}
\left.\beta_{l}=-V_{0} \frac{\partial p}{\partial V}\right)_{T} \tag{1}
\end{equation*}
$$

Where:
$V_{0}=$ Initial volume,
$V=$ Total volume,
$\mathrm{p}=$ Absolute pressure ( $\mathrm{p}_{\mathrm{abs}}$ ),
The compressibility modulus will always have a positive value, meanwhile $\partial p / \partial V)_{T}$ is always negative. Its value is not constant, tending to increase non-linearly with the pressure and to decrease with the temperature.

Consequently besides the isothermal compressibility modulus ( $\beta_{l_{T}}$ ) defined above, it is also possible to obtain the isentropic compressibility modulus ( $\beta_{l_{s}}$ ). The isentropic compressibility modulus relates to conditions in which the pressure oscillations are fast, not allowing the temperature balance to be reestablished.

Nevertheless, the isentropic and isothermal modules co-relate by the ratio of the specific heats under constant pressure $\left(c_{p}\right)$ and under constant volume $\left(c_{v}\right)$, in such a way that $\beta_{l_{S}}=\left(c_{p} / c_{v}\right) \beta_{l_{T}}$. According to Linsingen (2003), for mineral oils under usual operational temperatures, the ratio of specific heats ranges up to 1,02 and therefore, the difference between the two compressibility modules can be disregarded, specially in those applications in which the presence of air mixed in the oil and the elasticity of the components of the system are significant.

The isothermal compressibility modulus for a mineral oil with specific mass of $850 \mathrm{~kg} / \mathrm{m}^{3}$, under $40^{\circ} \mathrm{C}$, may range from $1,48 \times 10^{9} \mathrm{~Pa}\left(1,48 \times 10^{4} \mathrm{bar}\right)$, the atmospheric pressure, up to $1,8 \times 10^{9} \mathrm{~Pa}\left(1,8 \times 10^{4}\right.$ bar $)$, under a pressure of 40 MPa ( 400 bar ). On the other hand, an oil with $\rho=930 \mathrm{~kg} / \mathrm{m}^{3}$ under $40^{\circ} \mathrm{C}$ presents modulus between $1,78 \times 10^{9} \mathrm{~Pa}$ ( $1,78 \times 10^{4}$ bar) under atmospheric pressure and $2,1 \times 10^{4}$ bar under 400 bar.

The ISO 6073 standard stipulates a procedure for the prediction of the compressibility modulus of the petroleum derivative fluids exempt from bubbles (fresh air), and also includes methods for determining medium and instant values. But eventually, it is possible that gas or steam aggregates are blended with the liquid or retained in the grooves of the components. Due to the high compressibility of those, the volume reduction of the mixture fluid/gas (or steam) might be substantial, depending on its proportion.

Therefore in order to accomplish the system analysis, it is to be considered the compressibility modulus which incorporates the effect of the volumetric oscillation of the parts.

This file shows a simple experimental technique that measures the effective compressibility modulus range of oil with the pressure in a hydraulic system.

## 2. RELEVANT ASPECTS FOR THE DETERMINATION OF THE COMPRESSIBILITY MODULUS

There are many factors affecting the affective compressibility modulus, $\beta e$, or the equivalent compressibility of hydraulic oil. Among them, air content of the oil, oil pressure, oil temperature, tube rigidity, and interface condition between the oil and the air are some of the main factors. As shown by Magorien, if there is some air in a hydraulic system, the value of $\beta e$ will be reduced substantially.

Generally in any hydraulic systems, air exists in the oil either in entrained or dissolved form. Dissolved air is not normally a problem, except at very high pressures. However, entrained air is known to seriously affect $\beta e$, due to the size of the air bubbles in solution. Even if air is related to pressure and temperature, $\beta e$, will also depend on their parameters.

### 2.1. Thermal Expansion and oscillation of the specific mass with the temperature.

Oscillations of the specific mass under a given temperature and pressure are usually described by state equations. Nevertheless, for hydraulic fluids the equations are by far more complex than the ones for gases.

According to Linsingen (2003), for petroleum derivative fluids the coefficient of thermal expansion is kept approximately constant for a usual range of temperatures between 15 and $100^{\circ} \mathrm{C}$. However the values depend considerably on the specific mass of the considered oil, approximately 7,2 a $8,1 \times 10^{-4} /{ }^{\circ} \mathrm{C}$, for specific masses between 0,93 and $0,85 \mathrm{~g} / \mathrm{cm}^{3}$, respectively.

For usual calculations the expansion coefficient used is the average value of $7,5 \times 10^{-4} /{ }^{\circ} \mathrm{C}$. Which means that for each $10^{\circ} \mathrm{C}$ of temperature fluctuation there is an increase of approximately $0,75 \%$ in volume of the fluid mass considered. Under such conditions, considering the temperature of $15^{\circ} \mathrm{C}$, the oscillation of the specific mass with the temperature, as defined by Eq. (2), may be estimated by:

$$
\begin{equation*}
\rho=\rho_{0}\left[1-7,5 \times 10^{-4}\left(T-15^{\circ} C\right)\right] \tag{2}
\end{equation*}
$$

Where:

$$
\rho=\text { fluid density }
$$

$\rho_{0}=$ initial fluid density,
$T=$ fluid temperature .
The Fig. (1) shows the oscillations of specific mass under a given temperature as Eq. (2), for petroleum derivative fluids with specific mass between 0,85 and $0,93 \mathrm{~g} / \mathrm{cm}^{3}$ at $15^{\circ} \mathrm{C}$ :


Figure 1. Specific mass of mineral oils in relation to temperature and atmospheric pressure Source: Fundamentos de Sistemas hidráulicos, Irlan Von Linsingen

The volumetric expansion of hydraulic fluids is particularly important in those systems for which the fluid stays confined under certain operational conditions. This is the case of fluids confined in a channelling section between two closing valves, without leaks. Under such circumstances, the increase of temperature may cause a dangerous pressure elevation, because the compressibility is associated with the specific mass variation of the fluid in relation to the pressure that it is submitted to.

In the case of fluids, there is no analytical expression to represent the behavior of the specific mass in a system, but it is of common knowledge that the latter is influenced by pressure and by temperature, that is, $\rho=\rho(p, T)$.

Despite this, for relatively small oscillations, according to Merrit (1967), it is possible to reach an expression satisfactorily precise with a linear approximation by means of the differentiation of the expression $\rho=\rho(p, T)$, like:

$$
\begin{equation*}
\left.\left.d \rho=\frac{\partial \rho}{\partial p}\right)_{T} d p+\frac{\partial \rho}{\partial T}\right)_{p} d T \tag{3}
\end{equation*}
$$

In this situation it is possible to obtain an expression through the expansion in series of Taylor of Eq. (3) discarding terms of second order and superior, that is, in relation to a given initial state $\left(\rho_{0}, p_{0}, T_{0}\right)$ of the system, as follows, Eq. (4):

$$
\begin{equation*}
\left.\left.\rho-\rho_{0}=\frac{\partial \rho}{\partial p}\right)_{T}\left(p-p_{0}\right)+\frac{\partial \rho}{\partial T}\right)_{p}\left(T-T_{0}\right) \tag{4}
\end{equation*}
$$

That corresponds to the state equation in a liquid, linearized in an operation point.
Where $\rho=m / V$, the deriving partials may be rearranged conveniently, achieving Eq. (5):

$$
\begin{equation*}
\rho=\rho_{0}\left[1+\kappa\left(p-p_{0}\right)+\alpha\left(T-T_{0}\right)\right] \tag{5}
\end{equation*}
$$

The Eq. (5) is the linear state equation for a fluid, being $\kappa$ and $\alpha$ respectively obtained by:

$$
\begin{align*}
& \left.\kappa=-\frac{1}{V_{0}} \frac{\partial V}{\partial p}\right)_{T}  \tag{6}\\
& \left.\alpha=\frac{1}{V_{0}} \frac{\partial V}{\partial T}\right)_{p} \tag{7}
\end{align*}
$$

Where:
$V_{0}=$ initial volume,
$V=$ total volume of the fluid.
The Eq. (6) establishes the form of volume oscillation depending on the pressure, under a constant temperature, which means that, $\boldsymbol{K}$ is the isothermal coefficient of the fluid compressibility.

The Eq. (7) indicates the form of volume oscillation depending on the temperature under a given pressure, which means that $\alpha$ is the coefficient of isobaric thermal expansion.

### 2.2. Dissolution capability of gas

In hydraulic systems it is common the presence of gas, usually air, mixed and/or dissolved in the hydraulic fluid, possibly alternating its structure depending on the conditions to which such fluid is submitted to.

As previously demonstrated, bubbles mixed to the fluid may substantially affect the compressibility, compromising the system performance.

On the other hand, every fluid has the ability of keeping gas in solution, that is, of dissolving gas in proportions depending on the type and on the characteristics of the fluid.

Usually for mineral oils great part of the air mixed to the atmospheric pressure is dissolved when the pressure increases. The dissolution capability is directly proportional to a pressure of approximately 30 MPa ( 300 bar). According to the Dalton law, the percentage of dissolved gas volume under a constant temperature can be estimated using the expression:

$$
\begin{equation*}
V_{d}=S \frac{p}{p_{0}} V_{l} \tag{8}
\end{equation*}
$$

Where:
$\mathrm{V}_{\mathrm{d}}=$ dissolved volume,
$\mathrm{V}_{1}=$ fluid volume,
$\mathrm{p}_{0}=$ atmospheric pressure (in absolute value),
$\mathrm{p}=$ absolute pressure ( pabs ),
$\mathrm{S}=$ solubility constant.
The solubility constant depends on the fluid's type and characteristics but it can also be slightly influenced by the temperature and the viscosity. This constant determines the gas volumetric concentration under normal temperature and pressure conditions ( $1,013 \times 10^{5} \mathrm{~Pa}$ e $20^{\circ} \mathrm{C}$ ) reaching the following average values for different types of fluids, according to Linsingen (2003):

$$
\begin{aligned}
& \text { - mineral oil .................. } \mathrm{S}=0,06 \text { a } 0,12 \text {; } \\
& \text { - HFA ....................... } \mathrm{S}=0,02 \text { a } 0,05 \\
& \text { - HFC ....................... } \mathrm{S}=0,01 \text { a } 0,04 ; \\
& \text { - HFD ...................... } \mathrm{S}=0,012 \text { a } 0,04 .
\end{aligned}
$$

The above mentioned values indicate that the mineral oils (HL) may contain a larger amount of dissolved gas than the emulsions (HFA), the polymer watery solutions (HFC) and then the synthetic fluids (HFD).

The dissolved air usually does not affect the fluid properties. However, under given conditions, like in regions of low static pressure (pump suction, flow section reduction) the air may be released from the fluid (aeration), forming bubbles that may become unstable and implode when entering regions of higher pressure. Under such circumstances, this effect is similar to the cavity, being often named gas cavitations.

## 3. EFFECTIVE COMPRESSIBILITY MODULUS

Whenever a closed hydraulic system is submitted to pressure variation, the liquid volume tends to decrease due to its compressibility. In the same way, the components of the system tend o expand due to the elastic deformation of the material they consist of.

According to Linsingen (2003), in order to determine the effective compressibility modulus, it is to be considered a cylinder with a piston filled with a mixture of oil and air, as demonstrated in Fig. 2.


Figure 2. Volumetric oscillation of a system caused by pressure
Source: Fundamentos de Sistemas hidráulicos, Irlan Von Linsingen
Admitting that the gas is not dissolved in the fluid, but only forming mixed bubbles or retained volumes in the corners, once the solved gas has a negligible effect on the compressibility modulus.

For an initial condition of balance, the total volume is as follows:

$$
\begin{equation*}
V_{t}=V_{l}+V_{g}=V_{c} \tag{9}
\end{equation*}
$$

Where:
$V_{l}$ e $V_{g}$ are the initial volumes of fluid and gas and
$V_{c}$ is the cylinder inner volume
The increase of the internal pressure resulting of the application of a force F to the piston causes the fluid volume to decrease from $\Delta V_{\mathrm{l}}$, the gas volume from $\Delta V_{\mathrm{g}}$ and the cylinder volume to increase from $\Delta V_{\mathrm{c}}$ due to an elastic dilatation of the wall as a result of the pressure oscillation $\Delta p$, this is likewise valid for tubes, hoses and components in general. The volumetric oscillation will be as follows:

$$
\begin{equation*}
\Delta V_{t}=\Delta V_{c}-\Delta V_{l}-\Delta V_{g} \tag{10}
\end{equation*}
$$

Under these conditions, the compressibility modulus of the system will be according the Eq. (1),

$$
\begin{equation*}
\beta_{e}=-V_{t} \frac{\Delta p}{\Delta V_{t}} \tag{11}
\end{equation*}
$$

Considering the effect of the volumetric oscillation of the parts,

$$
\begin{align*}
& \beta_{l}=-V_{l} \frac{\Delta p}{\Delta V_{l}}  \tag{12}\\
& \beta_{g}=-V_{g} \frac{\Delta p}{\Delta V_{g}}  \tag{13}\\
& \beta_{c}=-V_{t} \frac{\Delta p}{\Delta V_{c}} \tag{14}
\end{align*}
$$

Combining these expressions with the Eq. 1(10), and rearranging it conveniently, the result is:

$$
\begin{align*}
& \frac{1}{\beta_{e}}=\frac{1}{\beta_{c}}+\left(\frac{V_{l}}{V_{t}}\right) \frac{1}{\beta_{l}}+\left(\frac{V_{g}}{V_{t}}\right) \frac{1}{\beta_{g}}  \tag{15}\\
& \Delta V_{t}=\left[\frac{V_{t}}{\beta_{c}}+\left(\frac{V_{l}}{\beta_{l}}+\frac{V_{g}}{\beta_{g}}\right)\right] \Delta p \tag{16}
\end{align*}
$$

or

$$
\begin{equation*}
\frac{1}{V_{t}} \frac{\Delta V_{t}}{\Delta p}=\frac{1}{\beta_{e}}=\frac{1}{\beta_{c}}+\frac{V_{l}}{V_{t}} \frac{1}{\beta_{l}}+\frac{V_{g}}{V_{t}} \frac{1}{\beta_{g}} \tag{17}
\end{equation*}
$$

In case the gas is solved in the fluid its presence practically does not cause any influence on the compressibility modulus. However if the gas has bubble shape the compressibility modulus of the mixture might suffer a significant reduction causing the system a series of problems. Therefore it is convenient that the gas volume percentage is minimum and ideally non-existent $V_{g} \ll V_{t}$. In this case, it is possible to admit that $V_{1} \cong V_{\mathrm{t}}$ and hence the Eq. (15) is reduced to:

$$
\begin{equation*}
\frac{1}{\beta_{e}}=\frac{1}{\beta_{c}}+\frac{1}{\beta_{l}}+\frac{V_{g}}{V_{t}}\left(\frac{1}{\beta_{g}}\right) \tag{18}
\end{equation*}
$$

If there is no presence of gas, so:

$$
\begin{equation*}
\frac{1}{\beta_{e}}=\frac{1}{\beta_{c}}+\frac{1}{\beta_{l}} \tag{19}
\end{equation*}
$$

In order to use appropriately the equations above it is necessary to consider the compressing process.
In case it is isothermal, meaning a process in which the heat is transferred through the walls of the element in analysis (quasi-steady process), the compressibility modulus of the gas equals the pressure level $\left(\beta_{g}=p\right)$.

In case it is adiabatic, meaning a process in which the temperature oscillates (there is no heat transference through the walls), so the ratio between the specific heats shall be considered in a such a way that $\beta_{g}=\left(c_{p} / c_{\nu}\right) p$.

In order that the air can be used $\left(c_{p} / c_{v}\right)=1,4$.
The experimental determination of the adiabatic compressibility modulus was achieved by the La Habra Laboratory of California, by means of an apparatus which allows the measurement of the density and the ultra sound speed of a fluid under a temperature ranging between $75^{\circ}$ up to $500^{\circ} F$ and under pressures between 1 atm and 5000 psig .

For usual calculations, the compressibility modulus of the steels used in hydraulic systems components is approximately $E_{m}=195,3 \times 10^{4} \operatorname{bar}\left(E_{m}=195,3 \times 10^{9} \mathrm{~Pa}\right)$. The determination of the compressibility modulus of the components depends on the particular geometry and on the tube material they are made of. As for the cylinders of thick walls, with external diameter $D_{e}$ and internal diameter $D_{i}$, width $e$, using a material with elasticity modulus $E_{m}$ and Poisson coefficient $v$ (for metals, $v$ ranges between 0,25 to 0,35 ) the bulk compressibility can be determined by the expression, according to Linsingen (2003):

$$
\begin{equation*}
\beta_{c}=\frac{E_{m} e\left(D_{e}+D_{i}\right)}{(1-v) D_{e}^{2}+(1-v) D_{i}^{2}} \tag{20}
\end{equation*}
$$

In the case of cylinder with walls of width $e=\left(D_{e}-D_{i}\right) / 2$, in which $D_{e}=$ external diameter and $D_{i}=$ internal diameter, using a material with elasticity modulus $E$ and Poisson modulus $\xi$, the following expression can be used:

$$
\begin{equation*}
\frac{1}{\beta_{c}}=\frac{2}{E}\left[\frac{(1+\xi) D_{e}^{2}+(1-\xi) D_{i}^{2}}{2 e\left(D_{e}+D_{i}\right)}\right] \tag{21}
\end{equation*}
$$

For steel tubes of thin walls, considering $\xi=0,25$, the Eq. (21) can be estimated as:

$$
\begin{equation*}
\beta_{c}=\frac{e E}{D_{i}} \tag{22}
\end{equation*}
$$

As for steel tubes of thick walls, in which $D_{e} \gg D_{i}$, the expression will be:

$$
\begin{equation*}
\beta_{c}=\frac{E}{2(1+\xi)} \cong \frac{E}{2,5} \tag{23}
\end{equation*}
$$

In case the width of the wall is equal to the internal radius ( $e=D_{i} / 2$ and $D_{e}=2 D_{i}$ ), so:

$$
\begin{equation*}
\beta_{c}=\frac{3 E}{2(5+3 \xi)} \cong \frac{E}{3,83} \tag{24}
\end{equation*}
$$

For wall width higher than the tube internal radius, the increase of the compressibility modulus is proportionally much smaller.

With the objective to reduce the noise in hydraulic systems and also to facilitate the installation, it has been frequently used hydraulic hoses made of synthetic rubber involving layers of steel frame. The compressibility modulus
of such hoses is relatively lower than the one of rigid tubes, being, according to McCloy \& Martin (1980), the ranging around 70 MPa to $350 \mathrm{MPa}\left(70 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right.$ to $\left.350 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)$.

A great amount of hydraulic systems use hoses, made of synthetic rubber involving layers of steel frame. Such components have a compressibility modulus considerably lower than the one of the metallic channellings, being, according to McCloy \& Martin (1980), the ranging around 70 MPa to 350 MPa ( 700 to 3500 bar).

The effective compressibility modulus of the fluid $\beta e$ considerably oscillates under the operational pressure in hydraulic systems in spite of its value being considered as constant in most published papers. In practice the effective compressibility modulus is difficult to determine (Merrit, 1967).

Jinghong et al (1994) established a theoretical model on which the effective compressibility modulus oscillates together with the fluid pressure. The developed model can be demonstrated by the following expression:

$$
\begin{equation*}
\beta e=\frac{\beta\left(1+p \cdot 10^{-5}\right)^{1+\frac{1}{\gamma}}}{\left(1+p \cdot 10^{-5}\right)^{1+1 \frac{\gamma}{\gamma}}+W \cdot 10^{-5} \cdot\left(1-c_{1} \cdot p\right)\left(\frac{\beta}{\gamma}-10^{5}-p\right)} \tag{25}
\end{equation*}
$$

Where:
$\beta=$ compressibility modulus of oil exempt of air [Pa]
$W=$ the amount of air by oil volume under atmospheric pressure (\%)
$\gamma=\frac{c_{p}}{c_{v}}=$ ratio of specific heats for the air in adiabatic processes
$c_{p}=$ specific heat under constant pressure [J/ kg.K]
$c_{v}=$ specific heat under constant volume [J/ kg.K]
$c_{l}=$ coefficient of bubbles volume oscillation due to the oscillation of the relation of free air and air dissolved in the oil $\left[\mathrm{Pa}^{-1}\right]$

The parameters are fixed when the oil temperature and the pumping conditions are kept constant and when the piping is considered stiff. According to Stringer (1976) the compressibility modulus can be considered constant for many oils with approximately $17 \times 10^{8} \mathrm{~N} / \mathrm{m}^{2}$, although the effective compressibility modulus $\beta e$ of a hydraulic oil on the inside of a system or of closed recipient is smaller than the assumed value due to the dilatation of the recipient and to a possible presence of air in the oil. Nevertheless when the oil is submitted to a pressure increase a fraction of the air dissolves into the oil, not affecting the $\beta e$. Merrit (1967) recommends the adoption of $\beta e=6,9.10^{8} \mathrm{~Pa}$. The values of $\beta e=10.10^{8} \mathrm{~Pa}$ are also accepted (Andrighetto, 1996).

## 4. EXPERIMENTAL APARATUS

A simple experimental technique measures the equivalent compressibility modulus of the oil with the pressure in a hydraulic system. The term "equivalent compressibility modulus of the oil" is used because we take into account both the elasticity effects of the component body and the oil compressibility.

Typical components that may be characterized with this technique include hydraulic lines and tubes, fuel tubes, fuel tubes with pneumatic dampers, accumulators, or any other component where the component adds a significant expansion effect. Remember that this technique is done in quasi-steady state and that dynamic effects may be important in your system (specifically rubber hoses).

The Fig. 3 presents the test used to measure the equivalent compressibility modulus. In this case, hydraulic oil with air tube is under investigation.


Figure 3. Compressibility modulus measurement
The characteristic of the pressure in the tested element in relation to the obtained volume gives the static stiffness of the element.

The equivalent compressibility modulus of the tested element plus the fluid is calculated using the following equation:

$$
\begin{align*}
& \frac{\Delta P}{\Delta V}=\frac{\beta(P)}{V(P)}  \tag{26}\\
& \beta(P)=V(P) \cdot \frac{\Delta P}{\Delta V} \tag{27}
\end{align*}
$$

## 5. RESULTS

The measurements for hydraulic oil with air tube are shown on Tab.1. The tests were made using oil with the compressibility modulus of $0,16 \mathrm{~Pa}$ ( 16000 bar ).

Table 1. Measured pressure and volume

| 1st measurement |  |
| :---: | :---: |
| $\mathbf{P}\left(\mathbf{x ~ 1 0}^{\mathbf{5}} \mathbf{~ P a}\right)$ | $\mathbf{V}(\mathbf{m} \boldsymbol{l})$ |
| 0 | 3.94 |
| 50 | 4.07 |
| 100 | 4.17 |
| 150 | 4.265 |
| 200 | 4.36 |
| 250 | 4.45 |
| 300 | 4.532 |
| 350 | 4.62 |
| 400 | 4.705 |
| 450 | 4.79 |
| 500 | 4.87 |

Equivalent Compressibility modulus of oil and air tube $\mathrm{P} \approx 30 \mathrm{MPa}$, see Fig 4.


Figure 4. Equivalent compressibility modulus of oil and air
As with any experimental work, repeating the process will help to reduce experimental error and refine your final values.

## 6. CONCLUSIONS

Oil pressure is known to have a significant effect on the $\beta e_{\text {value, especially }}$ at the lower range of pressure. One reason for the effect of pressure on $\beta e$ is that variation of oil pressure will change the ratio of entrained air content and dissolved air content in hydraulic oil. As pressure increases, some entrained air becomes dissolved air, hence the dependence of $\beta e$, on pressure is reduced.

The effective compressibility modulus $\beta e$, oscillates with the temperature and according to the amount of dissolved air that may be released by the pressure oscillation of the system. Oil temperature also has an influence on $\beta e$, because it affects the density of the air content, the size of small air bubbles in oil and therefore the equivalent compressibility of oil. Although the effect of oil temperature is very important for dynamic situations, this effect can be ignored when the oil temperature is approximately constant. The effect of tube rigidity on $\beta e$ can also be ignored if rigid tubes are assumed in hydraulic systems. Usually, at any hydraulic system the air is present in both oil in a dissolved form or in bubbles form. The dissolved air is not usually a problem, except at high pressures.

Many factors affecting $\beta e$ have been neglected, large errors are certainly pressure in the dynamic analysis of hydraulic systems.

These preliminary value correlations of compressibility modulus show a considerable potential for development of relations to the predictions of the interrelationships between these values obtained under conditions of hydraulic systems. The limits of pressure and temperature and types of flow under which they apply these correlations have not been fully explored.

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