WEIBULL PARAMETERS ANALISYS USING SIX DIFFERENT METHODS

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Abstract. *The research for new clean energy sources has intensified the use of the wind as an alternative way to the traditional energy sources. However, due to meteorological differences, the wind changes its characteristics. This can happen geographically, in some regions, and sporadically, in several periods of the year. Thereby, the use of mathematical and statistical tools to analyze the wind behavior, in order to evaluate the availability of wind's use in a given site, is clearly needed. The main objective of this work is to present a statistical analysis of the data collected in Paracuru, Ceará, in 2005. To achieve it, the wind speed data were supplied by the Secretaria de Infraestrutura do Estado do Ceará. The data has been collected every second; however, the data storage was the arithmetic mean speed by ten minutes via data logger. This interval is justified by the analysis of the spectrum of local turbulence, which has no peaks near. With the data, Weibull's fit of the frequency distribution was made through six distinct methods. A comparison between the methods was performed, by the analysis of the predicted power from the methods and the measured power and by the RMSE. These parameters have been used to support the statistical analysis and the computational simulation on wind turbine performance.*
Keywords*: wind energy, Weibull curve, wind power prediction.*

1. INTRODUCTION

The increase in energy demand in Brazil has intensified the use of wind to generate energy. The viability of a project involving wind power depends among other variables, on the behavior of wind speed at the site where it's intended to install the wind conversion system. Thus, it is important to check such behavior over time.

Accordingly, since the advantages cited in Hennessey (1977) and confirmed in Justus et. al (1978), many researchers use the two parameter Weibull probability distribution to describe the wind speed behavior. To perform this task, there are different methods for determining the parameters of the Weibull probability distribution.

This article compared six different methods for determining the parameters of the Weibull probability distribution for the city of Paracuru, Ceará, in 2005. The efficiency of the methods was tested through a statistical test (RMSE) and a physical test (Power Error). The main objective of this paper was to determine the method that best fit the data analyzed.

2. METHODOLOGY

2.1. Wind Speed Data

The wind speed data used to estimate the Weibull parameters of the city of Paracuru were supplied by the Secretaria de Infraestrutura do Estado do Ceará and can be accessed through the SEINFRA's homepage.

The anemometer tower in Paracuru was installed at the point of 60 m height with the following coordinates: 03º 24' 42,4" S e 38º 59' 2,8" W.

Three anemometers were used, measuring wind speed in two directions, registering the mean wind speed after each 10 minutes.

2.2. Methods for Evaluating Weibull Parameters

The probability density function $f(v)$ and the cumulative distribution function $F(v)$ of a two parameters Weibull distribution used to describe the wind speed are calculated as follows:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1}\exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (2)$$

where v is the wind speed, k is the dimensionless shape parameter of the distribution and c is the scale factor having the same unity as speed.

For a sample of velocity data collected, it's convenient to calculate the mean and standard deviation of the sample by the equations below, respectively:

$$\bar{v} = \frac{1}{n}\sum_{i=1}^{n} v_i \tag{3}$$

$$\delta = [\frac{1}{n-1}\sum_{i=1}^{n}(v_i - \bar{v})^2]^{1/2} \tag{4}$$

The parameters of the Weibull distribution can be determined from any of several existing methods in the literature. In this work, it was used the six methods discussed in Chang (2010).

2.2.1. Empirical Method

In the empirical method, the k parameter is determined by the empirical relation:

$$k = (\frac{\delta}{\bar{v}})^{-1.086} \tag{5}$$

While the c parameter is calculated as follows:

$$c = \frac{\bar{v}}{\Gamma(1+\frac{1}{k})} \tag{6}$$

Where the gamma function is described by the equation below:

$$\Gamma(x) = \int_0^\infty s^{x-1}\exp(-s)\,ds \tag{7}$$

2.2.2. Graphical Method

In the graphical method, Equation (2) is transformed, by taking double logarithm, resulting in the following equation:

$$\ln\{-\ln[1 - F(v)]\} = kln(v) - kln(c) \tag{8}$$

This equation describes a straight line where the abscissa is given by the values of $ln(v)$ and the ordinate is given by the values of $\ln\{-\ln[1 - F(v)]\}$. Using the wind speed data collected and the concept of least squares method, it is possible to plot the straight line that best fits the data. Thereby, the slope of this straight line is used to determine the k parameter and the intersection with the y axis is used to determine the c parameter.

2.2.3. Moment Method

In this method, the parameters are obtained through numerical iteration. According to this method, the mean speed and the standard deviation can be calculated as follows:

$$\bar{v} = c\Gamma(1 + \frac{1}{k}) \tag{9}$$

$$\delta = c[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)]^{1/2} \tag{10}$$

Thus, dividing Equation (9) by Equation (8), it remains an equation depending only on the k parameter:

$$\frac{\delta}{\bar{v}} = [\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1]^{1/2} \tag{11}$$

After numerical iteration to estimate the k parameter, it is possible to determine the c parameter using Equation (8) or Equation (9).

2.2.4. Energy Pattern Factor Method

In the energy pattern method (Akdag & Dinler, 2008), it is calculated the energy pattern factor as:

$$E_{pf} = \frac{\overline{v^3}}{\bar{v}^3} \quad (12)$$

Where $\overline{v^3}$ is obtained as:

$$\overline{v^3} = \frac{1}{n}\sum_{i=1}^{n} v_i^3 \quad (13)$$

The k parameter is then calculated from:

$$k = 1 + \frac{3.69}{{E_{pf}}^2} \quad (14)$$

While the c is obtained from Equation (6).

2.2.5. Maximum Likelihood Method

The maximum likelihood method also requires the use of numerical iteration for estimating both the k and c parameters. The k parameter is obtained by numerical iterations on the equation below:

$$k = [\frac{\sum_{i=1}^{n} v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} \ln(v_i)}{n}]^{-1} \quad (15)$$

With the value of k, it is possible to estimate c through the following equation:

$$c = (\frac{1}{n}\sum_{i=1}^{n} v_i^k)^{1/k} \quad (16)$$

2.2.6. Maximum Likelihood Modified Method

In the likelihood modified method, the data is organized in intervals. The frequency of the interval, $f(v)$, the number of intervals, n, and the bin center speed, v_i, are used. The parameter k is obtained by numerical iteration on the equation below:

$$k = [\frac{\sum_{i=1}^{n} v_i^k \ln(v_i) f(v_i)}{\sum_{i=1}^{n} v_i^k f(v_i)} - \frac{\sum_{i=1}^{n} \ln(v_i) f(v_i)}{f(v \geq 0)}]^{-1} \quad (17)$$

While c is estimated by iteration on the following equation:

$$c = (\frac{1}{f(v \geq 0)}\sum_{i=1}^{n} v_i^k)^{1/k} \quad (18)$$

2.2.7. RMSE Test

The statistical RMSE value is obtained as:

$$RMSE = [\frac{1}{n}\sum_{i=1}^{n}(y_i - x_i)^2]^{1/2} \quad (19)$$

Where n is the number of intervals, y_i is the observed frequency for the sample, x_i is the Weibull frequency. The lower the RMSE value, the better the method is considered.

2.2.7. Power Error

The power density supplied by wind in a site can be calculated through both the data sample and the Weibull distribution:

$$P_{da} = \frac{1}{2}\rho\overline{v^3} \qquad (20)$$

$$P_{dw} = \frac{1}{2}\rho c^3 \Gamma\left(1 + \frac{3}{k}\right) \qquad (21)$$

where ρ is the air density.

Thus the power error is defined as the difference between the power densities calculated by the Weibull distribution and expected by the data sample:

$$Power\ Error = \left|\frac{P_{dw} - P_{da}}{P_{da}}\right| \qquad (22)$$

Once the Weibull distribution is commercially accepted to estimate the power density for a site, it is important to classify a method based on the power error. The lower the value of the power error the better the method is to predict the amount of energy that a site can generate.

3. RESULTS AND DISCUSSION

The wind speed data for Paracuru, Ceará, in 2005 is analyzed in this paper. The main results are compiled in Table 1 and the graphics below:

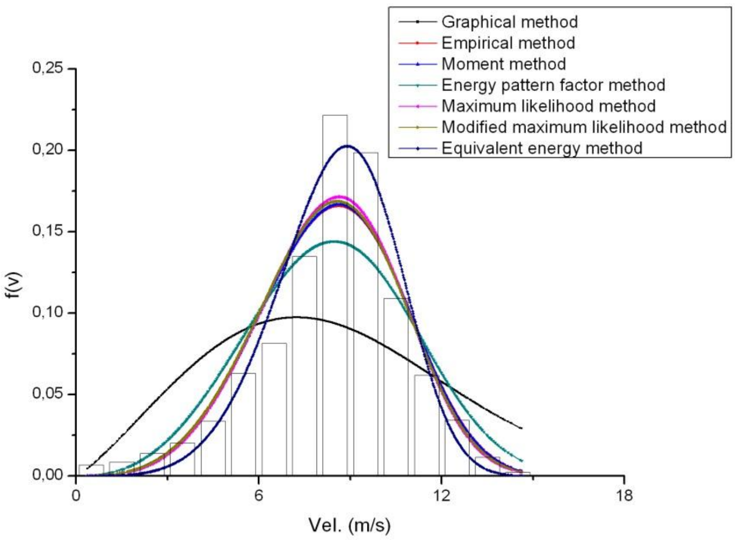

Figure 1. Weibull distribution – January, 2005

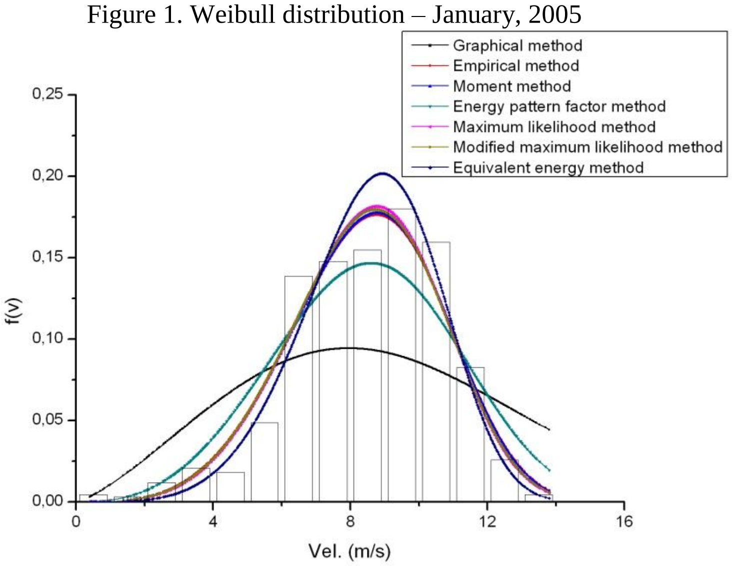

Figure 2. Weibull distribution – February, 2005

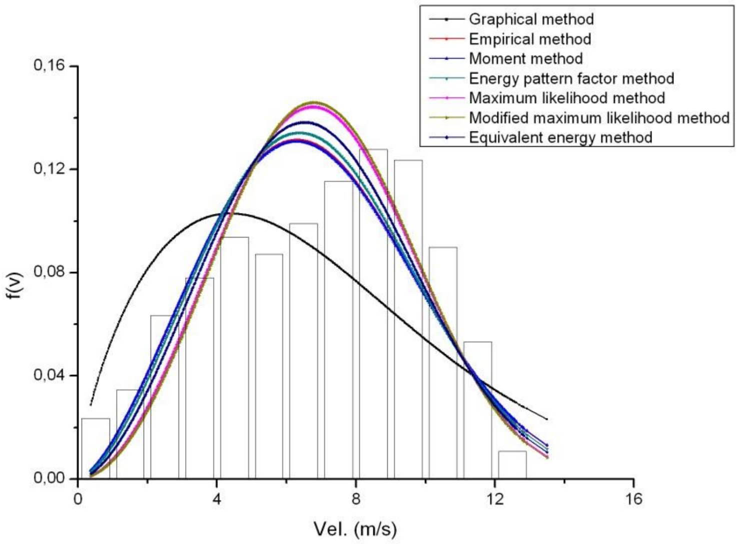

Figure 3. Weibull distribution – March, 2005

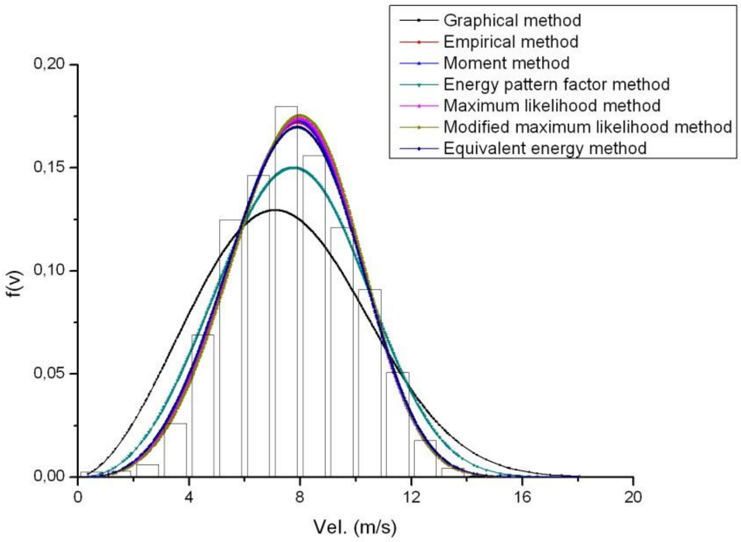

Figure 4. Weibull distribution – April, 2005

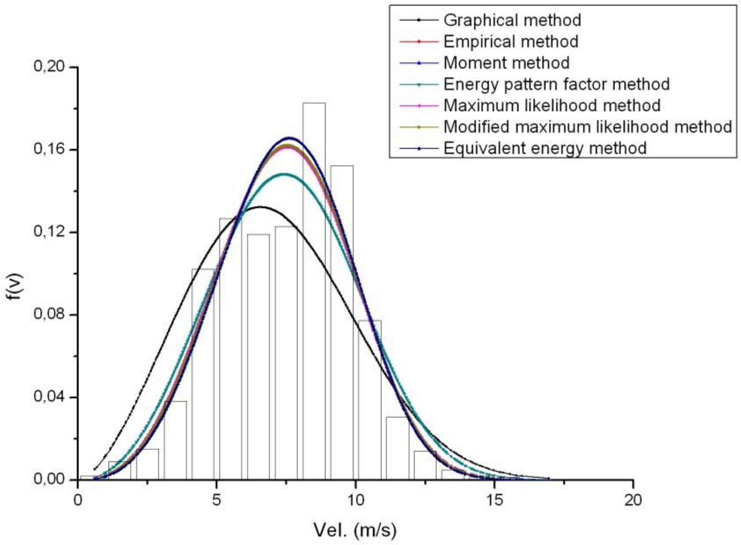

Figure 5. Weibull distribution – May, 2005

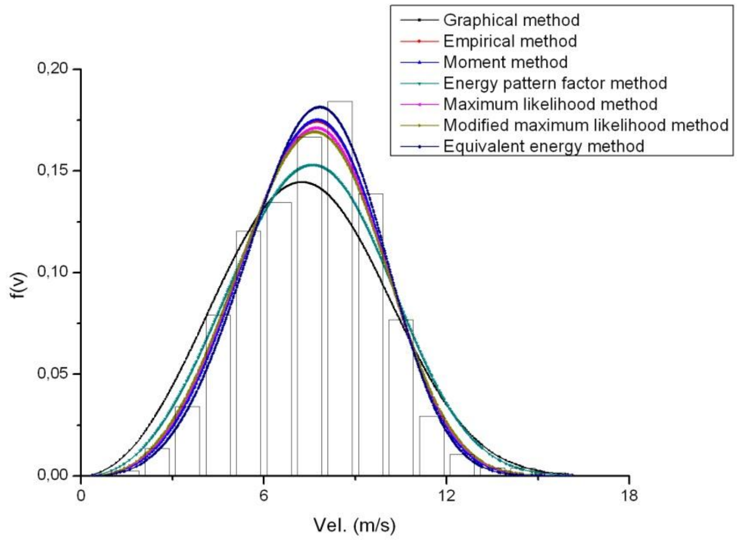

Figure 6. Weibull distribution – June, 2005

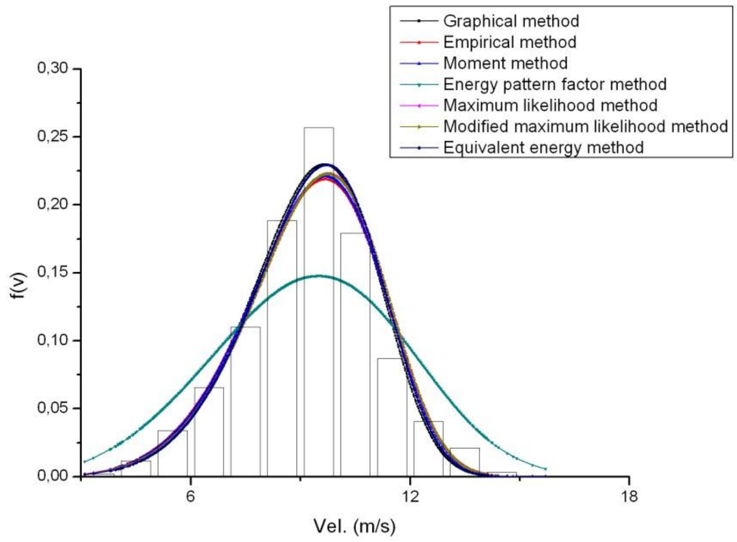

Figure 7. Weibull distribution – July, 2005

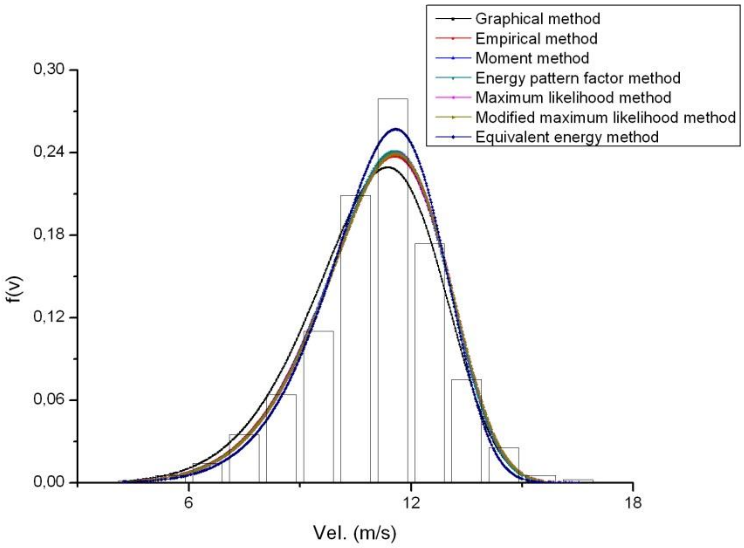

Figure 8. Weibull distribution – August, 2005

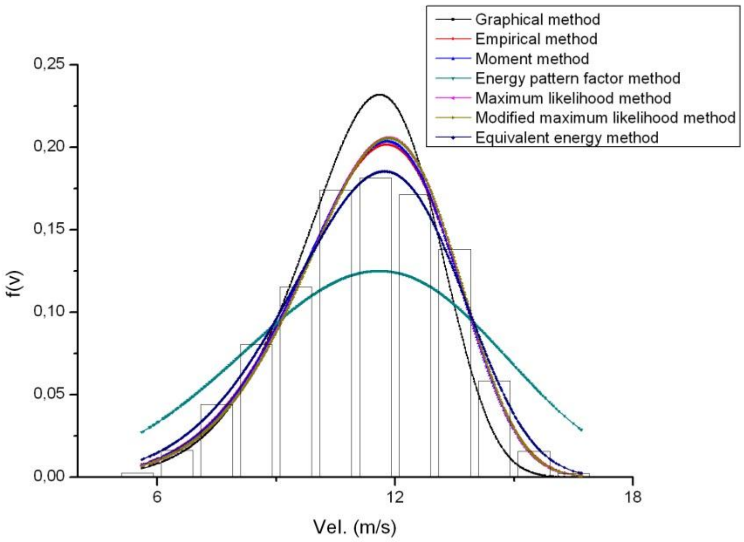

Figure 9. Weibull distribution – October, 2005

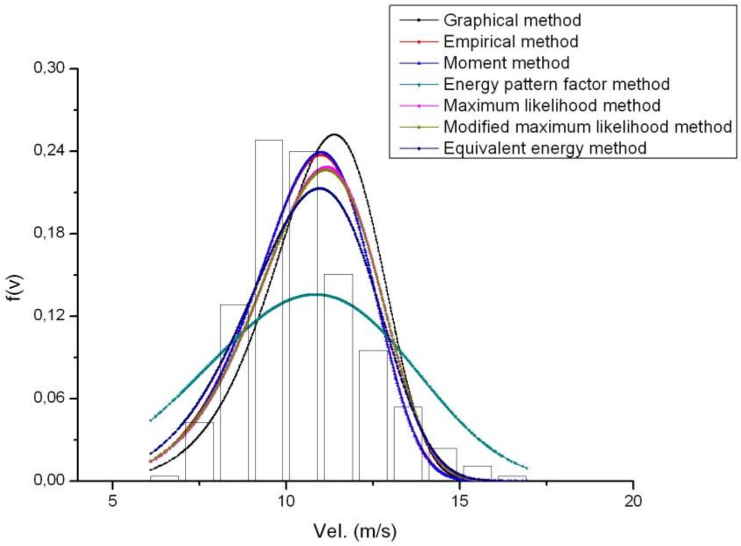

Figure 10. Weibull distribution – November, 2005

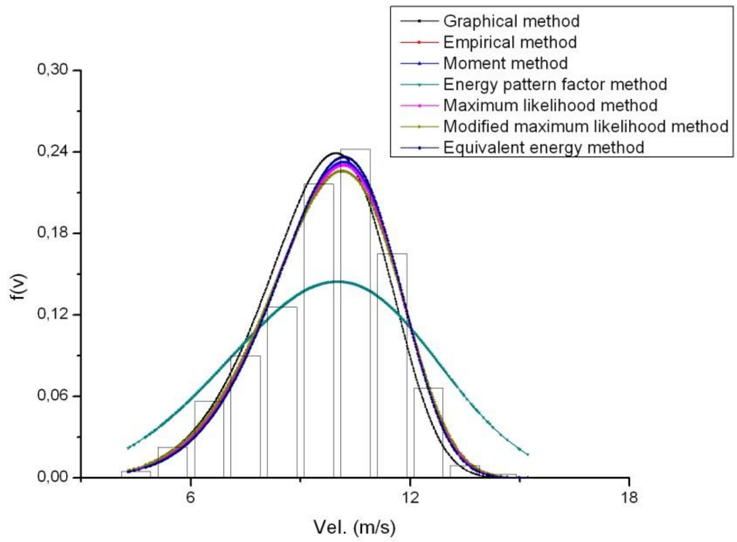

Figure 11. Weibull distribution – December, 2005

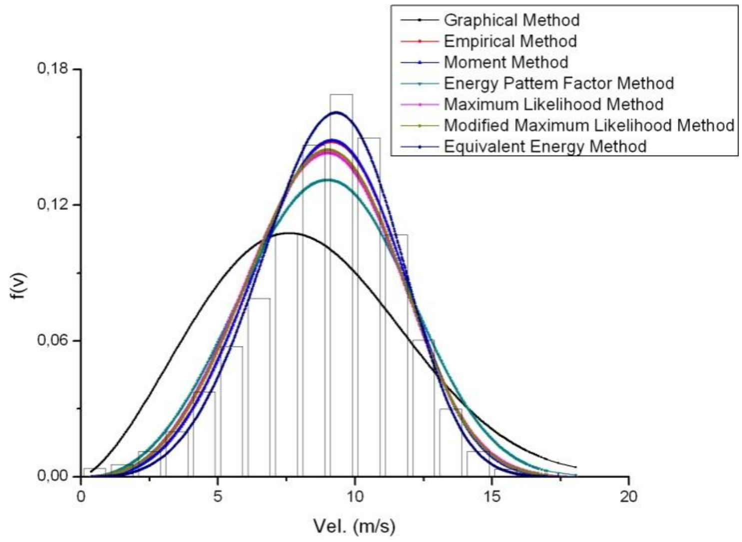

Figure 12. Weibull distribution – Annual curves, 2005

Month	Parameters	Empiric Method	Graphic Method	Moment Method	Energy Pattern Method	ML Method	MLM Method
January	k (-)	4,028600	2,221000	4,053630	3,490750	4,153000	4,078000
	c (m/s)	9,250460	9,480000	9,247190	9,323970	9,205000	9,194000
	RMSE	0,043040	0,037020	0,043230	0,039570	0,044140	0,043680
	Power Error	1,14E-02	4,28E-01	9,24E-03	7,03E-02	8,61E-03	9,12E-03
February	k (-)	4,33566	2,327000	4,367770	3,601260	4,451000	4,398000
	c (m/s)	9,318420	10,12500	9,314440	9,416500	9,274000	9,267400
	RMSE	0,047650	0,033470	0,047960	0,041280	0,048980	0,048540
	Power Error	8,69E-03	6,54E-01	6,37E-03	7,92E-02	9,14E-03	9,71E-03
March	k (-)	2,514280	1,676000	2,500350	2,580340	2,867000	2,912000
	c (m/s)	7,718490	7,545000	7,719560	7,713070	7,840000	7,854000
	RMSE	0,046670	0,039950	0,046490	0,047570	0,050450	0,050960
	Power Error	4,54E-01	2,81E-02	3,22E-02	9,37E-03	1,58E-03	3,00E-04
April	k (-)	3,852250	2,733000	3,872990	3,350190	3,916400	3,962000
	c (m/s)	8,559350	8,390000	8,556770	8,623960	8,593000	8,611000
	RMSE	0,029580	0,024280	0,029790	0,025240	0,030100	0,030510
	Power Error	1,34E-03	6,18E-02	3,36E-03	5,93E-02	7,08E-03	1,11E-02
May	k (-)	3,487660	2,733000	3,498990	3,188370	3,476500	3,500100
	c (m/s)	8,319350	8,390000	8,317920	8,357280	8,300000	8,311000
	RMSE	0,027550	0,024280	0,027650	0,025430	0,027590	0,027700
	Power Error	6,04E-02	6,18E-02	6,18E-02	1,97E-02	6,61E-02	6,42E-02
June	k (-)	3,823520	3,053000	3,843530	3,343050	3,743500	3,694000
	c (m/s)	8,385580	8,260000	8,383130	8,446260	8,370000	8,361000
	RMSE	0,036900	0,031760	0,037100	0,032650	0,036250	0,035850
	Power Error	1,58E-05	2,31E-02	2,00E-03	5,87E-02	9,01E-04	1,02E-03
July	k (-)	5,851110	6,116000	5,907370	3,968680	6,002000	5,997000
	c (m/s)	9,994010	9,931000	9,988690	10,21934	10,05000	10,05000
	RMSE	0,044130	0,046100	0,044530	0,034730	0,045120	0,045090
	Power Error	2,70E-04	1,94E-02	1,55E-03	1,10E-01	1,66E-02	1,66E-02
August	k (-)	7,526320	7,161000	7,586910	7,647270	7,601200	7,593400
	c (m/s)	11,78082	11,61400	11,77609	11,77144	11,80000	11,79900
	RMSE	0,041110	0,039890	0,041420	0,041720	0,041480	0,041440
	Power Error	9,44E-04	4,20E-02	5,65E-05	1,03E-03	6,10E-03	6,10E-03
October	k (-)	6,541340	7,388000	6,602220	4,088880	6,700000	6,694000
	c (m/s)	12,09054	11,84000	12,08455	12,41769	12,12000	12,13000
	RMSE	0,038850	0,046680	0,039300	0,027390	0,039860	0,039780
	Power Error	5,67E-04	5,89E-02	8,99E-04	1,21E-01	7,99E-03	1,05E-02
November	k (-)	7,165680	7,870000	7,227290	4,149190	7,002000	6,934000
	c (m/s)	11,23323	11,58600	11,22829	11,58283	11,39700	11,39200
	RMSE	0,046150	0,054750	0,046560	0,034180	0,046270	0,045730
	Power Error	3,22E-03	9,62E-02	4,38E-02	1,26E-01	4,06E-02	3,91E-02
December	k (-)	6,557000	6,455990	6,516510	4,081930	6,443000	6,315000
	c (m/s)	10,44739	10,22000	10,44215	10,72356	10,43000	10,42200
	RMSE	0,047500	0,050160	0,047960	0,035370	0,047520	0,046680
	Power Error	1,56E-03	6,24E-02	4,69E-05	1,20E-01	3,43E-03	5,65E-03
Annual	k (-)	3,832150	2,488000	3,852000	3,388210	3,658000	3,701000
	c (m/s)	9,897950	9,348000	9,895090	9,963980	9,800000	9,813000
	RMSE	0,037250	0,033200	0,037410	0,033980	0,036470	0,036710
	Power Error	8,71E-03	1,32E-02	6,73E-03	6,22E-02	1,05E-02	9,31E-03

Table 1. Values of k, c, RMSE and Power Error for Paracuru 2005

Graphically, it can be observed that the Weibull distribution has a good fit to the data in general. It is also possible to notice that the methods that best fit the data are the ones that use numerical iteration (moment, maximum likelihood, maximum likelihood modified).

It is possible to notice that the values of RMSE calculated are close to all methods and have low values, confirming the good fit of the obtained curves to the speed histogram. It can be verified that the method with the lowest value of RMSE is the graphical method, despite the fact that it obtained the worse fit to the speed histogram. It can be explained due to the fact that such method seeks to minimize the squared errors directly.

The analysis of power error brings values in the order of 10^{-2} and 10^{-3}. In general, it can be noticed that the methods that use numerical iterations presented the lowest power error. In special, the moment method presented the lowest error for the annual analysis.

4. CONCLUSION

It was performed and compared six different methods to the determination of the Weibull parameters for the city of Paracuru, Ceará, in the year of 2005. The obtained results allow concluding that the methods that use numerical iteration are the best for determining the parameters k and c of the Weibull distribution, according to the physical test (power error). For the annual analysis, the moment method is the best method according to the statistical (RMSE) and physical (power error) tests.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

AKDAG, SEYIT A., DINLER, ALI, “A new method to estimate Weibull parameters for wind energy applications”, Journal Energy Conversion and Management, 50, pp. 1761-1766, 2009

AKPINAR, E. KAVAK, AKPINAR, S., “Determination of the wind energy potential for Maden-Elazig”, Journal Energy Conversion and Management, 45, pp. 2901-2914, 2004

CHANG, TIAN PAU, “Performance comparison of six numerical methods in estimating Weibull parameters for wind energy application”, Journal Applied Energy, 88, pp. 272-282, 2010

HENNESSEY, J. P., “Some aspects of wind power statistics”, Journal Applied Meteorology, 16, pp. 119-128, 1977

JUSTUS, C.G, MIKHAIL, A., “Height variation of wind speed and wind distribution statistics”, ERDA ORO / 5108.76 / 4C60, 1976

ZHOU, WEI, YANG, HONGXING, FANG, ZHAOHONG, “Wind power potential and characteristic analysis of the Pearl River Delta region, China”, Journal Renewable Energy, 31, pp. 739-753, 2005

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