ANALYSIS OF DRYING IN SPHERICAL CAPILLARY-POROUS SOLIDS BASED ON LUIKOV EQUATIONS WITH PRESSURE GRADIENT AND INTEGRAL TRANSFORMS

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Abstract. *The Generalized Integral Transform Technique (GITT) is employed in the solution of the Luikov equations including pressure gradient effects for the drying of capillary-porous solids of spherical geometry. A parametric analysis is performed in order to investigate the influence of typical governing parameters for such physical situation. The results for temperature, moisture and pressure fields are produced with different values of the governing parameters illustrating the potential of the GITT approach in solving drying problems with significant pressure variations. Also, comparisons with previous results in the literature are also performed, in order to validate the numerical codes developed in the present work and to demonstrate the consistency of the final results.*

Keywords*: Drying, Luikov equations, Porous bodies, Spherical geometry, Integral transforms, Hybrid methods.*

1. INTRODUCTION

Simultaneous heat and mass transfer in capillary porous media has received significant attention in the literature due to its application in several fields, such as in drying of building materials, paper recycling, processes of melting and solidification of materials, food processing, control of thermal cycles in metallurgical processes, control of heating and dehumidification of computer chips, among many others.

The mathematical modeling of drying in capillary-porous bodies is described by the well known Luikov equations (Luikov, 1964; Luikov, 1966; Luikov, 1975; Luikov, 1981). This parabolic system for temperature and moisture potential, in its most usual form, has been previously handled through both analytical and numerical approaches. For linear problems, formal exact solutions were obtained by integral transforms in (Mikhailov, 1973) and numerical results for specific examples were later presented (Mikhailov and Shishedjiev, 1975). These and related contributions were also summarized in the work of (Mikhailov and Özisik, 1984), which deals with the basic mathematical tool behind such developments, the integral transform method. Later on, it was observed that several of the early computational works could be in error, particularly for those results reported at smaller values of the dimensionless time, due to the occurrence of complex eigenvalues not accounted for in such earlier contributions (Lobo *et al.*, 1987). For this reason, (Ribeiro *et al.*, 1993) proposed an alternative analytical solution to Luikov's equations in linear formulation, based on an extension of the ideas in the Generalized Integral Transform Technique, GITT (Cotta, 1993; Cotta and Mikhailov, 1997; Cotta, 1998). Through this approach, the complex eigenvalues are completely avoided by considering instead a pair of classical Sturm-Liouville problems for the temperature and moisture eigenfunctions expansions, which involve real quantities only. The excellent convergence characteristics of the proposed expansions were then illustrated for an application in contact drying of a moist porous sheet, and this work was later extended to nonlinear formulations (Ribeiro *et al.*, 1995) and inverse problem analysis (Dantas *et al.*, 2002; Dantas *et al.*, 2003; Dantas *et al.*, 2007).

Particularly when a highly intensive and fast drying process occurs, the pressure gradient within the porous structure of the solid body becomes important and should be included in the formulation of Luikov equations (Lewis and Ferguson, 1990; Irudayaraj *et al.*, 1992; Irudayaraj and Wu, 1994, 1996; Irudayaraj *et al.*, 1996; Chang and Weng, 2002; Datta, 2007). Therefore, this paper further advances the analysis of the drying process in porous solids, now considering the effect of capillary pressure gradients, employing the extended Luikov equations system to model the phenomenon, with three coupled equations for temperature, moisture content and gas pressure, and again utilizing the GITT approach. Numerical results are obtained for a spherical geometry and compared with those obtained with the subroutine NDSolve for numerical solution of partial differential equations from the *Mathematica* platform (Wolfram, 2005), and those obtained by Pandey *et al.* (2000), for a particular situation with negligible pressure effects.

2. ANALYSIS

2.1. Problem Formulation

The problem here studied is physically described by simultaneous heat and mass transfer within a spherical capillary porous solid, with initial conditions of uniform temperature and moisture, as illustrated in Figure 1. The porous solid is in contact with a stream of hot air that exchanges heat and moisture by diffusion and convection. The mathematical formulation of this problem is defined by the Luikov system of equations (Luikov, 1975; Luikov, 1980), which in dimensionless form is written as:

$$\frac{\partial\theta}{\partial\tau}=K_{11}\nabla^2\theta+K_{12}\nabla^2\phi+K_{13}\nabla^2P;\ \frac{\partial\phi}{\partial\tau}=K_{21}\nabla^2\theta+K_{22}\nabla^2\phi+K_{23}\nabla^2P;\ \frac{\partial P}{\partial\tau}=K_{31}\nabla^2\theta+K_{32}\nabla^2\phi+K_{33}\nabla^2P, \quad \text{in } 0<R<1,\quad \tau>0 \tag{1.a-c}$$

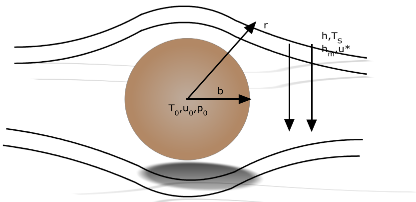


Figure 1.Geometric configuration of the drying problem in a spherical capillary porous solid.

where,

$$K_{11}=1+\varepsilon KoLuPn;\ K_{12}=-\varepsilon KoLu;\ K_{13}=\varepsilon BuLu_p \tag{2.a-c}$$

$$K_{21}=-LuPn;\ K_{22}=Lu;\ K_{23}=-\frac{BuLu_p}{Ko} \tag{2.d-f}$$

$$K_{31}=-\varepsilon\frac{KoLuPn}{Bu};\quad K_{32}=\varepsilon\frac{KoLu}{Bu};\quad K_{33}=(1-\varepsilon)Lu_p \tag{2.g-i}$$

with initial and boundary conditions

$$\theta(R,0)=\phi(R,0)=P(R,0)=0,\quad 0\le R\ge 1 \tag{3.a-c}$$

$$\frac{\partial\theta(0,\tau)}{\partial R}=\frac{\partial\phi(0,\tau)}{\partial R}=\frac{\partial P(0,\tau)}{\partial R}=0,\quad \tau>0 \tag{4.a-c}$$

$$\frac{\partial\theta(1,\tau)}{\partial R}+Bi\theta(1,\tau)+(1-\varepsilon)Bi_mKoLu\left[1-\phi(1,\tau)\right]=Bi,\quad \tau>0 \tag{5.a}$$

$$\frac{\partial\phi(1,\tau)}{\partial R}+Bi_m^*\phi(1,\tau)=Bi_m^*-BiPn\left[\theta(1,\tau)-1\right]+\frac{BuLu_p}{KoLu}\frac{\partial P(1,\tau)}{\partial R};\ P(1,\tau)=0,\quad \tau>0 \tag{5.b,c}$$

The dimensionless quantities used in Eqs. (1) to (5) are given by:

$$R=\frac{r}{b};\ \tau=\frac{\alpha t}{b^2};\ \theta(R,\tau)=\frac{T(r,t)-T_0}{T_s-T_0};\ \phi(R,\tau)=\frac{u_0-u(r,t)}{u_0-u^*};\ P(R,\tau)=\frac{p(r,t)-p_0}{p_0};\ Bi=\frac{hb}{k};\ Bi_m=\frac{h_mb}{k_m} \tag{6.a-g}$$

$$Bi_m^*=Bi_m\left[1-(1-\varepsilon)KoLuPn\right];\ Bu=\frac{r'c_p\,p_0}{c(T_s-T_0)};\ Ko=\frac{r'(u_0-u^*)}{c(T_s-T_0)};\ Lu=\frac{\alpha_m}{\alpha};\ Lu_p=\frac{\alpha_p}{\alpha};\ Pn=\frac{\delta(T_s-T_0)}{(u_0-u^*)} \tag{6.h-m}$$

The physical properties of the porous medium that appear in Eqs. (6) above are the thermal diffusivity (α), moisture diffusivity (α_m), vapor diffusion coefficient for filtration motion (α_p), thermogradient coefficient (δ), phase conversion factor (ε), specific heat (c), coefficient of humid air capacity (c_p), thermal conductivity (k), moisture conductivity (k_m), and latent heat of evaporation of water (r'). The remaining physical quantities that appear in Eqs. (6) are the sphere radius (b), heat transfer coefficient (h), mass transfer coefficient (h_m), initial pressure distribution in the medium (p_0), temperature of the surrounding air (T_s), initial temperature in the medium (T_0), moisture content of the surrounding air (u^*), and initial moisture content in the medium (u_0). In addition, *Bu*, *Ko*, *Lu*, Lu_p and *Pn* represent the Bulygin, Kossovich, Luikov, Luikov for filtration and Posnov numbers, respectively.

2.2. Solution methodology

The Generalized Integral Transform Technique (GITT) is a hybrid analytical-numerical solution methodology, well known for providing accurate error controlled results in different applications in diffusion and convection-diffusion (Cotta, 1993; Cotta and Mikhailov, 1997; Cotta, 1998). The advantage of this technique is due to its hybrid nature, from which an analytical solution is determined in all the independent variables involved, except for one, for which the solution is obtained numerically by the solution of the resulting ordinary differential system from the integral transformation process. This technique has been applied successfully to obtain solutions for different classes of problems including various non-linear situations, such as the Luikov equations for temperature and moisture content (Ribeiro *et al.*, 1993; Ribeiro *et al.*, 1995). Therefore, following the steps in the GITT approach, the extended Luikov system of equations in a spherical body as given by Eqs. (1) to (5) is first homogenized by application of simple filtering functions for the temperature and moisture fields. This filtering procedure improves the computational performance, since the boundary conditions for such potentials become homogeneous in the direction of the eigenfunction expansion. The filtered potentials are defined from:

$$\theta(R,\tau) = 1 + \theta_h(R,\tau); \quad \phi(R,\tau) = 1 + \phi_h(R,\tau) \tag{7.a,b}$$

Introducing Eqs. (7) into Eqs. (1) to (5), the following homogenous formulation is obtained:

$$\frac{\partial \theta_h}{\partial \tau} = K_{11}\nabla^2\theta_h + K_{12}\nabla^2\phi_h + K_{13}\nabla^2 P, \quad 0 < R < 1, \quad \tau > 0 \tag{8.a}$$

$$\frac{\partial \phi_h}{\partial \tau} = K_{21}\nabla^2\theta_h + K_{22}\nabla^2\phi_h + K_{23}\nabla^2 P, \quad 0 < R < 1, \quad \tau > 0 \tag{8.b}$$

$$\frac{\partial P}{\partial \tau} = K_{31}\nabla^2\theta_h + K_{32}\nabla^2\phi_h + K_{33}\nabla^2 P, \quad 0 < R < 1, \quad \tau > 0 \tag{8.c}$$

$$\theta_h(R,0) = \phi_h(R,0) = -1; \;\; P(R,0) = 0, \quad 0 \le R \ge 1 \tag{9.a-c}$$

$$\frac{\partial \theta_h(0,\tau)}{\partial R} = \frac{\partial \phi_h(0,\tau)}{\partial R} = \frac{\partial P(0,\tau)}{\partial R} = 0, \quad \tau > 0 \tag{10.a-c}$$

$$\frac{\partial \theta_h(1,\tau)}{\partial R} + Bi\theta_h(1,\tau) = (1-\varepsilon)Bi_m KoLu\phi_h(1,\tau), \quad \tau > 0 \tag{11.a}$$

$$\frac{\partial \phi_h(1,\tau)}{\partial R} + Bi_m^*\phi_h(1,\tau) = -BiPn\theta_h(1,\tau) + \frac{BuLu_p}{KoLu}\frac{\partial P(1,\tau)}{\partial R}, \quad \tau > 0 \tag{11.b}$$

$$P(1,\tau) = 0, \quad \tau > 0 \tag{11.c}$$

Following the GITT steps, the auxiliary eigenvalue problems for the temperature, moisture content and pressure fields are taken, respectively, as:

$$\frac{d}{dR}\left[R^2\frac{d\Gamma_i(R)}{dR}\right] + {\gamma_i}^2 R^2\Gamma_i(R) = 0, \quad 0 < R < 1 \tag{12.a}$$

$$\frac{d\Gamma_i(0)}{dR} = 0; \qquad \frac{d\Gamma_i}{dR} + Bi\Gamma_i(1) = 0 \tag{12.b,c}$$

$$\frac{d}{dR}\left[R^2\frac{d\chi_i(R)}{dR}\right] + {\nu_i}^2 R^2\chi_i(R) = 0, \quad 0 < R < 1 \tag{13.a}$$

$$\frac{d\chi_i(0)}{dR} = 0; \qquad \frac{d\chi_i}{dR} + Bi_m^*\chi_i(1) = 0 \tag{13.b,c}$$

$$\frac{d}{dR}\left[R^2\frac{d\psi_i(R)}{dR}\right] + {\mu_i}^2 R^2\psi_i(R) = 0, \quad 0 < R < 1 \tag{14.a}$$

$$\frac{d\psi_i(0)}{dR} = 0; \qquad \psi_i(1) = 0 \tag{14.b,c}$$

Such eigenvalue problems are analytically solved, to yield, the following eigenfunctions, eigenvalues, orthogonality properties and norms for the temperature, moisture and pressure expansions, respectively:

$$\Gamma_i(R)=\frac{sin(\gamma_i R)}{R};\quad tan(\gamma_i)=\frac{\gamma_i}{(1-Bi)},\ \text{i=1,2,3,...};\quad \int_0^1 R^2\Gamma_i\Gamma_j dR=\begin{cases}0, & i\neq j\\ L_i, & i=j\end{cases};\quad L_i=\int_0^1 R^2\Gamma_i^2 dR=\frac{1}{2} \tag{15.a-e}$$

$$\chi_i(R)=\frac{sin(\nu_i R)}{R};\quad tan(\nu_i)=\frac{\nu_i}{(1-Bi_m^*)},\ \text{i=1,2,3,...};\quad \int_0^1 R^2\chi_i\chi_j dR=\begin{cases}0, & i\neq j\\ M_i, & i=j\end{cases};\quad M_i=\int_0^1 R^2\chi_i^2 dR=\frac{1}{2} \tag{16.a-e}$$

$$\psi_i(R)=\frac{sin(\mu_i R)}{R};\quad \mu_i=i\pi;\ \text{i=1,2,3,...};\quad \int_0^1 R^2\psi_i\psi_j dR=\begin{cases}0, & i\neq j\\ N_i, & i=j\end{cases};\quad N_i=\int_0^1 R^2\psi_i^2 dR=\frac{1}{2} \tag{17.a-e}$$

After the choice of suitable auxiliary problems, the next step is the definition of the integral transform pairs, in the following form:

- For the temperature field:

$$\bar{\theta}_i(\tau)=\int_0^1 R^2\tilde{\Gamma}_i(R)\theta_h(R,\tau)dR,\quad \text{transform};\quad \theta_h(R,\tau)=\sum_{i=1}^{\infty}\tilde{\Gamma}_i(R)\bar{\theta}_i(\tau),\ \text{inverse};\quad \tilde{\Gamma}_i(R)=\frac{\Gamma_i(R)}{\sqrt{L_i}} \tag{18.a-c}$$

- For the moisture field:

$$\bar{\phi}_i(\tau)=\int_0^1 R^2\tilde{\chi}_i(R)\phi_h(R,\tau)dR,\quad \text{transform};\quad \phi_h(R,\tau)=\sum_{i=1}^{\infty}\tilde{\chi}_i(R)\bar{\phi}_i(\tau),\ \text{inverse};\quad \tilde{\chi}_i(R)=\frac{\chi_i(R)}{\sqrt{M_i}} \tag{19.a-c}$$

- For the pressure field:

$$\bar{P}_i(\tau)=\int_0^1 R^2\tilde{\psi}_i(R)P(R,\tau)dR,\quad \text{transform};\quad P(R,\tau)=\sum_{i=1}^{\infty}\tilde{\psi}_i(R)\bar{P}_i(\tau),\ \text{inverse};\quad \tilde{\psi}_i(R)=\frac{\psi_i(R)}{\sqrt{N_i}} \tag{20.a-c}$$

The next task is thus to actually promote the integral transformation of the filtered partial differential system given by Eqs. (8) to (11). For this purpose, Eqs. (8.a) to (8.c), and the initial conditions (9.a) to (9.c) are multiplied by $R^2\tilde{\Gamma}_i(R)$, $R^2\tilde{\chi}_i(R)$ and $R^2\tilde{\psi}_i(R)$, respectively, integrated over the domain [0,1] in R, and the inverse formulae given by Eqs. (18.b), (19.b) and (20.b) are employed in place of the potentials. After the required manipulations, the following coupled ordinary differential system results, for the calculation of the transformed potentials:

$$\frac{d\bar{\theta}_i}{d\tau}+K_{11}\gamma_i^2\bar{\theta}_i(\tau)+K_{12}\gamma_i^2\sum_{j=1}^{\infty}A_{ij}\phi_j(\tau)+K_{13}\gamma_i^2\sum_{j=1}^{\infty}B_{ij}\bar{P}_j(\tau)=\tilde{\Gamma}_i(1)KoLu\left\{\phi_h(1,\tau)(Bi_m-\varepsilon Bi)+\varepsilon BiPn\theta_h(1,\tau)\right\} \tag{21.a}$$

$$\frac{d\bar{\phi}_i}{d\tau}+K_{21}\nu_i^2\sum_{j=1}^{\infty}C_{ij}\bar{\theta}_j(\tau)+K_{22}\nu_i^2\bar{\phi}_i(\tau)+K_{23}\nu_i^2\sum_{j=1}^{\infty}D_{ij}\bar{P}_j(\tau)=-\tilde{\chi}_i(1)LuPn\left[\theta_h(1,\tau)Bi_m^*-(1-\varepsilon)Bi_mKoLu\phi_h(1,\tau)\right] \tag{21.b}$$

$$\frac{d\bar{P}_i}{d\tau}+K_{31}\mu_i^2\sum_{j=1}^{\infty}E_{ij}\bar{\theta}_j(\tau)+K_{32}\mu_i^2\sum_{j=1}^{\infty}F_{ij}\phi_j(\tau)=\tilde{\psi}_i'\frac{KoLu\varepsilon}{Bu}\left[Pn\theta_h(1,\tau)-\phi_h(1,\tau)\right] \tag{21.c}$$

$$\bar{\theta}_i(0)=\bar{f}_i;\quad \bar{\phi}_i(0)=\bar{g}_i;\quad \bar{P}_i(0)=0 \tag{22.a-c}$$

where,

$$A_{ij}=\int_0^1 R^2\tilde{\Gamma}_i(R)\tilde{\chi}_j(R)dR;\quad B_{ij}=\int_0^1 R^2\tilde{\Gamma}_i(R)\tilde{\psi}_j(R)dR;\quad C_{ij}=\int_0^1 R^2\tilde{\chi}_i(R)\tilde{\Gamma}_j(R)dR;\quad D_{ij}=\int_0^1 R^2\tilde{\chi}_i(R)\tilde{\psi}_j(R)dR \tag{23.a-d}$$

$$E_{ij}=\int_0^1 R^2\tilde{\psi}_i(R)\tilde{\Gamma}_j(R)dR;\quad F_{ij}=\int_0^1 R^2\tilde{\psi}_i(R)\tilde{\chi}_j(R)dR;\quad \bar{f}_i=-\int_0^1 R^2\tilde{\Gamma}_i(R)dR;\quad \bar{g}_i=-\int_0^1 R^2\tilde{\chi}_i(R)dR \tag{23.e-h}$$

2.3. Integral balance scheme

In order to improve the convergence behavior of the final solution, it is recommendable to avoid the evaluation of the boundary potentials that appear in Eqs. (21) and (22), directly through the inverse formulae, Eqs.(18.b, 19.b), since the chosen eigenvalue problems do not exactly match the original problem boundary conditions. One alternative is to employ an integral balance over the homogenized equations, so as to extract information from the derivatives of such potentials, which are then substituted into the boundary conditions, thus providing explicit expressions of the boundary potentials expected to offer improved convergence rates in comparison to the original inverse formulae (Cotta and Mikhailov, 1997). To implement this integral balance scheme, Eqs. (8) are integrated over the domain [$0,R$], to yield:

$$\sum_{i=1}^{\infty}\left(\int_0^R \eta^2\tilde{\Gamma}_i(\eta)d\eta\right)\frac{d\bar{\theta}_i}{d\tau} = R^2\left(K_{11}\frac{\partial\theta_h}{\partial R}+K_{12}\frac{\partial\phi_h}{\partial R}+K_{13}\frac{\partial P}{\partial R}\right) \quad (24.a)$$

$$\sum_{i=1}^{\infty}\left(\int_0^R \eta^2\tilde{\chi}_i(\eta)d\eta\right)\frac{d\bar{\phi}_i}{d\tau} = R^2\left(K_{21}\frac{\partial\theta_h}{\partial R}+K_{22}\frac{\partial\phi_h}{\partial R}+K_{23}\frac{\partial P}{\partial R}\right) \quad (24.b)$$

$$\sum_{i=1}^{\infty}\left(\int_0^R \eta^2\tilde{\psi}_i(\eta)d\eta\right)\frac{d\bar{P}_i}{d\tau} = R^2\left(K_{31}\frac{\partial\theta_h}{\partial R}+K_{32}\frac{\partial\phi_h}{\partial R}+K_{33}\frac{\partial P}{\partial R}\right) \quad (24.c)$$

Equations (24.a) and (24.b) are multiplied by K_{22} andK_{12}, respectively, the results are subtracted and the definitions given by Eqs. (2) are used, to obtain:

$$\frac{\partial\theta_h}{\partial R} = \sum_{i=1}^{\infty}G_i(R)\frac{d\bar{\theta}_i}{d\tau}+\varepsilon Ko\sum_{i=1}^{\infty}H_i(R)\frac{d\bar{\phi}_i}{d\tau} \quad (25.a)$$

Similarly, Eqs. (24.a) and (24.b) are multiplied by K_{21} and K_{11}, Eqs. (24.b) and (24.c) are multiplied by K_{31} and K_{21}, respectively, the results are subtracted and the definitions given by Eqs. (2) are used, to obtain:

$$\frac{\partial\phi_h}{\partial R} = Pn\sum_{i=1}^{\infty}G_i(R)\frac{d\bar{\theta}_i}{d\tau}+\frac{(1-\varepsilon+\varepsilon KoLuPn)}{Lu}\sum_{i=1}^{\infty}H_i(R)\frac{d\bar{\phi}_i}{d\tau}+\frac{Bu}{KoLu}\sum_{i=1}^{\infty}I_i(R)\frac{d\bar{P}_i}{d\tau} \quad (25.b)$$

$$\frac{\partial P}{\partial R} = -\frac{\varepsilon Ko}{BuLu_p}\sum_{i=1}^{\infty}H_i(R)\frac{d\bar{\phi}_i}{d\tau}+\frac{1}{Lu_p}\sum_{i=1}^{\infty}I_i(R)\frac{d\bar{P}_i}{d\tau} \quad (25.c)$$

where,

$$G_i(R)=\frac{1}{R^2}\int_0^R\eta^2\tilde{\Gamma}_i(\eta)d\eta;\quad H_i(R)=\frac{1}{R^2}\int_0^R\eta^2\tilde{\chi}_i(\eta)d\eta;\quad I_i(R)=\frac{1}{R^2}\int_0^R\eta^2\tilde{\psi}_i(\eta)d\eta \quad (26.a\text{-}c)$$

Evaluating Eqs. (25) at $R=1$, one obtains:

$$\left.\frac{\partial\theta_h}{\partial R}\right|_{R=1} = \sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\Gamma}_i dR\right)\frac{d\bar{\theta}_i}{d\tau}+\varepsilon Ko\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\chi}_i dR\right)\frac{d\bar{\phi}_i}{d\tau} \quad (27.a)$$

$$\left.\frac{\partial\phi_h}{\partial R}\right|_{R=1} = Pn\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\Gamma}_i dR\right)\frac{d\bar{\theta}_i}{d\tau}+\frac{(1-\varepsilon+\varepsilon KoLuPn)}{Lu}\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\chi}_i dR\right)\frac{d\bar{\phi}_i}{d\tau}+\frac{Bu}{KoLu}\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\psi}_i dR\right)\frac{d\bar{P}_i}{d\tau} \quad (27.b)$$

$$\left.\frac{\partial P}{\partial R}\right|_{R=1} = -\frac{\varepsilon Ko}{BuLu_p}\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\Gamma}_i dR\right)\frac{d\bar{\phi}_i}{d\tau}+\frac{1}{Lu_p}\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\psi}_i dR\right)\frac{d\bar{P}_i}{d\tau} \quad (27.c)$$

Introducing Eqs.(11.a) and (11.b) into Eqs. (27.a) and (27.b), after some algebraic manipulations, it results:

$$-Bi\theta_h(1,\tau)+(1-\varepsilon)Bi_m KoLu\phi_h(1,\tau) = \sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\Gamma}_i dR\right)\frac{d\bar{\theta}_i}{d\tau}+\varepsilon Ko\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\chi}_i dR\right)\frac{d\bar{\phi}_i}{d\tau} \quad (28.a)$$

$$-BiPn\theta_h(1,\tau)-Bi_m^*\phi_h(1,\tau) = Pn\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\Gamma}_i dR\right)\frac{d\bar{\theta}_i}{d\tau}+\frac{(1+\varepsilon KoLuPn)}{Lu}\sum_{i=1}^{\infty}\left(\int_0^1 R^2\tilde{\chi}_i dR\right)\frac{d\bar{\phi}_i}{d\tau} \quad (28.b)$$

Equations (28) are then solved, to yield:

$$\theta_h(1,\tau)=\frac{1}{Bi}\left[\sum_{i=1}^{\infty}\bar{f}_i\frac{d\bar{\theta}_i}{d\tau}+Ko\sum_{i=1}^{\infty}\bar{g}_i\frac{d\bar{\phi}_i}{d\tau}\right];\quad \phi_h(1,\tau)=\frac{1}{Bi_m Lu}\left[\sum_{i=1}^{\infty}\bar{g}_i\frac{d\bar{\phi}_i}{d\tau}\right] \quad (29.a,b)$$

Equations (29) complete the system of equations given by Eqs. (21) and (22), which will be integrated with appropriate subroutines for initial value problems that might present stiffness, such as the subroutine DIVPAG from the IMSL Library (1991).

In order to obtain expressions with better computational performance than the inverse formulae, Eqs.(18.b), (19.b) and (20.b), throughout the space variable domain, the process of integral balance is continued by integrating Eqs. (25) over the domain [*1,R*], after employing Eqs. (29) and (11.c), resulting in:

$$\theta_h(R,\tau)=\frac{1}{Bi}\left[\sum_{i=1}^{\infty}\bar{f}_i\frac{d\bar{\theta}_i}{d\tau}+Ko\sum_{i=1}^{\infty}\bar{g}_i\frac{d\bar{\phi}_i}{d\tau}\right]+\sum_{i=1}^{\infty}J_i(R)\frac{d\bar{\theta}_i}{d\tau}+\varepsilon Ko\sum_{i=1}^{\infty}K_i(R)\frac{d\bar{\phi}_i}{d\tau} \tag{30.a}$$

$$\phi_h(R,\tau)=\frac{1}{Bi_m Lu}\left[\sum_{i=1}^{\infty}\bar{g}_i\frac{d\bar{\phi}_i}{d\tau}\right]+Pn\sum_{i=1}^{\infty}J_i(R)\frac{d\bar{\theta}_i}{d\tau}+\frac{(1-\varepsilon+\varepsilon KoLuPn)}{Lu}\sum_{i=1}^{\infty}K_i(R)\frac{d\bar{\phi}_i}{d\tau}+\frac{Bu}{KoLu}\sum_{i=1}^{\infty}O_i(R)\frac{d\bar{P}_i}{d\tau} \tag{30.b}$$

$$P(R,\tau)=-\frac{\varepsilon Ko}{BuLu_p}\sum_{i=1}^{\infty}K_i(R)\frac{d\bar{\phi}_i}{d\tau}+\frac{1}{Lu_p}\sum_{i=1}^{\infty}O_i(R)\frac{d\bar{P}_i}{d\tau} \tag{30.c}$$

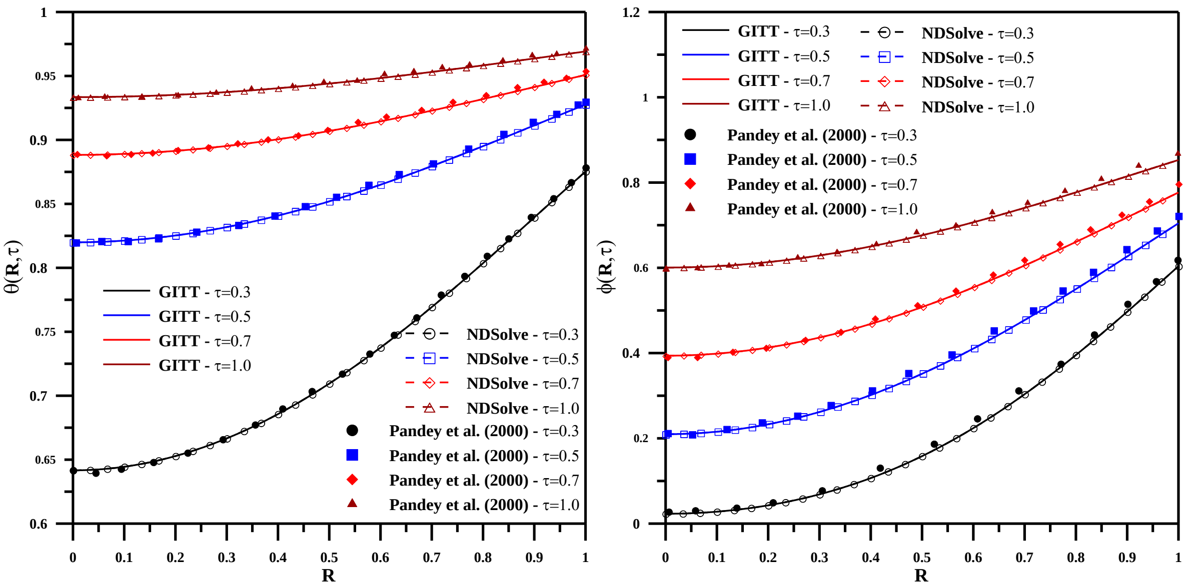
where,

$$J_i(R)=\int_R^1 G_i(\eta)d\eta;\quad K_i(R)=\int_R^1 H_i(\eta)d\eta;\quad O_i(R)=\int_R^1 I_i(\eta)d\eta \tag{31.a-c}$$

3. RESULTS AND DISCUSSION

Numerical results for the temperature, moisture and pressure fields were computed for typical values of the governing parameters, so as to illustrate the advanced methodology. For this purpose, a computational code was developed in the programming language FORTRAN 95/2003. The subroutine DIVPAG from the IMSL Library (1991) was used to handle the coupled system of ODEs, Eqs. (21) and (22), with a relative error target of 10^{-8}. Equal truncation orders in all three eigenfunction expansions, $N=90$, were utilized for computation of the transformed potentials.

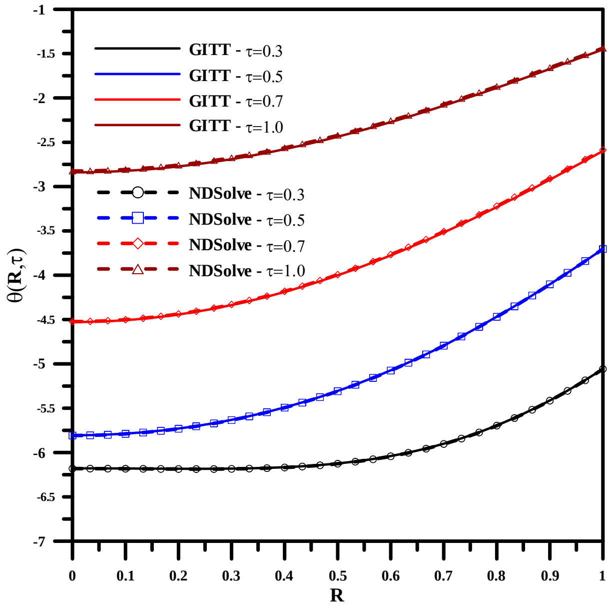
First, the computational code was verified using the previously solved problem without pressure gradient ($Lu_p\to 0$ or $Bu\to\infty$). The values of the dimensionless parameters were taken from the work of Pandey *et al.* (2000), i.e. $\varepsilon=0.25$, $Bi=5$, $Bi_m=2.5$, $Ko=1.2$, $Lu=0.3$, $Pn=0.25$, in order to allow for the direct comparison against their results. A set of comparisons is provided in Figs. 2.a,b for the dimensionless temperature and moisture, respectively. In both figures one can observe the excellent agreement of the present results with those obtained with the numerical routine NDsolve from the *Mathematica* platform (Wolfram, 2005), as well as with those results obtained by Pandey *et al.* (2000).



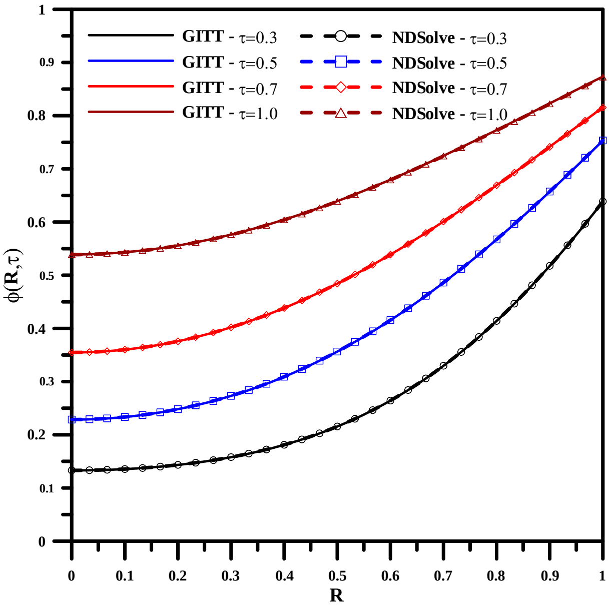
(a) (b)

Figure2. Comparison of the present dimensionless temperature and moisture distributions for the case without pressure gradient against those obtained with the routine NDSolve (Wolfram, 2005) and those of Pandey *et al.* (2000).

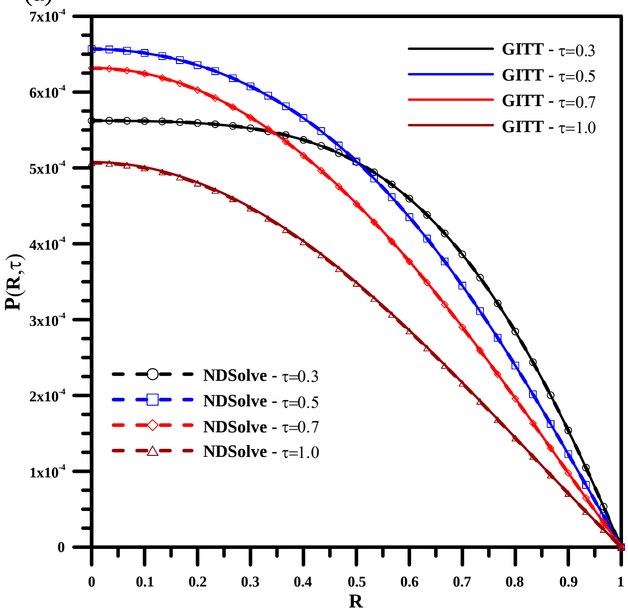
The analysis now proceeds to the situation when the pressure gradient becomes important. The values of the governing parameters were taken as $\varepsilon=0.2$, $Bi=2.5$, $Bi_m=3.3$, $Bu=3432$, $Ko=49$, $Lu=0.24$, $Lu_p=0.5$, $Pn=0.084$. Figures 3.a-c show results of the dimensionless temperature, moisture, and pressure, respectively, along the radial position at different times. One can observe the more pronounced temperature and moisture gradients that are established in the vicinity of the sphere boundary, along most of the transient period, while both potentials move towards the equilibrium values provided by the external air temperature and moisture content. It should be recalled that the dimensionless moisture content increases with time but, on the other hand, dimensional moisture content decreases with time as a result of the drying process. It is also observed that the pressure decreases towards the edge of the sphere, which occurs primarily due to the process of moisture evaporation and to the resistance of the solid body. In addition, in the more internal regions, pressure first increases within the earlier transient, corresponding to the increased amount of vapor in the porous matrix due to the heat transfer process. As time increases further, the capillary transfer of the mixture vapor-gas increases towards the edge, causing an overall decrease in the pressure. It should be noticed that for the test-case of the present analysis, where the Luikov filtration number (Lu_p, ratio between the filtration capacity and the thermal diffusivity) is 0.5, the ability to transfer thermal energy is greater than the filtering capacity of medium. In all three sets of curves, the numerical results achieved with the Method of Lines implemented in routine NDSolve are also presented, providing a practically perfect agreement to the graph scale against the GITT results.



(a)



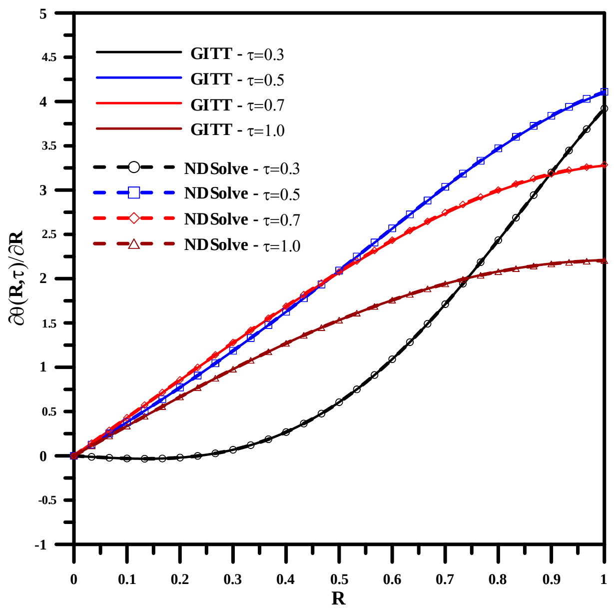
(b)



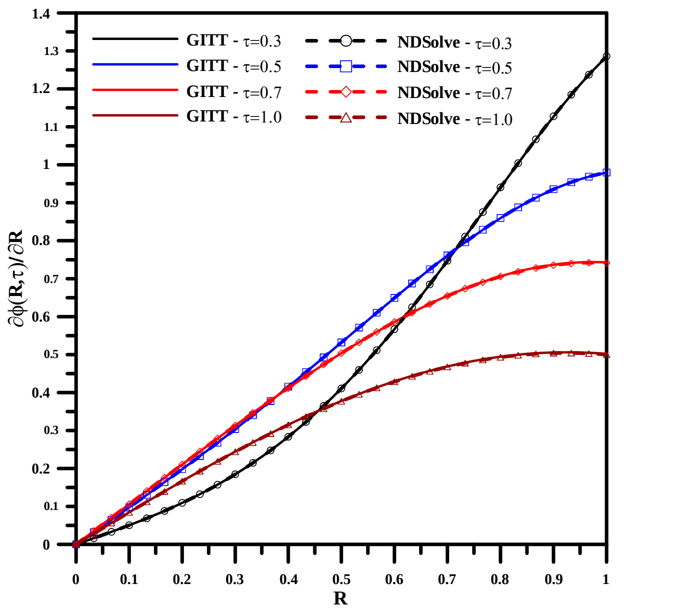
(c)

Figures 3. Comparison of the dimensionless potentials distributions along the radial position at different dimensionless times: (a) temperature; (b) moisture; (c) pressure.

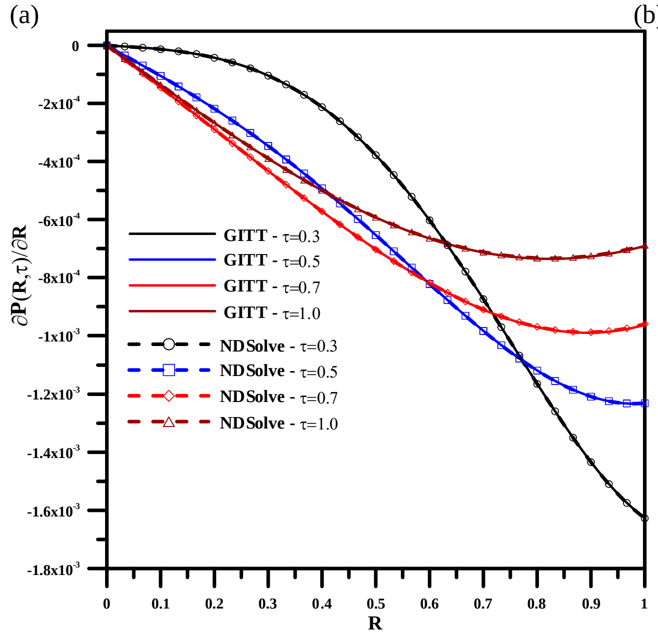
Figures 4.a-c show the temperature, moisture and pressure gradients, respectively, as a function of the dimensionless radial position at different dimensionless times. One shared and expected behavior among all the potentials gradients, is the progressive reduction towards the equilibrium with the external environment, when all the gradients are null. One can also observe that the largest temperature gradient at the edge of the sphere does not occur at the smallest time value, but rather sometime within the middle of the transient process. As for moisture, the large gradients achieved at the early transient are due to the effective evaporation of surface moisture. However, at each time value, the largest gradients (temperature, moisture and pressure) are always at the boundary. Within the more internal region of the sphere, it can be noticed that gradients start increasing to a certain level, to finally start decreasing to the zero value corresponding to the equilibrium values of the potentials as time progresses further. Again, no significant difference has been observed between the results from the present GITT implementation and the numerical method of routine NDSolve (Wolfram, 2005).



(a)



(b)



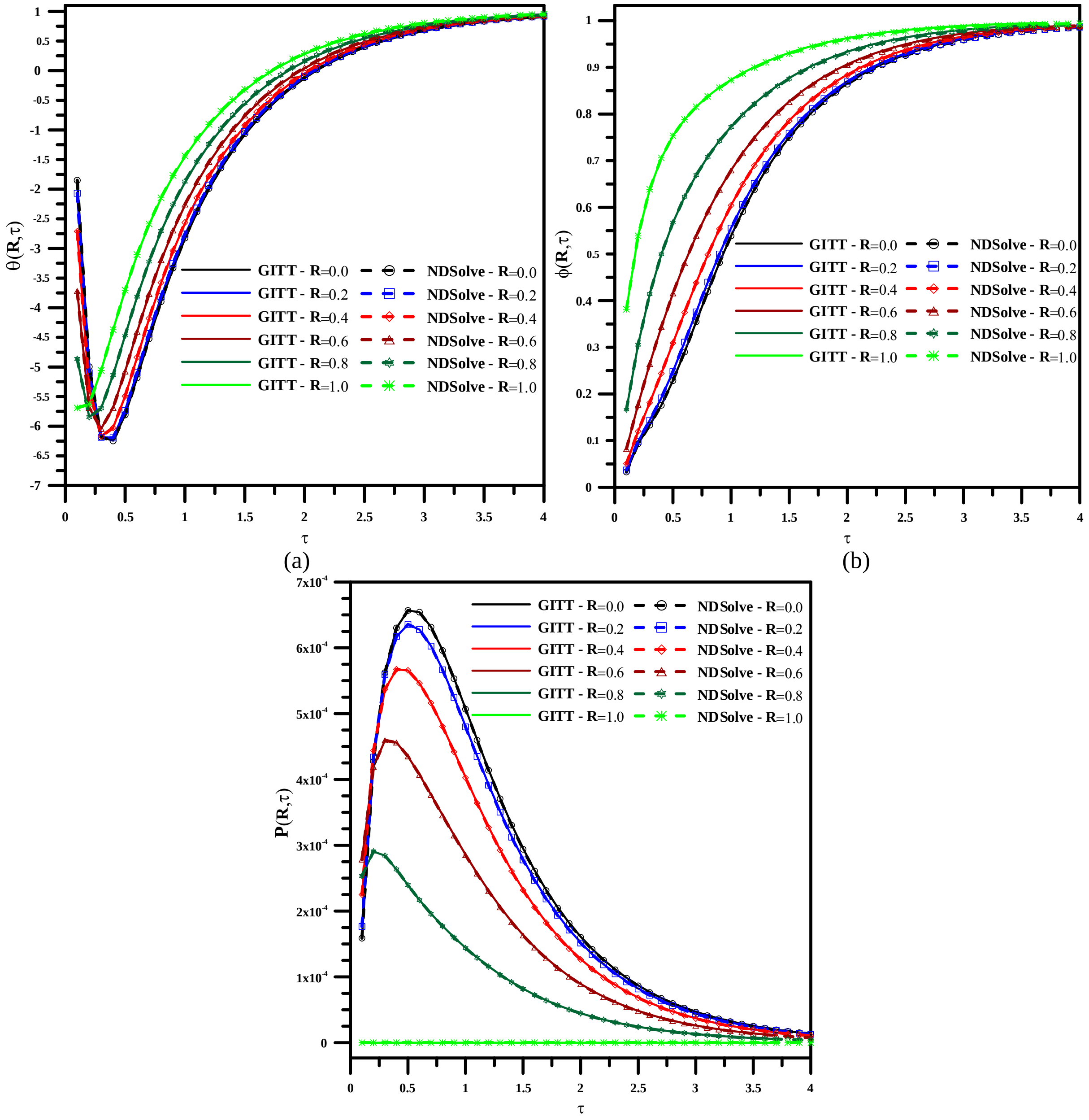
(c)

Figures 4. Comparison of the dimensionless potential gradients as function of the radial position at different dimensionless times: (a) temperature; (b) moisture; (c) pressure.

Figures 5.a-c present the time evolution of the dimensionless temperature, moisture and pressure, respectively, at selected radial positions, R=0.0, 0.2, 0.4, 0.6, 0.8 and 1.0. Again, the expected tendency to the equilibrium values for each of the potentials as time progresses has been reproduced, with the dimensionless temperature and moisture increasing, and the pressure decreasing, from center to surface of the sphere during most of the transient period.

Noticeable exceptions can be clearly deduced from Figures 5.a and 5.c, for the earlier transient period, when temperature first decreases while pressure increases, due to the prevailing moisture evaporation and latent heat transfer, with the consequent water vapor production and pressure increase.

Finally, it should be remarked that the comparison of the present GITT results with those of the routine NDSolve are in excellent agreement for all figures analyzed, once again verifying the computational code here developed.



Figures 5. Comparison of the dimensionless potentials evolution with time at different dimensionless radial positions: (a) temperature; (b) moisture; (c) pressure.

4. CONCLUSIONS

The Generalized Integral Transform Technique was applied in the hybrid numerical-analytical solution of the extended Luikov equations including pressure gradient effects in spherical coordinates. The excellent agreement of the present results with a previously reported numerical implementation for a simpler problem and with those of the numerical routine NDSolve for the same problem demonstrates the consistency of the proposed methodology and verifies the constructed code. A physical analysis was undertaken for typical values of the governing parameters, so as to illustrate the influence of the drying process in the temperature, moisture and pressure fields.

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