Experimental Modal Analysis of Gantry-Type Structure in Presence of Randomness

Neilson Moura da Silva, neilson_moura@hotmail.com
Waldir Correa de Oliveira Netto, wcnetto@hotmail.com
Alberto Carlos Guimarães Castro Diniz, adiniz@unb.br
Mario Olavo Magno de Carvalho, molavo@unb.br
Marcus Vinicius Girão de Morais, mumorais@gmail.com
University of Brasilia, - UnB, Brasilia, DF, 70910-900, Brazil

Abstract. *The Experimental modal analysis results for a Gantry-Type Structure, taking into account the influence of randomness in the parameters involved, is presented. A modal synthesis of substructures, using FRF, is used. The Experimental results using substructuration are compared to global structure modal analysis. The aim to this work is to exemplify important concepts of experimental substructuring techniques and structural modeling considering random parameters. Special attention is concerned to the uncertainty quantification of the model parameters. This study enabled us to evaluate the influence of randomness in the structure parameters on the dispersion of the system response data, thus improving the reliability of experimental results, leading to more robust predictive models.*

Keywords: *substructuration, modal synthesis, stochastic modeling, uncertainty quantification*

1. INTRODUCTION

The objective of this work is to exemplify important concepts of experimental substructuring techniques and structural modeling considering random parameters.

The stochastic modeling aims to develop robust models, that include random aspects, applying statistics principles in the quantification of uncertainties of the problem and in the propagation of these uncertainties at the response of the system. An important step in stochastic modeling is the quantification of the involved uncertainties. Determining the statistical characteristics of the input parameters or the model is essential to obtain reliable results, consistent with the physics of the studied problem. It is very common to adopt a Gaussian distribution to characterize random parameters, but in many problems this hypothesis is far from reality and other types of distributions must be adopted. Differential equations with random coefficients are used to model structures and systems with some kind of randomness in geometry, material properties or boundary conditions (T.T.Soong, 1973). The theory of he random matrices is used to represent the Randomness in the model (Mehta, 2002) .

Substructuring allows the study of complex structures that can be divided in subparts to a better representability of the model and a optimization of the calculus process, reducing the degrees of freedom of the model and allowing the usage of different modeling tools.

The Substructuration methods appeared in the end of Sixties returns being an interesting subject of researches with an increasing use in industrial applications. Having been considered dispensable ahead of the great capacity of calculation of the modern computers, it is verified, currently, a great growth in the use of the substructures in dynamic analysis of industrial equipment formed by subparts.

The aim of the dynamic substructuring techniques is to describe the dynamic behaviour of a whole structure, by analysing independently its subparts. The behaviour of each substructure can be obtained by simulation whereas of another one, for experimental measurements. In way to couple the substructures and determine the behaviour of the global structure is used a compatible space of generalized coordinates to describe all substructures. In the dynamic analysis of structures the space of vibrations modes (eigenmodes) is traditionally used. The experimental modal analysis is a way to determine the dynamic characteristics of the substructures.

The possibility of treat independently substructures using different tools also is applied in the study of structures with random parameters. Substructuration methods allow that each type of uncertainty can be associated with a substructure (Dessombz *et al.*, 1999). The propagation of uncertainties from physicalto component modal coordinates can be treated independently and the deterministic components do not require reanalysis (Hinke *et al.*, 2009). This characteristic is also used to advantage experimental studies (Voormeeren *et al.*, 2010) and industrial applications (Brow, 1998).

This work presents a substructuration methodology, modal analysis and synthesis of a structure considering random properties. The studied structure is showed in the Figure 1. It is a gantry-type structure with six identical modules, but, by another hand, with random stiffness. This structure will be divided in substructures and then the experimental modal analysis of this parts, then modal synthesis will be done. The results will be compared with the ones obtained by modal analysis of the complete structure.

Figure 1. The structure

Figure 2. The module

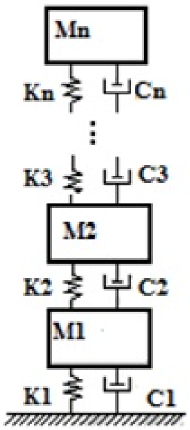

Figure 3. The model

2. MODAL SYNTHESIS WITH FRF

There are a lot of methods of modal synthesis and substructuring. A few of these methods are used in numerical simulations and others in experimental testing. The modal synthesis with Frequency Response Function (FRF) is a simple, intuitive and easy method to apply.

Using the experimental FRFs of the substructures, it is possible to get the FRFs of the entire structure, using a suitable technique, that consider the compatibilities between the interface areas of the substructures. The method was proposed by Crawley *et al.* (1984) and Otte *et al.* (1991), and that can be represented by the Figure 4.

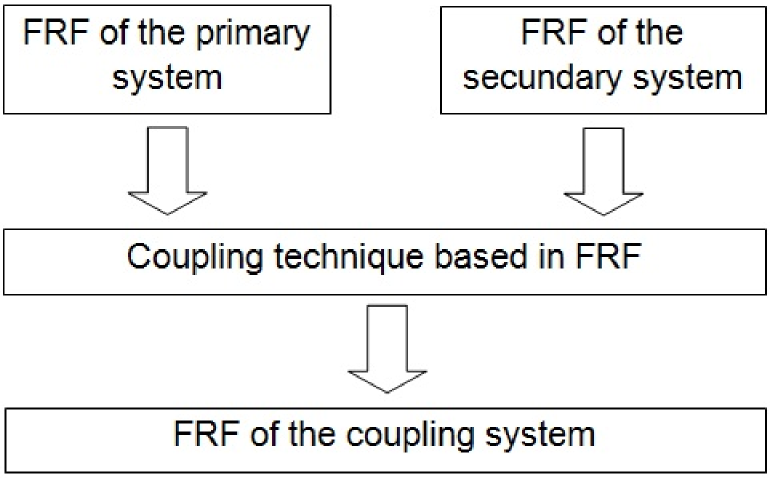

Figure 4. Algorithm to substructure coupling technique based on FRF

When the structure is divided in more than two substructures, the coupling technique is applied in pair of two substructures, and then, the new resultant substructure now is called primary substructure that is coupled to another secondary substructure. The process is applied until the entire structure is totally represented.

Considering the subscripts A, B and C, refers to a primary substructure, secondary substructure and entire structure, respectively, and now, the equations of movement can be wrote.

$$\{X_A\} = [H_A]\{F_A\} \tag{1}$$

$$\{X_B\} = [H_B]\{F_B\} \tag{2}$$

$$\{X_C\} = [H_C]\{F_C\} \tag{3}$$

Where $X_A(\omega)$, $X_B(\omega)$ e $X_C(\omega)$ are the vectors of the harmonics displacements, F_A, F_B e F_C are the vectors of the excitation forces and $[H_A(\omega)]$, $[H_B(\omega)]$ e $[H_C(\omega)]$ are the Frequency Response Function matrices.

One condition of displacement compatibility and one condition of force balance is applied, it means that.

$$\{X_A\} = \{X_B\} = \{X_C\} \tag{4}$$

$$\{F_A\} = \{F_B\} = \{F_C\} \tag{5}$$

Considering that the subscripts R, T and S refers to the internal coordinates of the primary substructure, internal coordinates of the secondary substructure and the shared coordinates between the two substructures, respectively, and then, leaving out the term ω, the coupled structure FRF matrix, with subscripts C, can be wrote using the FRFs of the substructures A and B.

The formation below shows that the inverse of the sum of the shared displacements matrix and the quality of the results depends on this inverse matrix a lot (Cuppens *et al.*, 2000).

$$\begin{bmatrix} H_{C_{RR}} & H_{C_{RS}} & H_{C_{RT}} \\ H_{C_{SR}} & H_{C_{SS}} & H_{C_{ST}} \\ H_{C_{TR}} & H_{C_{TS}} & H_{C_{TT}} \end{bmatrix} = \begin{bmatrix} H_{A_{RR}} & H_{A_{RS}} & 0 \\ H_{A_{SR}} & H_{A_{SS}} & H_{C_{ST}} \\ 0 & 0 & H_{B_{TT}} \end{bmatrix} - \begin{bmatrix} H_{A_{RS}} \\ H_{A_{SS}} \\ H_{B_{TS}} \end{bmatrix} \begin{bmatrix} H_{A_{SS}} + H_{B_{SS}} \end{bmatrix}^{-1} \begin{bmatrix} H_{A_{RS}} \\ H_{A_{SS}} \\ H_{B_{TS}} \end{bmatrix}^{T} \tag{6}$$

The modal synthesis method using FRFs will be applied in a modal synthesis of the gantry-type structure, using two substructures and their experimental FRFs.

3. DETERMINATION OF THE STIFFNESS PROBABILITY DISTRIBUTION IN THE MODEL

The uncertainties that is present in predictive model of mechanical systems is associated to approximations of the proposed model (hypotheses of physical behavior, linearization, boundary conditions) to the randomness of the loading applied to the system and to the uncertainties of its parameters (Soize, 2005). The randomness present in the loading in dynamical systems are part of the grand area of the mechanical random vibration. The uncertainties related to the parameters of the model requires a stochastic treatment “parametric”, where the random parameters are modeled by random variables, while the uncertainties of the model requires a different approach, referred to in literature as“nonparametric” (Soize, 2005).

This work, consider just the randomness associated to the parameters of the model. Under this approach, after identifying the random parameters of the system, these should be treated as random variables defined on a limited support and consistent with the phenomenon studied. So it must determine, based on available information, a probability distribution that accurately represents the variable and is suitable for the physical problem studied (Soize, 2005).

Considering the stiffness of the modules like a random variable, a series of experiments was performed to determinate the probability distribution of the stiffnes. Each part, called “floor’, was tested individually using the apparatus showed in the Figure 5.

Each module was rigidly fixed at a inertial base and was submitted to a known force, that was applied by pneumatic actuator. A comparator clock was used to measure the displacement of the free border. Gradually increasing the applied force, the displacement was measured and then, the stiffness was calculated using a curve fitting. Ten tests were realized to each module with sixteen points to fit a curve for each of the measurements.

A lot of works has adopted a Gaussian distribution to represent the randomness of the stiffness. So, some adherence tests were made in the results obtained with the normal distribution. The Figure 6 and Figure 7 shows the results of the normal tests using the software Action$^{\circledR}$, that has a package of distribution tests. The figures below, shows the results of the normal tests to the third and the fifth module.

With these results, we were led to believe that the normal distribution of probability represents well the randomness, but is well known that at the limits, the normal distribution has negative values that do not happen in physic reality. Because of this the Gaussian distribution could not be used to represent the stiffness.

In the Table 1 can be seen the results of the normal distribution tests and of the Chi-Square (Gamma) distribution test. Despite the P-Values of the Gaussian distribution can be acceptable, the P-Values of the Chi-Square distribution are better, as can be seen in Table 1.

The P-values were obtained using the software Action$^{\circledR}$. The importance of conducting this type of statistical analysis is in determiner a distribution model that best represents the random variable of interest, and approaches to the physical reality. In the case of stiffness, the model chi-square is more appropriate than the Normal template, because the model Chi-square there are negative values of the random variable.

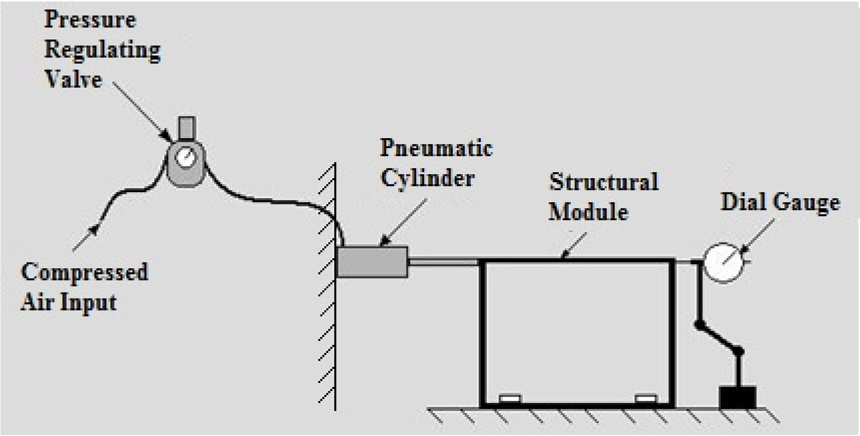

Figure 5. Schematic of the experiment to obtain the stiffness of the floors of the gantry-type structure

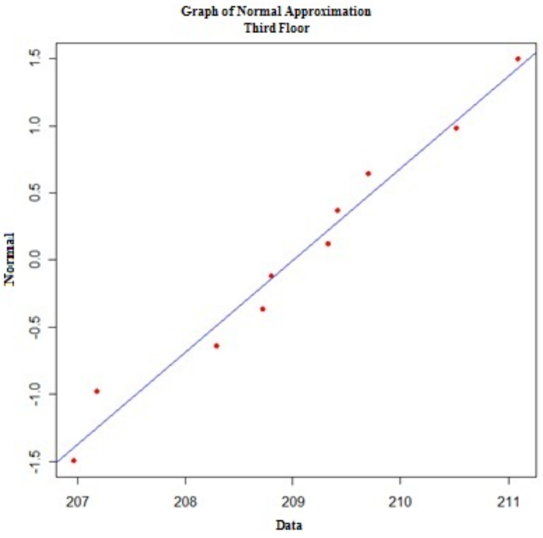

Figure 6. Adherence to the normal distribution to third floor

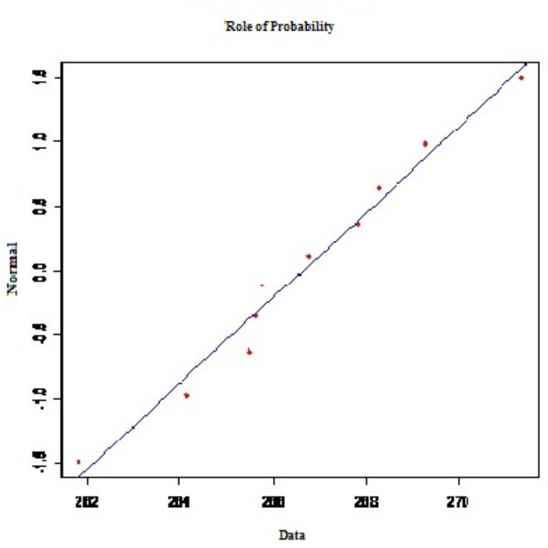

Figure 7. Adherence to the normal distribution to fifth floor

Table 1. *P-Values of the normal and Chi-Square distribution tests.*

Floor	P-Value Normal Distribution	P-Value Gamma Distribution	Floor	P-Value Normal Distribution	P-Value Gamma Distribution
2	0.23	0.99	5	0.99	0.99
3	0.86	0.99	6	0.55	0.99
4	032	099	7	053	099

4. MODAL ANALYSIS OF THE COMPLETE STRUCTURE

In order to get a reference for evaluation of results was performed to analyze the modal structure complete (six floors). We performed a numerical and experimental modal analysis.

For the numerical modal analysis was considered a lumped model with six degrees of freedom and only the transverse modes were considered. The results of experimental modal analysis agreed with this hypothesis since it was found that the boards of the floors acted as a rigid body can be modeled as concentrated mass and the columns as springs. The numerical modal analysis was done using a program developed in MATLAB®. Functions Frequency response experiments were obtained using a force transducer B&K type 8200, an accelerometer B&K type 4366, two amplifiers load B&K type 2635 and Nexus, a power amplifier B&K type 2712 and a Shaker B&K type 4808. The signals were captured by an acquisition system from National Instruments formed by an acquisition board and an interface NI9234 NI cDAQ-9174, connected to a notebook. The excitation signal was generated by an HP 35665A analyzer, using a random signal like Burst Chirp in a frequency range of 0 to 400Hz. Figure 11 shows the routine developed in LabView®, used with virtual instrument for acquisition and signal processing and generation of FRFs.

For statistical purposes, the measurements were repeated in order to memorize ten samples per floor. Figure 8 have the FRF obtained exciting the structure at first floor and measuring the acceleration on the same floor but on the opposite side of the structure. The shape of the FRF indicates that the floor plate acts as a rigid body. In Figure 9 we have the FRF measurement of acceleration on the third floor. Table 2 values we obtained for the natural frequencies in experimental and numerical analysis. The FRF obtained numerically for the first floor is shown in Figure 10.

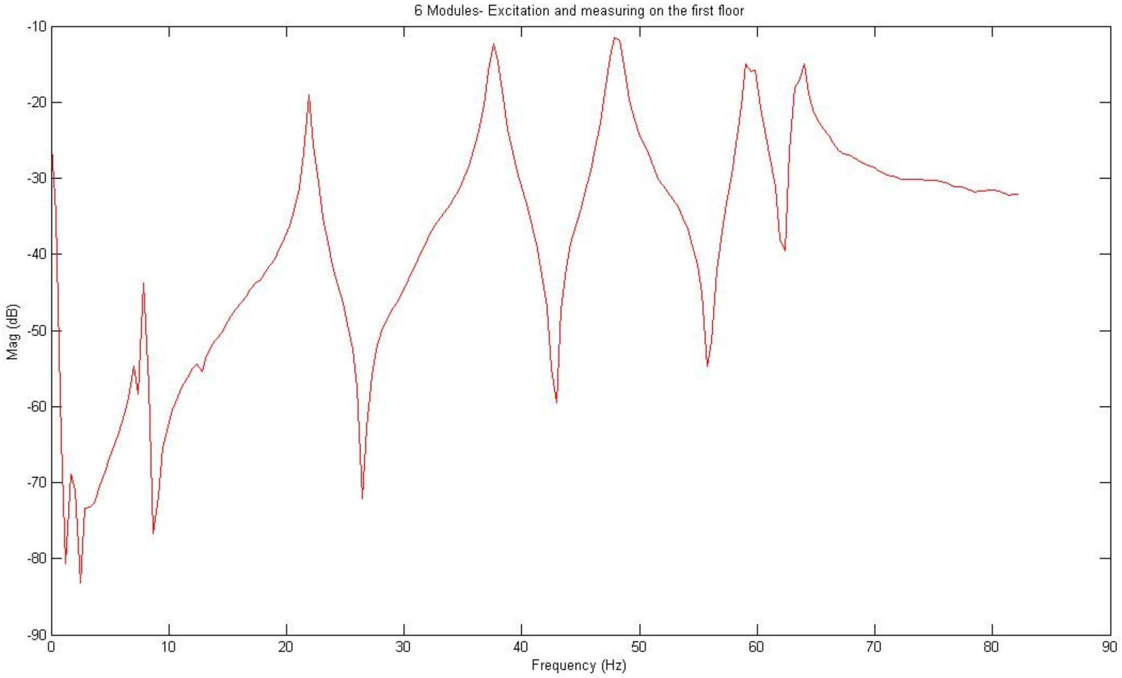

Figure 8. FRF obtained exciting the structure at first floor and measuring the acceleration on the same floor

Table 2. *Values for the natural frequencies in experimental and numerical analysis.*

	Frequency (Hz)					
Mode	1°	2°	3°	4°	5°	6°
Numerical	7.70	22.70	34.70	45.90	54.70	58.60
Experimental	7.85	21.88	37.57	48.31	59.87	64.00

5. RESULTS OBTAINED USING SUBSTRUCTURING

To apply the modal synthesis method of substructures, was performed a modal analysis of primary and secondary substructures. The primary substructure was formed by four floors and clamped in the base. In this configuration, the excitation was applied to the first floor using a shaker and the responses were measured in each one of the other floors. The FRF for the other floor was obtained by applying the force using a impact hammer. The second substructure was suspended to obtain a condition of free interface. The force of excitation was applied to the bottom of the substructure using the shaker. The responses were measured in all degrees of freedom considered.

To reach the FRFs of the complete structure, the modal synthesis was performed using the Frequency Response Functions of primary and secondary substructures. These results were compared to those obtained by experimental modal analysis of the complete structure.

Figure 12 shows the results obtained by modal analysis of the complete structure and by modal synthesis of the substructures FRFs for the excitation and the output measured in the first floor. In this figure, we can see that the difference between the magnitudes are less than 3.5%. But the difference between the modal frequencies are higher than 5%. Considering the FRFs to other floors, the average difference between the natural frequencies obtained by modal synthesis subsetruturas and those obtained experimentally are shown in Table 3

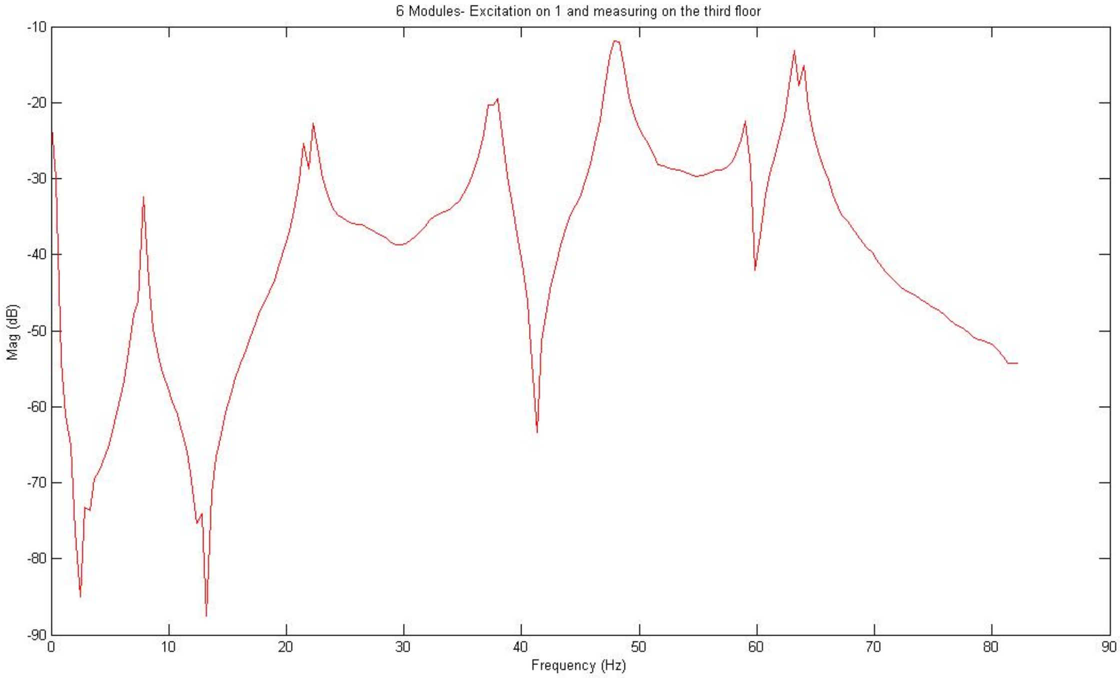

Figure 9. FRF measurement of acceleration on the third floor

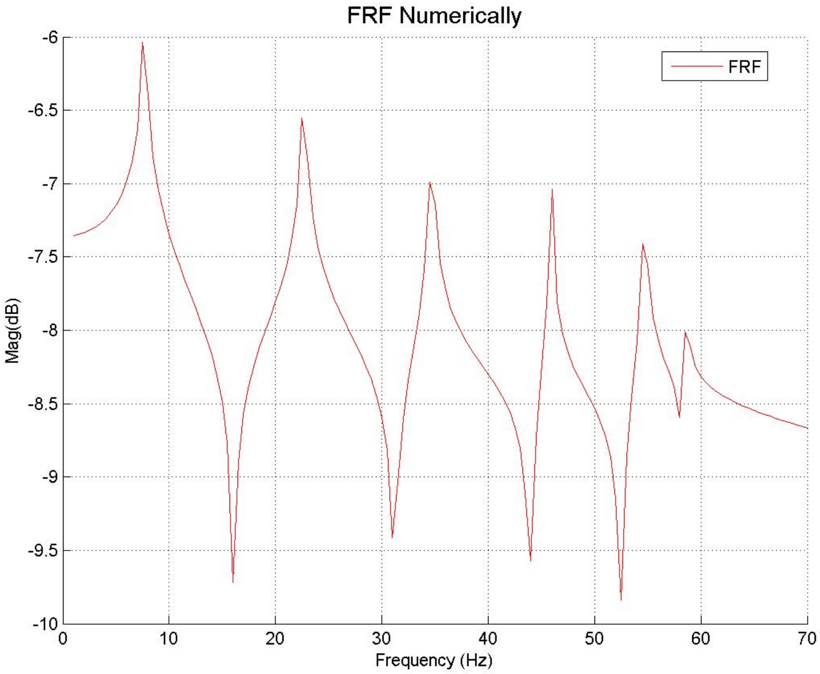

Figure 10. FRF obtained numerically for the first floor

An analysis of these results suggests that the major source of difference between the modal synthesis method and experimental modal analysis is in the test of the secondary substructure, which attempted to simulate the condition of free interface. Inaccuracies appear when considering the free interface between the substructures with respect to model of the complete structure (MacNeal, 1971).

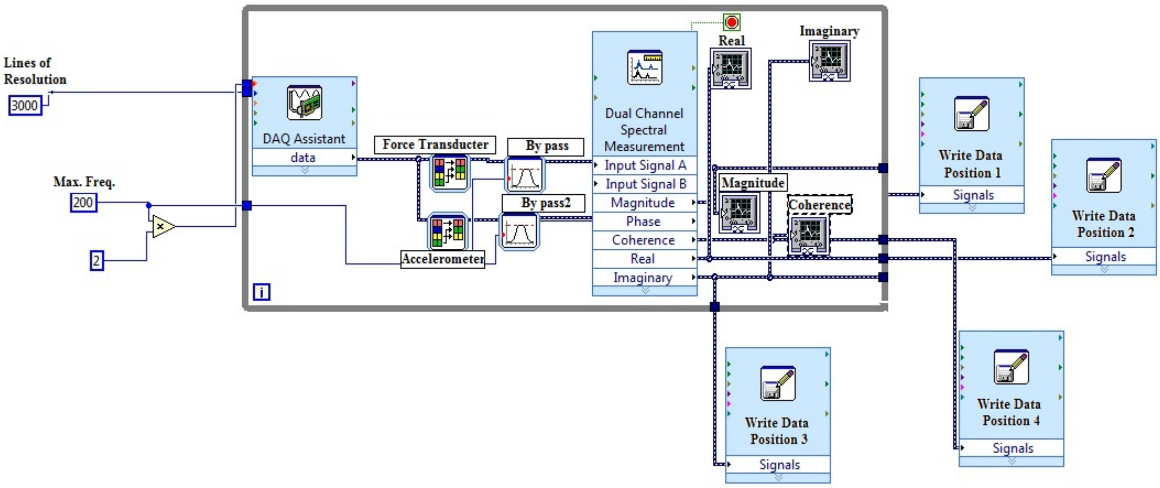

Figure 11. Program made in LabView® data acquisition

Table 3. *Natural frequencies obtained by experimental modal analysis and modal synthesis of substructures*

	Frequency (Hz)					
Mode	1°	2°	3°	4°	5°	6°
Experimental Modal Analysis	7.85	21.88	37.57	48.31	59.87	64.00
Modal Synthesis	11.19	25.44	34.53	45.88	51.69	62.10
Difference (%)	42.51	16.27	8.10	5.03	13.67	2.97

6. CONCLUSIONS

In this work we applied an experimental modal synthesis method using Frequency Response Functions (FRF). The study took into account the effects of randomness present in a gantry-type structure.

In order to apply the method of modal synthesis of substructures, we performed a modal analysis of primary and secondary substructures. The primary substructure (clamped in the base) was assembled using four structural modules. The secondary substructure, with two structural modules, was suspended to simulate the condition of free interface. The Modal Synthesis of the complete structure was performed using the results obtained from primary and secondary substructures modal analysis. So the FRF of the global structure was determined using the FRF of each substructure.

These results were compared wich those obtained by experimental modal analysis of the global structure. The results indicate differences between experimental modal analysis and modal synthesis method. An analysis of these differences shows that the major cause of the deviation between the modal synthesis method and experimental modal analysis is in the test for secondary substructure, which attempted to simulate the condition of free interface. We conclude that this method is a good way to get the response of the structure, which should however observe carefully the boundary conditions.

The study found that the stiffness of the structural modules was random and to quantify the associated uncertainty was made a series of tests to measure the stiffness of each module. We obtained thus the probability distribution of the stiffness of the structure. It was found that the experimental results followed the expected tendecy by the theory of stochastic mechanical models and was obtained a distribution of type Gamma.

The obtained results shown that despite the randomness of the stiffness of the structure, it is not large enough to significantly influence the dynamic response of the structure. The experimental modal analysis showed that the natural frequencies and vibration shape modes presented no significant variation caused by the randomness of structural stiffness.

7. REFERENCES

Brow, A., 1998. “Development of a probabilistic dynamic synthesis method for the analysis of nondeterministic structures”. Technical Report NASA/TM-1998-208473.

Crawley, J.R., Rochlin, G., Klosterman, A. and Vold, H., 1984. “Direct structural modifications using frequency response functions”. *2nd International Modal Analisys Conference*, pp. 58–65.

Cuppens, K., Sas, P. and Hermans, L., 2000. “Evaluation of the frf based substructuring and modal synthesis technique applied to vehicle fe data”. *Proceedings of the Twenty Fifth International Conference on Noise and Vibration Engineering (ISMA)), Katholieke Univ. Leuven, Belgium.*

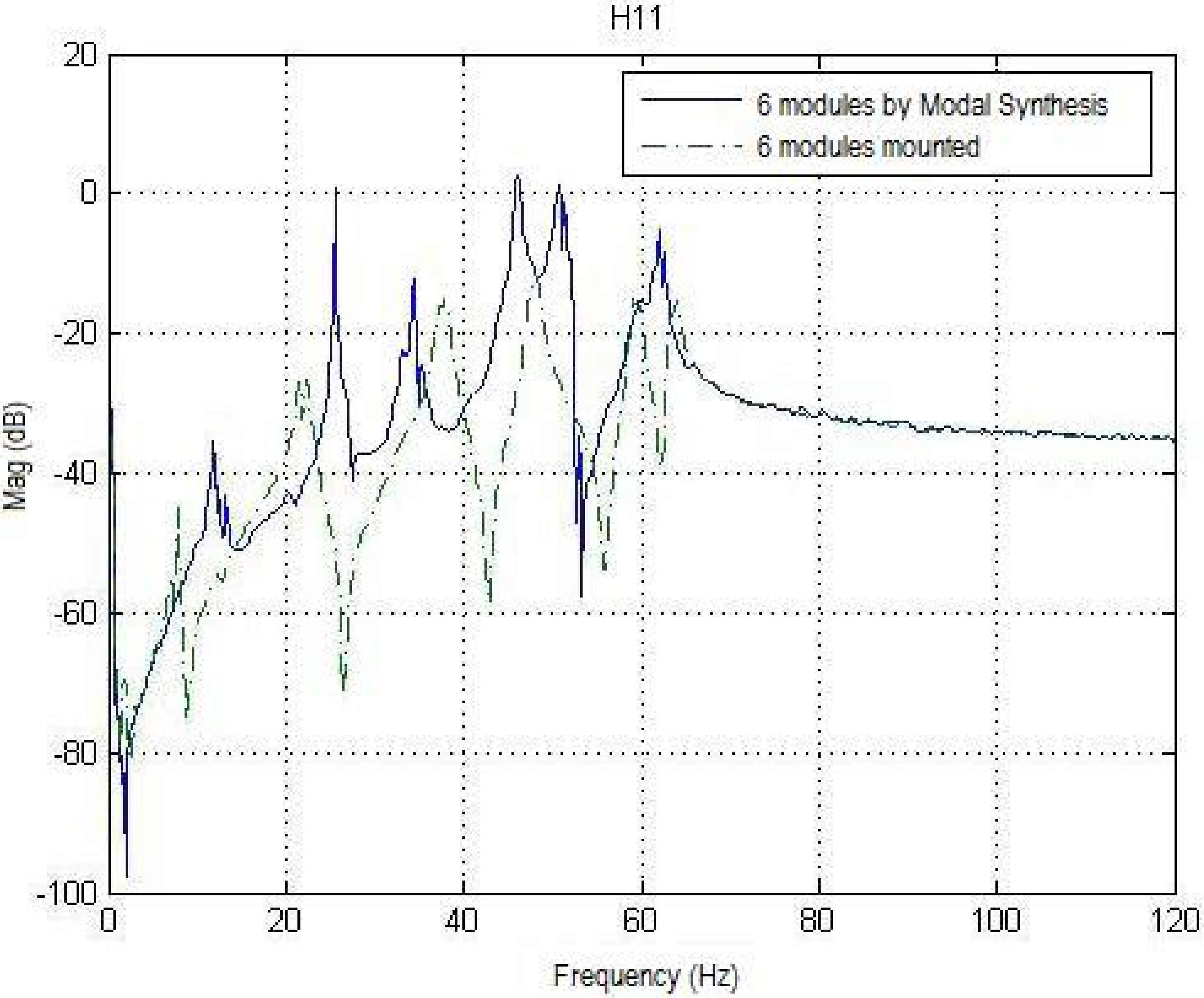

Figure 12. Comparison between the results obtained by modal analysis of the complete structure and synthesis of modal FRFs of the substructures

Dessombz, O., Diniz, A.C., Thouverez, F. and Jezequel, L., 1999. “Analysis of stochastic structures : Perturbation method and projection on homogeneous chaos”. In ABCM, ed., *Proceeding of the International Modal Analysis Conference*. pp. 1195–1201.

Hinke, L., Dohnal, F., Mace, B., Waters, T. and Ferguson, N., 2009. “Component mode synthesis as framework for uncertainty analysis”. *Journal of Sound and Vibration*, Vol. 324, pp. 161–178.

MacNeal, R., 1971. “A hybrid of component mode synthesis”. *Computers and Structures*, Vol. 1, pp. 581–601.

Mehta, M.L., 2002. *Random Matrices*. Academic Press, New York, USA, 3rd edition.

Otte, D., Grangier, H., Leuridan, J. and Aquilina, R., 1991. “Prediction of the dynamics of structural assemblies using measuring frf-data: Some improved data enhancement thecniques”. *Proceedings of the International Modal Analysis Conference, USA*, pp. 909–917.

Soize, C.A., 2005. “A comprhensive overview of a non-parametric probabilistic approach of model un- certainties for predictive models in structural dynamics”. *Journal of Sound and Vibration*, pp. 623–652.

T.T.Soong, 1973. *Random Differential Equations in Science and Engineering*. Academic Press, New York, USA.

Voormeeren, S., Klerk, D. and Rixen, D., 2010. “Uncertainty quantification in experimental frequency based substructuring”. *Mechanical Systems and Signal Processing*, Vol. 24, No. 1, pp. 106–118. doi:10.1016/j-ymssp.2009.01.016.

8. Responsibility notice

The authors are the only responsible for the printed material included in this paper