

TRANSVERSAL RESPONSE OF A RIGID CIRCULAR FOUNDATION EMBEDDED ON A TRANSVERSELY ISOTROPIC BI-MATERIAL INTERFACE

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Abstract. *This paper examines the transversal response of a rigid circular foundation embedded in a viscoelastic, transversely isotropic bi-material interface. The Cauchy-Navier equations, which describe the behavior of the aforementioned media, are solved by using Hankel integral transforms. A boundary-value problem corresponding to the case of a distributed transversal ring load in the interface of the two materials is introduced. The model of embedded disc is formulated in terms of a discretized integral equation, which couples the rigid displacement of the disc with the tractions acting over its surface. The disc is discretized by a number of annular discs, and over each of these elements the traction is considered to be constant. The system of the resulting discretized integral equations is solved numerically, which gives the tractions over each elementary disc. The weighted summation of these results by the respective area of the elements gives the total force applied over the disc corresponding to a unitary rigid displacement. The dynamic compliance of the media-inclusion system is shown in this paper for different constructions. The present solutions contribute to the study of the dynamic response of deeply buried foundations and anchors in non-homogeneous interfaces.*

Keywords*: Green's Functions, Soil-foundation Interaction, Transverse Isotropy, Deeply Buried Foundations.*

1. INTRODUCTION

The study of the interaction of rigid foundations with transversely isotropic materials is a branch of the theory of elasticity which has important practical applications in earthquake engineering and seismology. The case of a rigid foundation embedded in the interface of two different transversely isotropic materials is of particular interest to the study of foundations deeply buried in soil.

The present study is concerned with the steady-state transversal response of a rigid annular or solid circular disc embedded in the interface of two bonded viscoelastic, transversely isotropic, three-dimensional half-spaces. Figure 1 illustrates the present problem.

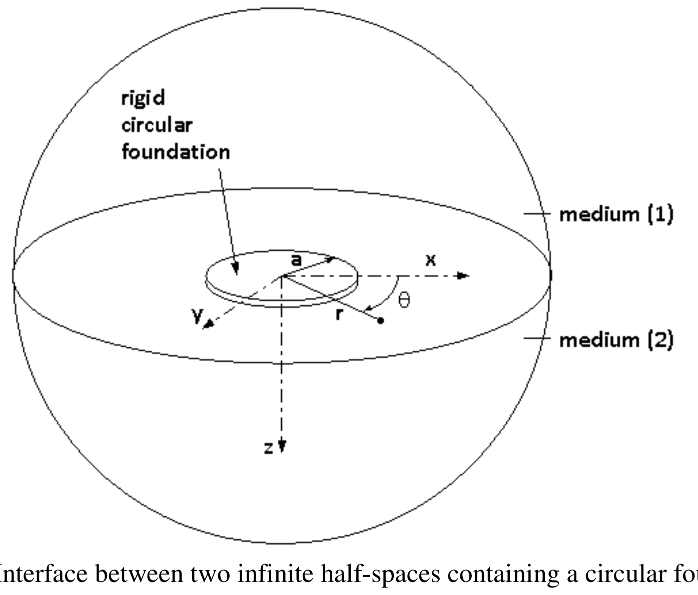

Figure 1. Interface between two infinite half-spaces containing a circular foundation.

The first section of the paper introduces the Cauchy-Navier equations, which describe the behavior of the aforementioned media. This system of equations is solved by using Hankel integral transforms. Hankel transforms are the most suitable transforms in the present case because cylindrical coordinates are used. The viscoelastic behavior of the medium is introduced by Christensen's elastic-viscoelastic correspondence principle. In the following section, a boundary-value problem corresponding to the case of a distributed transversal ring load in the interface of the two half-spaces is introduced. Next, the model of embedded disc is formulated in terms of an integral equation, the kernel of which corresponds to the influence function regarding those buried ring load. This integral equation couples the rigid displacement of the disc with the tractions acting over its surface. The disc is discretized by a number of concentric rigid annular discs. The traction over each of these elements is considered to be constant. Because the disc is rigid, the displacement of all these elements is the same. The system of the resulting discretized integral equations is solved numerically, which gives the tractions over each elementary disc. The weighted summation of these results by the respective area of the elements gives the total force applied over the disc corresponding to a unitary rigid displacement. Finally, some numerical results are presented in the form of dynamic compliance of the disc. The paper shows the convergence of the solution for increasing discretization levels, for different types of transversely isotropic materials, for varying inner radiuses of an annular disc with unitary outer radius, and for different combinations of two materials at the interface.

2. GOVERNING EQUATIONS

Consider two transversely isotropic elastic half-spaces bonded together throughout an infinite plane. A cylindrical coordinate system O(r, θ, z) is adopted, the z-axis of which is perpendicular to the plane that interfaces the two media (see Fig. 1). The equations of motion in these media are expressed by:

$$
\begin{aligned}
&c_{11}\left(\frac{\partial^2}{\partial r^2}u_r+\frac{1}{r}\frac{\partial}{\partial r}u_r-\frac{u_r}{r^2}\right)+\frac{c_{11}-c_{12}}{2}\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}u_r+c_{44}\frac{\partial^2}{\partial z^2}u_r\\
&+\frac{c_{11}+c_{12}}{2}\left(\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}u_\theta+\frac{1}{r^2}\frac{\partial}{\partial\theta}u_\theta\right)-2c_{11}\frac{1}{r^2}\frac{\partial}{\partial\theta}u_\theta+\left(c_{13}+c_{44}\right)\frac{\partial^2}{\partial r\partial z}u_z=\rho\frac{\partial^2}{\partial t^2}u_r
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&\frac{c_{11}-c_{12}}{2}\left(\frac{\partial^2}{\partial r^2}u_\theta+\frac{1}{r}\frac{\partial}{\partial r}u_\theta-\frac{u_\theta}{r^2}\right)+c_{11}\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}u_\theta+c_{44}\frac{\partial^2}{\partial z^2}u_\theta\\
&+\frac{c_{11}+c_{12}}{2}\left(\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}u_r-\frac{1}{r^2}\frac{\partial}{\partial\theta}u_r\right)+2c_{11}\frac{1}{r^2}\frac{\partial}{\partial\theta}u_r+\left(c_{13}+c_{44}\right)\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}u_z=\rho\frac{\partial^2}{\partial t^2}u_\theta
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
&c_{44}\left(\frac{\partial^2}{\partial r^2}u_z+\frac{1}{r}\frac{\partial}{\partial r}u_z+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}u_z\right)+c_{33}\frac{\partial^2}{\partial z^2}u_z\\
&+\left(c_{13}+c_{44}\right)\left(\frac{\partial^2}{\partial r\partial z}u_r+\frac{1}{r}\frac{\partial}{\partial z}u_z+\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}u_\theta\right)=\rho\frac{\partial^2}{\partial t^2}u_z
\end{aligned}
\tag{3}
$$

In Eqs. (1) to (3), ρ is the density of the medium and c_{ij} are material constants of the transversely isotropic material.

The solution of these coupled equations leads to the displacement field of the transversely isotropic media, expressed by (see Appendix for full development of the solution):

$$
\begin{aligned}
&u_r(r,\theta,z)=\int_0^\infty\left(g_1Ae^{-\delta\xi_1z}+g_1Be^{\delta\xi_1z}+g_2Ce^{-\delta\xi_2z}+g_2De^{\delta\xi_2z}\right)\cdot J_0(\lambda r)\lambda d\lambda\\
&+\int_0^\infty\left(g_3Ae^{-\delta\xi_1z}+g_3Be^{\delta\xi_1z}+g_4Ce^{-\delta\xi_2z}+g_4De^{\delta\xi_2z}+g_5Ee^{-\delta\xi_3z}+g_5Fe^{\delta\xi_3z}\right)\cdot J_1(\lambda r)\lambda d\lambda
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&u_\theta(r,\theta,z)\\
&=\int_0^\infty\left(g_6Ae^{-\delta\xi_1z}+g_6Be^{\delta\xi_1z}+g_7Ce^{-\delta\xi_2z}+g_7De^{\delta\xi_2z}+g_8Ee^{-\delta\xi_3z}+g_8Fe^{\delta\xi_3z}\right)\cdot J_1(\lambda r)\lambda d\lambda\\
&+\int_0^\infty\left(g_9Ee^{-\delta\xi_3z}+g_9Fe^{\delta\xi_3z}\right)\cdot J_0(\lambda r)\lambda d\lambda
\end{aligned}
\tag{5}
$$

$$
u_Z(r,\theta,z)=\int_0^\infty\left(-g_{10}Ae^{-\delta\xi_1z}+g_{10}Be^{\delta\xi_1z}-g_{11}Ce^{-\delta\xi_2z}+g_{11}De^{\delta\xi_2z}\right)\cdot J_1(\lambda r)\lambda d\lambda
\tag{6}
$$

In which,

$$g_{1,2} = \delta\zeta\vartheta_{1,2}\cos(\theta)\,,\ g_{3,4} = -\frac{1}{r}\vartheta_{1,2}\cos(\theta)\,,\ g_5 = \frac{1}{r}\cos(\theta) \tag{7}$$

$$g_{6,7} = -\frac{1}{r}\vartheta_{1,2}\sin(\theta)\,,\ g_8 = \frac{1}{r}\sin(\theta)\,,\ g_9 = -\delta\zeta\sin(\theta) \tag{8}$$

$$g_{10,11} = \delta\xi_{1,2}\cos(\theta) \tag{9}$$

$$\vartheta_{1,2} = \frac{\alpha\xi_{1,2}^2 - \zeta^2 + 1}{\kappa\zeta^2} \tag{10}$$

$$\zeta = \lambda / \delta \tag{11}$$

$$\xi_{1,2}(\zeta) = \frac{1}{\sqrt{2\alpha}}\left(\gamma\zeta^2 - 1 - \alpha \pm \sqrt{\Phi}\right)^{\frac{1}{2}} \tag{12}$$

$$\Phi(\zeta) = \left(\gamma\zeta^2 - 1 - \alpha\right)^2 - 4\alpha\left(\beta\zeta^4 - \beta\zeta^2 - \zeta^2 + 1\right) \tag{13}$$

$$\xi_3 = \pm\sqrt{\varsigma\zeta^2 - 1} \tag{14}$$

$$\alpha = \frac{c_{33}}{c_{44}}\,,\ \beta = \frac{c_{11}}{c_{44}}\,,\ \kappa = \frac{c_{13} + c_{44}}{c_{44}}\,,\ \varsigma = \frac{c_{11} - c_{12}}{2c_{44}}\,,\ \delta = \frac{\rho a^2}{c_{44}}\omega^2 \text{ and } \gamma = 1 + \alpha\beta - \kappa^2 \tag{15}$$

The stress field of the transversely isotropic media is derived from Eqs. (4) to (6):

$$\frac{\sigma_{ZZ}}{c_{44}\cos(\theta)} = \int_0^\infty \left(b_{21}Ae^{-\delta\xi_1 z} + b_{21}Be^{\delta\xi_1 z} + b_{22}Ce^{-\delta\xi_2 z} + b_{22}De^{\delta\xi_2 z}\right)J_1(\lambda r)\lambda d\lambda \tag{16}$$

$$\begin{aligned}\frac{\sigma_{\theta Z}}{c_{44}\sin(\theta)} &= \int_0^\infty \left(h_1Ae^{-\delta\xi_1 z} - h_1Be^{\delta\xi_1 z} + h_2Ce^{-\delta\xi_2 z} - h_2De^{\delta\xi_2 z} + h_3Ee^{-\delta\xi_3 z} - h_3Fe^{\delta\xi_3 z}\right)J_0(\lambda r)\lambda d\lambda \\ &+ \int_0^\infty \left(h_1Ae^{-\delta\xi_1 z} - h_1Be^{\delta\xi_1 z} + h_2Ce^{-\delta\xi_2 z} - h_2De^{\delta\xi_2 z} - h_3Ee^{-\delta\xi_3 z} + h_3Fe^{\delta\xi_3 z}\right)J_2(\lambda r)\lambda d\lambda\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\sigma_{\theta Z}}{c_{44}\sin(\theta)} &= \int_0^\infty \left(h_1Ae^{-\delta\xi_1 z} - h_1Be^{\delta\xi_1 z} + h_2Ce^{-\delta\xi_2 z} - h_2De^{\delta\xi_2 z} + h_3Ee^{-\delta\xi_3 z} - h_3Fe^{\delta\xi_3 z}\right)J_0(\lambda r)\lambda d\lambda \\ &+ \int_0^\infty \left(h_1Ae^{-\delta\xi_1 z} - h_1Be^{\delta\xi_1 z} + h_2Ce^{-\delta\xi_2 z} - h_2De^{\delta\xi_2 z} - h_3Ee^{-\delta\xi_3 z} + h_3Fe^{\delta\xi_3 z}\right)J_2(\lambda r)\lambda d\lambda\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{\sigma_{RZ}}{c_{44}\cos(\theta)} &= -\int_0^\infty \left(h_1Ae^{-\delta\xi_1 z} - h_1Be^{\delta\xi_1 z} + h_2Ce^{-\delta\xi_2 z} - h_2De^{\delta\xi_2 z} + h_3Ee^{-\delta\xi_3 z} - h_3Fe^{\delta\xi_3 z}\right)J_0(\lambda r)\lambda d\lambda \\ &- \int_0^\infty \left(-h_1Ae^{-\delta\xi_1 z} + h_1Be^{\delta\xi_1 z} - h_2Ce^{-\delta\xi_2 z} + h_2De^{\delta\xi_2 z} + h_3Ee^{-\delta\xi_3 z} - h_3Fe^{\delta\xi_3 z}\right)J_2(\lambda r)\lambda d\lambda\end{aligned} \tag{19}$$

In which,

$$b_{2i} = \alpha\delta^2\xi_i^2 - (\kappa - 1)\delta^2\zeta^2\vartheta_i \tag{20}$$

$$h_{1,2} = (1 + \vartheta_i)\delta\xi_i\frac{\delta\zeta}{2}\,,\ h_3 = \delta\xi_3\frac{\delta\zeta}{2}\,,\ h_{4,5} = \vartheta_{1,2}\frac{\delta\zeta}{2} \text{ and } h_6 = \frac{\delta\zeta}{2} \tag{21}$$

3. INFLUENCE FUNCTIONS

In this section, boundary-value problems regarding interior distributed annular loads are considered. The loads are applied at the interface between the media (1) and (2) (see Fig. 1). Figure 2 depicts a transversal load, uniformly distributed over the area of an annular disc of inner and outer radius s_1 and s_2.

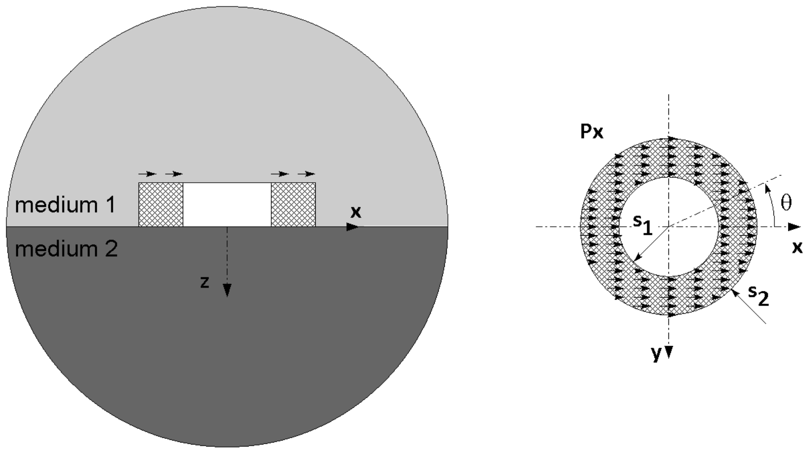

Figure 2. Distributed transversal load applied on an annular area.

The continuity conditions at the infinite bonded interface between media (1) and (2) are established as follows:

$$\frac{u_R^{(1)}(r,z=0)}{\cos(\theta)}-\frac{u_\theta^{(1)}(r,z=0)}{\sin(\theta)}=\frac{u_R^{(2)}(r,z=0)}{\cos(\theta)}-\frac{u_\theta^{(2)}(r,z=0)}{\sin(\theta)} \tag{52}$$

$$\frac{u_R^{(1)}(r,z=0)}{\cos(\theta)}+\frac{u_\theta^{(1)}(r,z=0)}{\sin(\theta)}=\frac{u_R^{(2)}(r,z=0)}{\cos(\theta)}+\frac{u_\theta^{(2)}(r,z=0)}{\sin(\theta)} \tag{53}$$

$$\frac{u_Z^{(1)}(r,z=0)}{\cos(\theta)}=\frac{u_Z^{(2)}(r,z=0)}{\cos(\theta)} \tag{54}$$

The upper indices (1) or (2) in Eqs. (52) to (54) indicate the medium which each component of displacement refers to. Consider for example the component $u^{(m)}{}_\theta(r,\theta,z)$, m=1,2. The arbitrary constants A, B, C, D, E and F in each case (m=1 or m=2) are selected so that the amplitude of the displacement vanishes with increasing depth in z.

$$\begin{aligned} u_\theta^{(1)}(r,\theta,z) &= \delta^{(1)2}\int_0^\infty \left(g_6^{(1)}B^{(1)}e^{\delta^{(1)}\xi_1^{(1)}z}+g_7^{(1)}D^{(1)}e^{\delta^{(1)}\xi_2^{(1)}z}+g_8^{(1)}F^{(1)}e^{\delta^{(1)}\xi_3^{(1)}z}\right)\cdot J_1\left(\delta^{(1)}\zeta r\right)\zeta d\zeta \\ &\quad +\delta^{(1)2}\int_0^\infty \left(g_9^{(1)}F^{(1)}e^{\delta^{(1)}\xi_3^{(1)}z}\right)\cdot J_0\left(\delta^{(1)}\zeta r\right)\zeta d\zeta \end{aligned} \tag{55}$$

$$\begin{aligned} u_\theta^{(2)}(r,\theta,z) &= \delta^{(2)2}\int_0^\infty \left(g_6^{(2)}A^{(2)}e^{-\delta^{(2)}\xi_1^{(2)}z}+g_7^{(2)}C^{(2)}e^{-\delta^{(2)}\xi_2^{(2)}z}+g_8^{(2)}E^{(2)}e^{-\delta^{(2)}\xi_3^{(2)}z}\right)\cdot J_1\left(\delta^{(2)}\zeta r\right)\zeta d\zeta \\ &\quad +\delta^{(2)2}\int_0^\infty \left(g_9^{(2)}E^{(2)}e^{-\delta^{(2)}\xi_3^{(2)}z}\right)\cdot J_0\left(\delta^{(2)}\zeta r\right)\zeta d\zeta \end{aligned} \tag{56}$$

Notice that the material constants and other parameters also depend on the respective medium (for example, $g^{(m)}{}_6$ and $\xi^{(m)}{}_1$).

For the distributed transversal load depicted in Fig. 2, the stress boundary conditions are:

$$\frac{\sigma_{ZZ}^{(1)}(r,z=0)}{\cos(\theta)}=\frac{\sigma_{ZZ}^{(2)}(r,z=0)}{\cos(\theta)} \tag{57}$$

$$\frac{\sigma_{RZ}^{(2)}(r,z=0)}{\cos(\theta)}+\frac{\sigma_{\theta Z}^{(2)}(r,z=0)}{\sin(\theta)}=\frac{\sigma_{RZ}^{(1)}(r,z=0)}{\cos(\theta)}+\frac{\sigma_{\theta Z}^{(1)}(r,z=0)}{\sin(\theta)} \tag{58}$$

$$\begin{aligned} &\frac{\sigma_{RZ}^{(2)}(r,z=0)}{\cos(\theta)}-\frac{\sigma_{\theta Z}^{(2)}(r,z=0)}{\sin(\theta)} \\ &=\frac{\sigma_{RZ}^{(1)}(r,z=0)}{\cos(\theta)}-\frac{\sigma_{\theta Z}^{(1)}(r,z=0)}{\sin(\theta)}+2\int_0^\infty \left\{s_2J_1(\lambda s_2)-s_1J_1(\lambda s_1)\right\}J_0(\lambda r)d\lambda \end{aligned} \tag{59}$$

The system of six equations comprising Eqs. (52) to (54) and Eqs. (57) to (59) can be solved to determine the arbitrary constants $A^{(2)}$, $B^{(1)}$, $C^{(2)}$, $D^{(1)}$, $E^{(2)}$ and $F^{(1)}$. Although this system can be solved analytically with the aid of some mathematical software, the solution for the constants is inconveniently long to be included in a program. In this present paper, this system of equations is solved numerically whenever necessary in the computer code.

4. TRANSVERSAL VIBRATIONS OF AN EMBEDDED RIGID DISC

Consider the harmonic excitation of a rigid disc of radius *a*, with zero thickness and no mass, embedded in the interface of two infinite half-spaces as shown in Fig. 1. It is assumed that the disc experiences time-harmonic displacements due to the loads applied. The relationship between unknown tractions and the displacement of the disc can be expressed in terms of the following integral equation:

$$\int_0^a G_{rr}\left(r,\theta,z=0,\omega\right)T_r\left(r,\theta,z=0,\omega\right)dr = U_r\left(r,\theta,z=0,\omega\right) \tag{60}$$

In Eq. (60), Tr denote a jump in tractions in the radial directio; Ur denote the radial displacements at a point of coordinates (r, θ) at the interface between the two materials (z=0), and Grr denote the transversal displacements due to transversal loads of unitary intensity Px=1 (see Fig. 2). The components Grr are obtained from Eq. (34) when the boundary conditions in Eqs. (57) to (59) are considered.

In this work, the coupled equation system expressed by Eq. (60) is solved by discretizing the surface of the disc into M concentric annular disc of inner and outer radiuses s_{1k} and s_{2k}, k=1,M. It is assumed that Tr is constant within each of these elementary discs.

In the case in which a transversal load is applied uniformly over the surface of the rigid disc, the rigid transversal displacement of each elementary disc k (k=1,M) is the same, Δ_0. In this case, the discretized version of Eq. (60) becomes:

$$\sum_{k=1}^{M} G_{rr}\left(r_k,\theta=0,z=0,\omega\right)T_r\left(r_k,\theta=0,z=0,\omega\right)dr = \Delta_0 \tag{61}$$

The tractions $Tr(r_k)$ due to a unitary displacement are obtained from Eq. (61) by making Δ_0=1. The total force acting on the surface of the disc corresponding to this case of unitary displacement is obtained by:

$$F(\omega) = \sum_{k=1}^{M} \pi\left(s_{2k}^2 - s_{1k}^2\right)T_r\left(r_k,\omega\right) \tag{62}$$

The transversal dynamic compliance of the system comprising the two infinite half-spaces and the embedded rigid disc is obtained for each frequency as:

$$C_R(\omega) = \Delta_0 a E^{(m)} / F(\omega) \tag{63}$$

In Eq. (63), $E^{(m)}$ is the Young's modulus of one of the two materials (m=1,2).

5. NUMERICAL RESULTS

The hardest computational task that arises in the solution of the embedded disc comes from the numerical integration of Grr from Eq. (60). The present case of two interfacing infinite half-spaces is characterized by the existence of interface waves, which possess an infinite number of true singularities to be integrated (Stoneley, 1924; Graff, 1974). In the present paper, however, no special attention is given to the implementation of integration methods or to the behavior of the integrand. A numerical solver of improper integrals, based on globally adaptive quadratures, which is freely available in numerical packages of the Fortran programming language, is used for this purpose. The issue of the singularities is avoided by the inclusion of a small damping in all the material constants according to Christensen's elastic-viscoelastic principle (Christensen, 2010).

Selvadurai and Singh (1984) presented an analytical solution for the static transversal compliance (C'_R) of a rigid circular plate buried in an isotropic full-space. Their solution is given by $C'_R=(7-8\nu)/[64(1-\nu)]$. Table 1 shows the error between their solutions and the ones obtained by the present program, as well as the convergence of the present solution with increasing discretization M. In these results, a homogeneous full-space with μ=1 and ν=0.25 is considered.

Table 1. Comparison of results with an analytical solution for the static problem.

M	$C_R(\omega=0)/C'_R$
5	1.034255919578913
10	1.015344450609305
20	1.006348607114810
35	1.001912669708005
50	1.000454275559764

The next results are presented in terms of the normalized compliance $C^*_R=C_R/C_R(\omega=0)$. The discretization of M=20 disc is chosen, because it is enough to allow an error of less than 1% with the analytical solution (see Table 1).

Figure 3 shows the influence of the inner radius b in the normalized compliance of the annular disc. A homogeneous isotropic medium with μ=1.0 and ν=0.25 is considered.

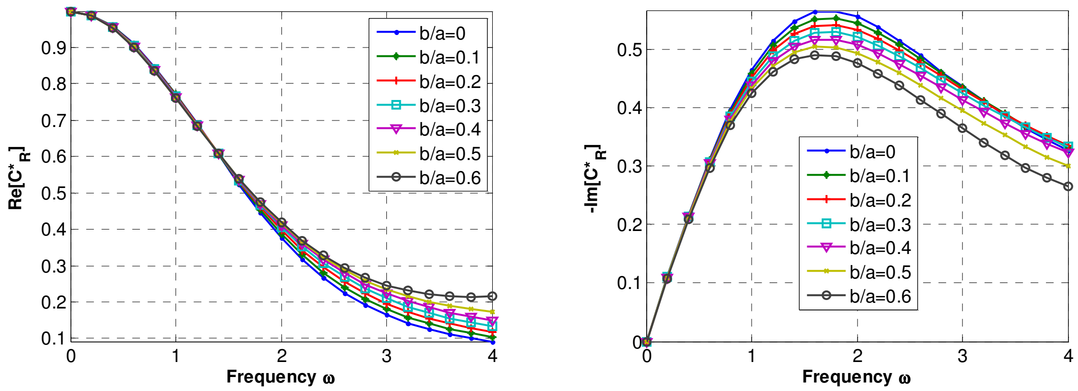

Figure 3. Influence of the inner radius b in the normalized compliance of the disc for the case of transversal load.

Finally, Fig. 4 shows the compliance of the system for some combinations of two transversely isotropic materials at the interface. In all six cases, the material of the bottom layer, medium (2), is Beryl Rock (see Table 2). The material of the upper layer, medium (1), is chosen from Table 2.

Table 2. Material constants for some isotropic and transversely isotropic materials, with $c'_{ij} = c_{ij}/c_{44}$ (Wang, 1992).

Material	c'_{11}	c'_{12}	c'_{13}	c'_{33}	c_{44} (10^4 MN/m^2)
Beryl Rock	4.13	1.47	1.01	3.62	1.00
Silty Clay	2.11	0.43	0.47	2.58	2.70
Layered Soil	4.46	1.56	1.24	3.26	1.40
Clay	4.70	1.70	1.20	3.30	0.01
Ice (257K)	4.22	2.03	1.62	4.53	0.32
Isotropic[1]	3.00	1.00	1.00	3.00	0.99997

1: Isotropic material considering μ=1.0 and ν=0.25.

In order to show more clearly the difference between the combinations of materials, the normalized compliance C^*_R in Fig. 4 is also normalized by the homogeneous case, in which both media (1) and (2) are Beryl Rock.

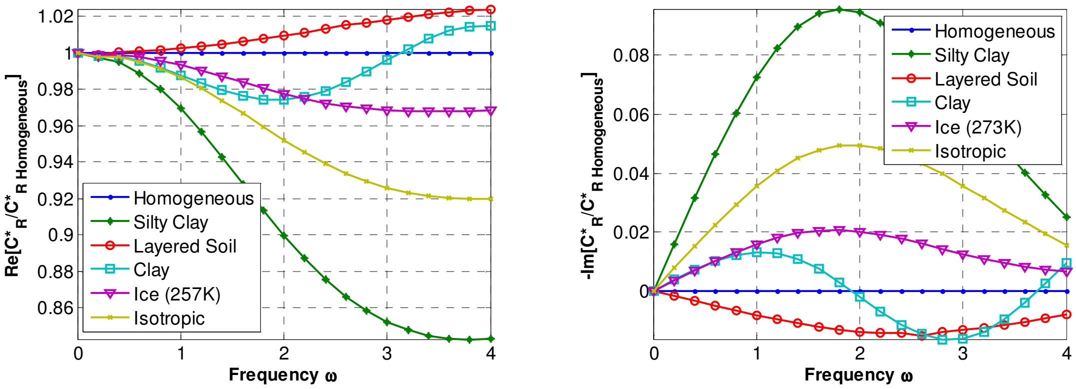

Figure 4. Normalized compliance of the system for different combinations of transversely isotropic bi-material interfaces for the case of transversal load.

6. CONCLUSION

A set of influence functions for displacements and stresses of two bonded transversely isotropic half-spaces subjected to time-harmonic circular loads has been introduced. These equations have been presented in terms of semi-infinite integrals which were solved using globally adaptive numerical quadratures. The problem of a circular rigid inclusion in the interface of the two half-spaces was formulated as coupled integral equations containing those influence functions and the displacements of the disc. A system of equations was obtained from these equations by considering the disc to be formed by a number of annular elementary discs. This system is solved for different frequencies, which yields the dynamic compliance function of the rigid circular inclusion under transversal loads. The solutions presented in this paper characterize the response of deeply buried foundations and anchors in bi-material interfaces.

7. ACKNOWLEDGEMENTS

The research leading to this article has been funded by Capes, CNPq, Fapesp and Faepex/Unicamp. This is gratefully acknowledged.

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