AN OPTIMIZATION METHODOLOGY OF A RADIO-CONTROLLED-AIRCRAFT WING COMPOSED BY FRAME STRUCTURES, CONSIDERING UNCERTAINTIES IN GEOMETRICAL AND PHYSICAL PARAMETERS WITH STRESS, DISPLACEMENTS AND NATURAL FRENQUENCIES CONSTRAINTS

Tiago dos Santos, t.santos@brturbo.com.br
Grupo de Mecânica Aplicada, Universidade Federal do Rio Grande do Sul, Rua Sarmento Leite, 425, Porto Alegre, RS, Brasil, http://www-gmap.mecanica.ufrgs.br

Leandro Luis Corso, llcorso@yahoo.com.br
Oscar A. G. de Suarez, oagsuarez@ucs.br
Departamento de Engenharia Mecânica, Universidade de Caxias do Sul, Rua Francisco Getúlio Vargas, 1130 Caxias do Sul, RS, Brasil, http://www.ucs.br

Herbert Martins Gomes, herbert@mecanica.ufrgs.br
Grupo de Mecânica Aplicada, Universidade Federal do Rio Grande do Sul, Rua Sarmento Leite, 425, Porto Alegre, RS, Brasil, http://www-gmap.mecanica.ufrgs.br

Abstract. *This paper presents a methodology for parametric structural optimization of frame structures on the wing of a radio-controlled aircraft which takes into account uncertainties. It is used the Genetic method for global search. The paper aim at minimizing mass subjected to stress, displacement and natural frequencies constraints. The design variables are cross sectional area values. So, it is performed a size optimization the structure. The uncertainties of the bar areas due to manufacturing process were considered. The result shows that the methodology is suitable for this kind of problem and may be applied for such family of problems. The results show the efficiency oh the methodology, which could be applied in similar cases.*

Keywords*: uncertainty, optimization, reliability based design optimization.*

1. INTRODUCTION

Although the use of optimization methods are increasing, in practical terms, the design parameters and/or design variables used in deterministic optimization procedures can present uncertainties. It means that analyzing the responses obtained by deterministic optimization procedures in terms of failure probability, they can have a level not acceptable.

This is due to the uncertainties of design parameters and / or design variables. One way to link such uncertainties in an optimization problem is the utilization of reliability index β , according to Agarwal (2004). This procedure is called Reliability Based Design Optimization (RBDO); in this case, the reliability index becomes a constraint to the optimization problem.

It is known that optimization procedures is complex in terms of calculation (Haldar and Mahadevan (1999), Burton and Hajela (2003)), whether maximization or minimization of nonlinear functions, which involves nonlinear constraints and also constraints on the value of the reliability index. Additionally, it can present a high computational cost. These difficulties mean that many times the results are not satisfactory.

When the simulations are related to heavy structural analysis this makes the use of the method worthless, mainly when the structural analysis are required exhaustively as in the RBDO. According to Yang, Chuang et al. (2005), Cheng, Xu et al. (2006) and Pereira (2007), this same problem can happen with the Importance Sampling technique for the reliability evaluations, for this reason it was chosen another methods to serve the objectives. A good alternative is to use FORM (First Order Reliability Method), which is capable to solve the majority of the practical cases, where the problem has multiple failure modes but well-known linearly and smooth functions. In this paper, the RBDO methodology was implemented in order to find the optimum parameters for minimum weight, maximum allowable deflections and limited stress in truss members. It was used the FORM method for the reliability index (Beta index) calculations and a Genetic Algorithm with floating point codification for the optimization. Such choice was taken in order to investigate the algorithm behavior and to ease the numerical implementation. Such method deals with cost functions with non-linear behavior, non-smooth or with functions where gradients are not defined. It is presented three simple examples of reliability based design optimization of trusses, as well as comparisons with literature results.

The variables are discrete and includes uncertainty in the design of the wing, which has limits of displacement, stress and natural frequency.

2. RELIABILITY BASED DESIGN OPTIMIZATION (RBDO)

RBDO is an optimization process which aims at the minimization/maximization of cost function and that satisfies reliability constraints as initial conditions. For this reason, it is necessary perform probability analysis during the optimization process. Besides, the design variables can be probabilistic parameters so that the optimization task became more complex.

The simpler and common formulation in the RBDO implementation is separated in two levels:

(a) An outer loop to perform the optimization, where the design variables are taken into account;

(b) An inner loop to perform the reliability analysis.

Generally speaking, an optimization model can be defined in the following way: Minimize (or Maximize) a cost function subjected to constraints. In a mathematical notation:

$$Minimize\ f(vp;\ p) \tag{1}$$

Subjected to

$$\begin{aligned} gi(vh;\ p) &= 0 \qquad i = 1...nr \\ gi(vh;\ p) &< 0 \qquad i = nr + 1...nr \\ vhl < vh &< vhu \qquad i = 1...nv \end{aligned} \tag{2}$$

where vh are the design variables, p are the constant parameters of the problem, g_i is the *i-th* model constraint, vhl and vhu are respectively the lower and upper limit for the design variables. The deterministic optimization does not consider uncertainties in the design variables. Using the RBDO methodology, the constraints in the deterministic formulation are changed by probabilistic constraints. By the probability theory, it is well-known that the reliability index can be written as function of the failure probability:

$$\beta = -\Phi(P_f(vp;p)) \tag{3}$$

where Φ is the cumulative probability function. In this work, the constraints can be stated as:

$$g_i^r(vp;p) = (\beta_c^f - \beta_e^f) \text{ where } g_i^r(vp;p) \le 0 \qquad i=1...m \tag{4}$$

$$g^d(vp;p) = (\beta_c^d - \beta_e^d) \text{ where: } g^d(vp;p) \le 0 \tag{5}$$

$$g^f(vp;p) = (\beta_c^f - \beta_e^f) \text{ where: } g^f(vp;p) \le 0 \tag{6}$$

where g_i^r are the strength constraints for each of the i-th truss member, g_i^d are the displacements constraints for the structure and g_i^f is the natural frequency constrain. This means that the reliability index pre-set in the input parameters (βe) for both stress and displacements must be greater than the calculated reliability index (βc) during the optimizations. In order to find β it is used FORM. In this work, vh refers to cross sectional areas of truss members. vhl and vhu are the corresponding lower and upper limits for member's cross section.

3. GENETIC ALGORITHM IMPLEMENTATION

Genetic Algorithms (GA) are optimization techniques based on the Darwin's Theory of evolution and survival of the fittest. The Darwin's Theory of Natural Selection (1859) *apud* Goldberg (1989) says that "… any being, if it varies slightly in any manner profitable to itself, will have better chance of surviving…". GA simulates the evolutionary process numerically. They represent the parameters in a given problem by encoding them into a string. As in genetics, genes are constituted by chromosomes. Similarly, in simple GA, encoded strings are composed of bits. A string of bits can be decoded to the respective problem parameter value and the total evaluation of the string of bits for and individual may be weighted following some fitness function representing the phenotype to that string of bits.

A simple genetic algorithm consists of three basic operations (Holland (1975)), these being reproduction, crossover and mutation. The algorithm begins with a population of individuals each of them representing a possible solution of the problem. The individuals, as in nature, perform the three basic operations and evolve in generations where the Darwin's Theory prevails, or in other words, a population of individuals more adapted emerges as natural selection.

The floating point Coded Genetic Algorithms assumes real values to each variable. The main differences in this method are found on the crossover operator. There are several methods to deal with the floating point Coded Genetic Algorithms crossover such as flat crossover, simple crossover, arithmetical crossover, Wright's crossover, linear BGA crossover, etc. In this paper the BLX-α was used because it uses an initial exploration of the parameters field followed by an exploitation phase to improve resolution. It may be described by Eq. (7).

$$\Delta = max[b_i(k), b_{i+1}(k)] - min[b_i(k), b_{i+1}(k)] \tag{7}$$
$$b(k) = random\{min[b_i(k), b_{i+1}(k)] - \alpha\Delta,\ max[b_i(k), b_{i+1}(k)] + \alpha\Delta\}$$

where, *i* and *i+1* are referred to two parents' chromosomes, α means a decreasing exploration parameter and random means a random number in the respective interval. Fig. (1) summarizes the main steps followed by a Floating Point Coded basic Genetic Algorithm to maximize functions.

As indicated by Fig. (1), the algorithm starts generating a random set of individual that will form the population. In the following, individual are selected and picked on pairs according its fitness (objective function). This is accomplished by a probabilistic raffle called roulette wheel. At this point crossover will occurs, mixing chromosomes from two individual generating two offspring with characteristics inherited from its parents. The reproduction will be promising with a probability of 'Pc'. At last, from the offspring population, some chromosomes of some individual will suffer mutation under a probabilistic rate of 'Pm' usually set as a low value (1% or less) as found in nature. Then, eventually, some best individual belonging to the parent set will bypass the natural selection and will be introduced in the offspring set through an elitism procedure and exchanged by the less fitted offspring. This procedure assures that best solutions are hold and not lost in the probabilistic selection. The generations will succeed until a convergence criterion being reached. In this paper the stopping criteria is the diversity on the population set evaluated by the standard deviation of the objective function (fitness).

Step i) Initialize Time *t=0,* Initialize Population size: "*m*", mutation probability: "*Pm*", crossover probability: "*Pc*", number of individual chromosomes: "*nc*", allowed limits for each chromosome: "*Pmax(nc), Pmin(nc)*".
Step ii) Generate Initial Population: $B_0 = (b_{1,0}, b_{2,0}, ..., b_{m,0})$
Step iii) While Stopping Condition is not fulfilled
Step iii-1) "Proportional Selection"
Loop *i=1* to *m*
x=random (0,1)
k=1
While *k<m* and *x*< $\sum_{j=1}^{k} f(b_{j,t}) / \sum_{j=1}^{m} f(b_{j,t})$
k=k+1
$b_{i,t+1} = b_{k,t}$
End While
End Loop
Step iii-2) "One Point Crossover"
Loop *i=1* to *m-1* step 2
If random *(0,1)<Pc* then
$\alpha = 0.5$
$\Delta = max[b_{i,t}(k), b_{i+1,t}(k)] - min[b_{i,t}(k), b_{i+1,t}(k)]$
$b_{i,t+1}(k) = random\{min[b_{i,t}(k), b_{i+1,t}(k)] - \alpha\Delta,\ max[b_{i,t}(k), b_{i+1,t}(k)] + \alpha\Delta\}$
$b_{i+1,t+1}(k) = random\{min[b_{i,t}(k), b_{i+1,t}(k)] - \alpha\Delta,\ max[b_{i,t}(k), b_{i+1,t}(k)] + \alpha\Delta\}$
End If
End Loop
Step iii-3) "Offspring Mutation"
Loop *i=1* to *m*
If random *(0,1)<Pm* then
*k=random (0,1)*nc*
$b_{i,t+1}(k) = random\{P_{max}(k), P_{min}(k)\}$
End if
End Loop
End Loop

Figure 1. Pseudo-Code for the floating point genetic algorithm codification.

4. FORM : FIRST ORDER RELIABILITY ANALYSIS

A mathematical expression for the failure of a system may be stated according to Eq. (8).

$$M(x) = g(X_1, X_2, ..., X_n) \le 0 \tag{8}$$

where means the safety margin and are the vector of n probabilistic variable that affects the material strength. means failure and means that the system is in the safe domain. The failure probability can be calculated using the jointly probability density function as (Melchers (2001)):

$$P_f = \iint \cdots \int_D f_x(X_1, X_2, ..., X_n) dX_1 dX_2 ... dX_3 \tag{9}$$

where D is the domain where $M \leq 0$. Taking into consideration a truss structure, where the failure function is defined by the stress σ_i in a member related to the allowed stress $\sigma_{\lim}$, the safety margin (limit state function) is written as:

$$M = 1 - \sigma_i / \sigma_{\lim} \tag{10}$$

This methodology can be used to compression stresses, displacements or any other function that indicates undesirable condition violations. Integration of Eq. (9) is difficult. Besides, sometimes statistical parameters of $f_x(\mathbf{X})$ are not known *a priori*. For this reason, it is usual use the first and the second moments (mean value and standard value) to calculate the reliability index β of the safety margin *M*. The known FORM (*First Order Second Moment*) uses an approximation of the limit state function in the vicinity of the design point in order to evaluate the β index. Besides, the FORM allows calculating the reliability index independent of the expression used as safety margin. For non-correlated variables, the random variables X_i can be transformed into standard non-correlated variables U_i taking:

$$U_i = \Phi^{-1}\left[F_{x_i}(X_i)\right] \tag{11}$$

where $F_{x_i}(X_i)$ and Φ are the cumulative distribution function and the Standard cumulative distribution function for variable X_i, respectively. In this way the safety margin in the real space can be transformed to the non-correlated standard space U :

$$H(\mathbf{U}) = M(\mathbf{X}) \tag{12}$$

A first order approximation of the limit state function on point $\mathbf{U}^*$ can be drawn and the gradient descent method can be used to find the smallest distance from approximated limit state function $H(\mathbf{U}) = 0$ to the origin of the non-correlated standard space $\mathbf{U}$. The $\mathbf{U}^*$ point is called design point and the reliability index β can be calculated as the Euclidian distance to the origin of the non-correlated standard normal space as:

$$\beta = \min(\mathbf{U}^{*T} \cdot \mathbf{U}^*)^{1/2} \tag{13}$$

In order to solve this problem Eq. (14) it is used an iterative solution proposed by Rackwitz – Fiessler (1978) *apud* Haldar and Mahadevan (1999), which can be described as:

$$U^*_{i\,k+1} = \left[\nabla M(U^*_{i\,k})^t U^*_{i\,k} - M(U^*_{i\,k})\right] \nabla M(U^*_{i\,k}) / \left|\nabla M(U^*_{i\,k})\right|^2 \tag{14}$$

where ∇M is the limit state function gradient (safety margin), U is the vector of probabilistic variables in the non-correlated normal space.

In Eq. (14) all the variables in the real space are considered non-correlated. If there exist any correlation it may be used a Cholesky decomposition of the covariance matrix in order to transform from real space to non-correlated Normal space (Haldar and Mahadevan (1999)).

5. RELIABILITY BASED DESIGN OPTIMIZATION ON A RADIO-CONTROLLED-AIRCRAFT WING

The wing is the main lifting element of aircraft and usually has as its secondary function, host components necessary to flight. The main loads acting on the wing are loads of lift, drag and the structure self weight.

During the aircraft design that aims to ensure the integrity and functionality of the structure under several operating conditions for maximum structural efficiency, in other words, to develop the lightest possible structure that attends the design requirements.

5.1 Boundary conditions of the structural analysis

In the analysis performed on one of the wings, were prescribed the displacements of clamped wing points at the fuselage. Equivalent nodal loads were applied on the structure, based on the pressure distributions obtained for the profiles and the load factor. In Fig. (2) is shown the discrete structure under the boundary conditions described.

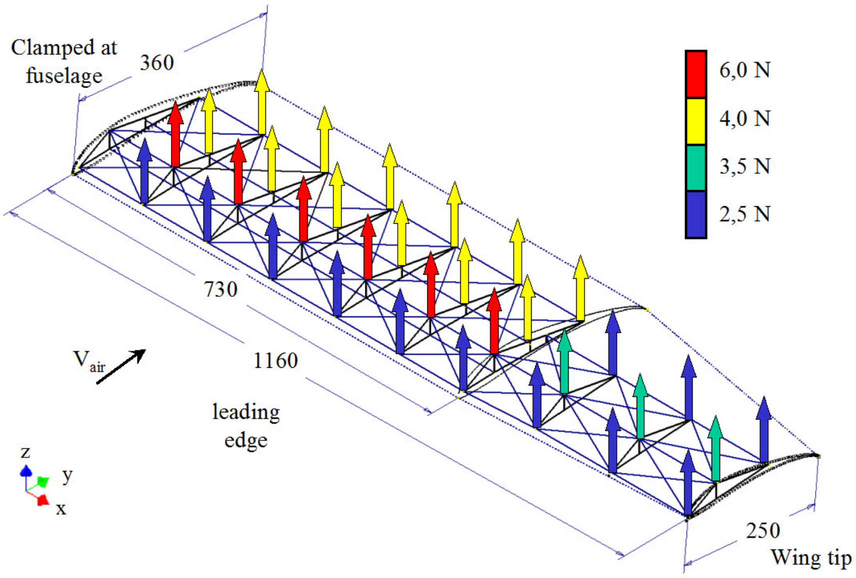

Figure 2. Discrete Structure, in [mm], and its boundary conditions.

The load acting on the wing was computed from the pressure distribution obtained for each airfoil. The numerical tests were performed for a condition of level flight at speed of 18 m/s (in this situation the wing is at an angle of attack α=6 °). Being applied on the values a maximum load factor $n_{máx}=2,5$, obtained in a *v-n* diagram developed as the *FAR part 23*. The specific mass considered for the air was $\rho=1,10\ kg/m^3$. In Fig. (3) are shown the gradient and the pressure distribution coefficient c_p for the profile of the straight wing piece (*Selig 1223* with slot and slotted flap). The curves show the values of c_p for the upper camber, lower camber and the sum. Figure (4) shows the analysis results for the airfoil *Selig 1223 clean*.

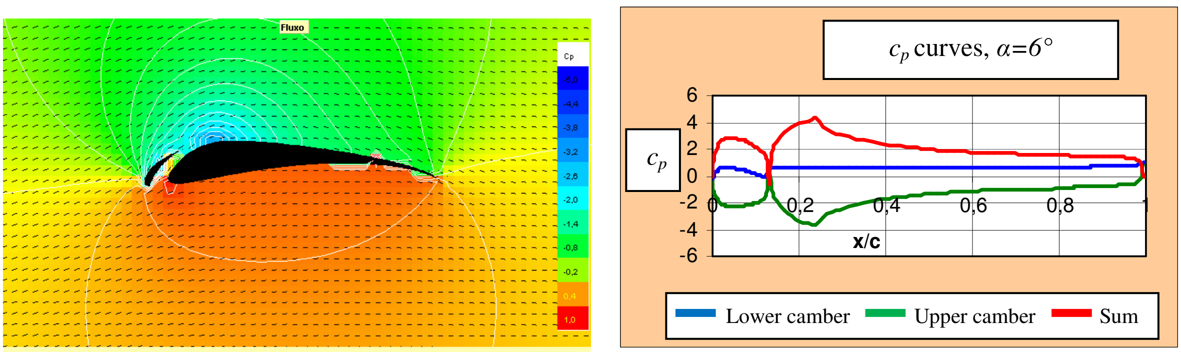

Figure 3. (a) Gradient and (b) c_p curve for *Selig 1223* with slot and slotted flap.

Were considered the effects of lift and drag. The loads were applied perpendicularly to the surface, according to Raymer (1989).

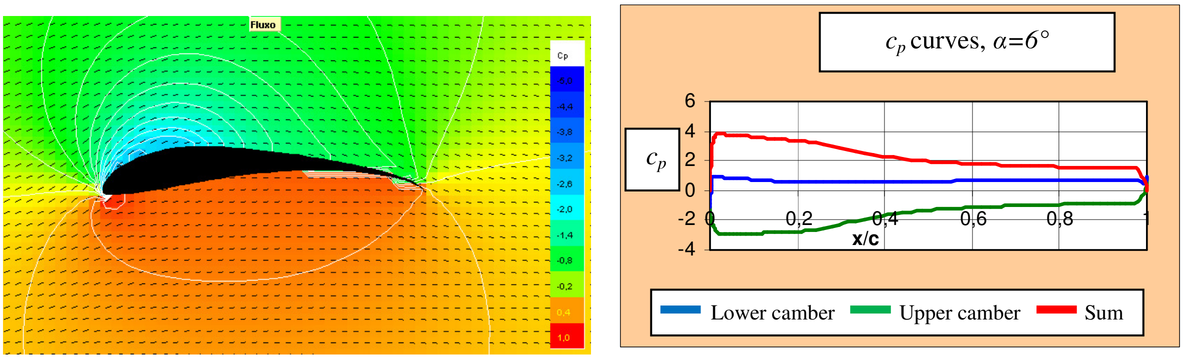

Figure 4. (a) Gradient and (b) c_p curve for *Selig 1223 clean.*

5.2 Optimization procedure for this case

In this case, outer diameters of tubular frames vary independently, assuming discrete values $D = [3, 5, 6, 8, 10]$ mm, which depends on available diameters on the market and are 0.5 mm thick.

The total number of design variables is 198, which is the number of bars of the wing. The limits for reliability scores for stress, displacement and natural frequency were assumed to be 3. The Tab. (1) presents the properties and constraints used for the problem. The uncertainties in the model are on diameters (which are design variables), on tensile strength that shows average of 633.31 MPa and variation coefficient of 0.02, on elasticity modulus of the material which has a variation coefficient of 0.02; for all the uncertainties were used a lognormal distribution. A general mathematical model for optimization used in this work can be seen in equation (15), where β_{re} is the reliability index value found for the stress value, β_{de} is the reliability index value found for the displacement, β_{fe} is the reliability index value found for the natural frequency, β_r is limit value of reliability index for the strength, β_d is limit value of reliability index for the displacement and β_f is limit value of reliability index for the natural frequency.

Properties	Values	Units
E (Elasticity Modulus)	90e3	MPa
ρ (density of a material)	2e-3	g/mm3
Limit of strength	633,31	MPa
Limit of maximum displacement at wing	80	mm
Limit of the first natural frequency	1	Hertz
Reliability index constrains	$\beta_{re} \geq 3$, $\beta_{de} \geq 3$ and $\beta_{fe} \geq 3$	-

Table 1. Parameters for optimization.

$$\text{Minimize } f(\mathbf{d}) = \sum_{i=1}^{\substack{bars\\numbers}} \rho L A(\mathbf{d})_i \qquad (15)$$

subject to

$$\beta_{re} \geq \beta_r(\mathbf{d})$$
$$\beta_{de} \geq \beta_d(\mathbf{d})$$
$$\beta_{fe} \geq \beta_f(\mathbf{d})$$
$$\mathbf{d}_L \leq \mathbf{d} \leq \mathbf{d}_U$$

The vector $(\mathbf{d})$ represents the set of design variables, which, in this case, are the bars areas.

6. RESULTS

The method was performed five times because the AG is a method in which the initial population is generated randomly. The same value was obtained on three times for the objective function given below. The convergence occurred by the variation coefficient of the best individuals of the last five generations have been lower than specified as a tolerance for convergence. The number of generations was 56 and the number of individuals used in each generation was 200.

The obtained graphic, Fig. (5), shows the convergence iterations, indicating the best and mean values of frames mass into population for the GA method.

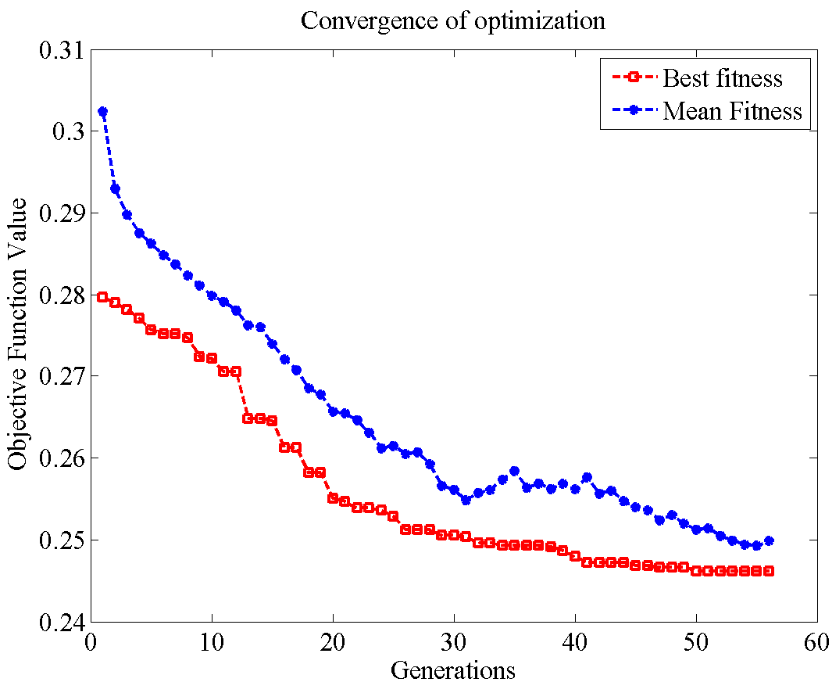

Figure 5. Convergence figure for Best and Mean values of truss weight for GA method.

A comparison between the current values used in the wing and the optimized values can be performed, where the reliability indexes were also calculated for both sets of parameters, see Tab. (2).

Parameters	Current	Optimized	Units
Stress	164,078	199,711	MPa
Displacements	30,4134	54,955	mm
Frequency	1,029	1,323	Hertz
β_{re}	6,719	5,988	-
β_{de}	7,222	3,001	-
β_{fe}	1,659	3,224	-
Objective Function	371	246	g

Table 2. Values obtained with sets of parameters (actual and optimized).

With the set of parameters obtained with optimization, the mass reduction was 33.7% of current value, decreasing from 371 g to 246 g. Regarding the current set of parameters, it is possible to realize that it has high levels of reliability for stress and displacement, but the value of reliability index for frequency was low, which shows a reasonable failure probability for this project.

The values of reliability indices for the design configuration with optimized parameters show that the limit value for the displacement is the design control, because the reliability index for stress and natural frequency are higher than value 3, while the reliability index displacement has almost reached the constraint.

Figure (6) shows the final geometry, where the diameters found with the optimization can be seen.

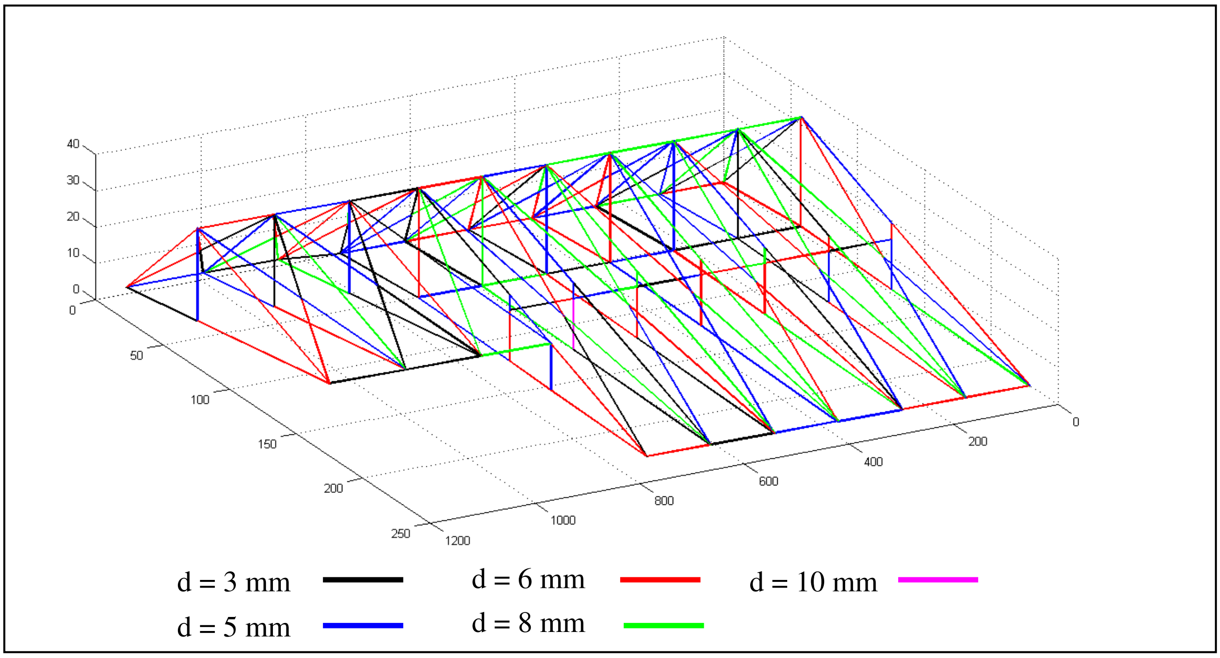

Figure 6. Wing geometry obtained with the reliability based design optimization.

7. CONCLUSIONS

Verifying the behavior of the reliability-based optimization methodology for the wing of the aircraft using genetic algorithms, proved that the method is efficient, since the mass reduction of the wing was considerable and the limits to restrictions were not violated.

The presented result for the case shows that the method can be used in similar cases. Although the optimization method has proven effectiveness in this case, the use of the reliability index constraints can generate a high computational cost, because many evaluates of function state limit are required.

For these cases, it is recommended the use of a hybrid method in the optimization procedure, thinking in terms of accelerating the convergence of the procedure.

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