FINITE ELEMENT MODELING OF COUPLED ELASTOPLASTICITY-ORTHOTROPIC DAMAGE FOR METALS

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Abstract. *In this paper, a three-dimensional ductile damage model based on principles of continuum mechanics is analyzed. The hypothesis of strain equivalence and the concept of effective stress, according to a methodology based on the thermodynamics of irreversible processes govern the model. The theories of elastoplasticity and damage (Lemaitre model) are coupled to perform a numerical simulation of the evolution of damage in structures through the finite element method (FEM). Postulates of damage mechanics in solid medium are used to incorporate the damage as an internal variable to the model. An isotropic scalar damage variable is firstly used and later the anisotropy in the distribution of microcracks is introduced through the orthotropic damage which is represented by a second order symmetric tensor. The program developed is based on FEM and the constitutive model of Lemaitre is suitable for metallic materials, considering an isotropic material behavior with linear isotropic hardening and von Mises yield criteria. The corresponding numerical algorithm integrating the constitutive equations is based on a prediction step (elastic trial state) and a correction step (plastic/damage corrector state), and the implementation of the numerical simulation was performed using the MATLAB®. The algorithm for integration and mapping of return based on the constitutive model are presented, as well as the numerical results.*

Keywords*: orthotropic damage, damage mechanics, elastoplasticity, Finite Element Method.*

1. INTRODUCTION

Continuum Damage Mechanics (CDM) represents a local approach to detect failure in a material and it is one of the most promising tools to predict macro-crack initiation and propagation (Doghri, 1995), treating the damaged material as a macroscopically homogeneous one (Chaboche, 1981). Fracture mechanics became one of its leading branches. It was based on the analysis of existing cracks, representing the damage that is the deterioration of material which occurs prior to failure (Lemaitre, 1996).

The objective of this paper is coupling the theories of plasticity and damage to perform numerical simulations of the evolution of damage in structures through the finite element method (FEM). Only the kinematics of small strains and displacements are considered.

The routines developed are adequate to metals and materials whit isotropic behavior and linear isotropic hardening. The numerical implementation was realized using the MATLAB® 7.6.0 program.

Isotropic and orthotropic damage is implemented. The importance of the orthotropic damage is in situations where, for example, two or more highly directional loads are applied sequentially. In such cases, each load will cause micro-cracks to grow in one preferential direction, affecting the material response to subsequent loads in different directions. Therefore, the usual isotropy hypotheses may offer a good first approximation, but may lead to substantial errors in many practical applications (Souza Neto et al. 2008). In this way, one of the goals of this paper is to compare the evolutions of isotropic and orthotropic damages.

2. CONTINUUM DAMAGE MECHANICS

The Continuum Damage Mechanics (CDM) is a branch of continuum solid mechanics, where it is possible to formulate continuum constitutive models capable to accounting for the internal deterioration of the solids.

The formulation of damage presented is based on the principle of continuum mechanics, on the strain equivalence hypothesis and on the effective stress concept $\bar{\sigma}$, which L.M. Kachanov as the pioneer in this study (Katan e Voyiadjis, 2002; Ladevèze, 1983; Armero e Ollers, 2000). Since the original model proposed by Kachanov (1958) and Rabotnov (1963), it did not take long before the concept of internal damage variable was generalized to three-dimensional situations.

The three-dimensional damage in ductile metals coupled which plasticity will be modeled in this paper utilizing the Lemaitre's ductile damage model (Lemaitre e Desmorat, 2005; Souza Neto et al. 2008).

2.1. Thermodynamics of damage

The starting point of this theory is the assumption that the free energy, taken as a thermodynamic potential, is a function of the set of internal state variables $\left\{\boldsymbol{\varepsilon}^p,\ \alpha, D\right\}$ (Souza Neto et al. 2008), as in Eq. (1):

$$\psi = \psi\left(\boldsymbol{\varepsilon}^e,\ \alpha, D\right) \tag{1}$$

2.1.1. Dissipation potential and evolution laws

The evolution laws of the internal variables $\left\{\boldsymbol{\varepsilon}^p,\ \alpha, D\right\}$ are derived from a dissipation potencial φ which is a function of associated variables $\left\{\boldsymbol{\sigma}, q, Y\right\}$ (Lemaitre, 1985). The dissipation potencial is assumed to be written as a sum of parcelas which effects of plasticity and hardening φ^p and damage φ^d , as in Eq. (2):

$$\varphi = \varphi^p\left(\boldsymbol{\sigma}, q; D\right) + \varphi^d\left(Y; D\right) \tag{2}$$

The evolution laws of plasticity with coupled damage are derived from this potential by mean of a scalar multiplier γ which is always positive,

$$\dot{\boldsymbol{\varepsilon}}^p = \gamma\frac{\partial\varphi}{\partial\boldsymbol{\sigma}}, \quad \dot{\alpha} = -\gamma\frac{\partial\varphi}{\partial q}, \quad \dot{D} = -\gamma\frac{\partial\varphi}{\partial Y} \tag{3}$$

2.2. Isotropic damage

In this case, the damage variable D is a scalar and presents the same value in all directions (Omerspahic, 2007).

In the present theory the postulated form for the elastic-damage potential is given by Eq. (4) (Lemaitre e Desmorat, 2005). The energy density release rate may be written by Eq. (5):

$$\rho\psi^{ed}\left(\boldsymbol{\varepsilon}^e, D\right) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \left(1-D\right)\boldsymbol{C} : \boldsymbol{\varepsilon}^e = \frac{1+\nu}{2E}\frac{\boldsymbol{\sigma}:\boldsymbol{\sigma}}{1-D} - \frac{\nu}{2E}\frac{tr\left(\boldsymbol{\sigma}\right)^2}{1-D} \tag{4}$$

$$-Y = \rho\frac{\partial\psi}{\partial D}\left(\boldsymbol{\varepsilon}^e, D\right) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \boldsymbol{C} : \boldsymbol{\varepsilon}^e = \frac{\varsigma^2 R_v}{2E(1-D)^2} \tag{5}$$

introcucing the triaxiality function $R_v = 2/3\left(1+\nu\right) + 3\left(1-2\nu\right)\left(p/\varsigma\right)^2$, where $\boldsymbol{C}$ is the isotropic elasticity tensor, $p = (1/3)\,tr\left(\boldsymbol{\sigma}\right)$ is the volumetric pressure, $\varsigma = \sqrt{(3/2)\,\boldsymbol{s}:\boldsymbol{s}}$ is the von Mises equivalent stress, $\boldsymbol{s} = \boldsymbol{\sigma} - p\boldsymbol{I}$ is the stress deviator, ν is the Poisson ratio and E is the Young modulus of elasticity.

The elasticity law is given by Eq. (6), where $\tilde{\boldsymbol{C}} = (1-D)\boldsymbol{C}$ is the effective elasticity tensor, which considers the effect of damage.

$$\boldsymbol{\sigma} = \rho\frac{\partial\psi^{ed}}{\partial\varepsilon^e} = \left(1-D\right)\boldsymbol{C} : \boldsymbol{\varepsilon}^e = \tilde{\boldsymbol{C}} : \boldsymbol{\varepsilon}^e \tag{6}$$

The effective stress tensor $\bar{\boldsymbol{\sigma}}$ may be written by Eq. (7). Equation (8) shows the following von Mises form adoptad for the yield function f .

$$\bar{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{\left(1-D\right)} \tag{7}$$

$$f\left(\boldsymbol{\sigma}, q, D\right) = \bar{\varsigma} - \left(\sigma_y + q\right) \tag{8}$$

where $\bar{\varsigma} = \sqrt{\left(3/2\right)\bar{\boldsymbol{s}} : \bar{\boldsymbol{s}}} = \sqrt{3J_2\left(\bar{\boldsymbol{s}}\right)}$, $q = H'\alpha$, H' is the isotropic hardening modulus and σ_y is the initial yield stress of the material.

The plastic flow rule is given by Eq. (9), where $\boldsymbol{N}$ is the flow vector given by Eq. (10) (Souza Neto et al. 2008) and J_2 is the second invariant of the stress deviator $\boldsymbol{s}$. The evolution lei of the internal variable associated to the isotropic hardening is given by Eq. (11).

$$\dot{\varepsilon}^p = \gamma \frac{\partial \varphi}{\partial \boldsymbol{\sigma}} = \gamma \frac{\partial \varphi^p}{\partial \boldsymbol{\sigma}} = \gamma \boldsymbol{N} \tag{9}$$

$$\boldsymbol{N} = \sqrt{\frac{3}{2}} \frac{\boldsymbol{s}}{\left(1\text{-}D\right)\left\|\boldsymbol{s}\right\|} = \frac{3}{2} \frac{\boldsymbol{s}}{\left(1\text{-}D\right)\sqrt{3J_2\left(\boldsymbol{s}\right)}} \tag{10}$$

$$\dot{\alpha} = -\gamma \frac{\partial \varphi}{\partial q} = -\gamma \frac{\partial \varphi^p}{\partial q} = \gamma \tag{11}$$

By taking the Equation (11), the evolution law for the accumulated plastic strain is given by Eq. (12) and the Equation (13) give the damage evolution law, where r and s are material parameters.

$$\dot{\varepsilon}^p_{ac} = \sqrt{\frac{2}{3}} \left\|\dot{\boldsymbol{\varepsilon}}^p\right\| = \frac{\gamma}{1-D} \tag{12}$$

$$\dot{D} = -\gamma \frac{\partial \varphi^d}{\partial Y} = \left(\frac{-Y}{r}\right)^s \dot{\varepsilon}^p_{ac} \tag{13}$$

The hypothesis of associate plasticity is adopted and the yield function is taken as plastic potential.

This model admits that damage growth starts only at a critical value of accumulated plastic strain. This value will be called the damage threshold and denoted by ε_D .

The condition to a mesocrack starts is when the corresponding damage reaches a critical value D_c. The critical damage is a material parameter.

2.3. Orthotropic damage

The orthotropic damage model used here is an extension of isotropic ductile damage model described above.

The largest generality for a damage variable is a representation by a fourth order tensor, but for practical applications a symmetric second order tensor is often used (Desmorat e Cantournet, 2007). As shown by microscopic observations, the second order damage tensor is mainly driven by the plastic strain which make it orthotropic (Lemaitre e Desmorat, 2001).

The forth order effective elasticity tensor and the effective stress tensor are defined by Eq. 14 e Eq.15, respectively.

$$\tilde{\boldsymbol{C}} = \boldsymbol{M}^{-1} : \boldsymbol{C} \tag{14}$$

$$\bar{\boldsymbol{\sigma}} = \boldsymbol{M}\boldsymbol{\sigma} \tag{15}$$

The forth order tensor $\boldsymbol{M}$ may be written by Eq. (16), where δ_{ij} é o delta de Kronecker, $d_H = \eta D_H$, η is a material parameter associated with the variation of Poisson's ratio due to damage and often for metals $\eta \approx 3$, $D_H = \left(1/3\right) tr\left(\boldsymbol{D}\right)$:

$$M_{ijkl} = H_{ik}H_{lj} - \frac{1}{3}\left[H_{kl}^2\delta_{ij} + H_{ij}^2\delta_{kl}\right] + \frac{1}{9}H_{pp}^2\delta_{ij}\delta_{kl} + \frac{1}{3\left(1-d_H\right)}\delta_{ij}\delta_{kl} \tag{16}$$

In the above expression, $\boldsymbol{H}$ is the second order tensor defined by:

$$H_{ij} \equiv \left(\boldsymbol{I} - \boldsymbol{D}\right)_{ij}^{-1/2} \tag{17}$$

The effective stress tensor also is defined as the sum of its volumetric and deviatoric parts, as

$$\bar{\boldsymbol{\sigma}} = \bar{\boldsymbol{s}} + \bar{p}\boldsymbol{I} \tag{18}$$

where $\bar{\boldsymbol{s}}$ e $\bar{p}$ are, respectively, the effective stress deviator and effective volumetric pressure defined by

$$\bar{\boldsymbol{s}} \equiv dev\left[\boldsymbol{HsH}\right], \quad \bar{p} = \frac{p}{1-d_H} \tag{19}$$

The energy density release rate tensor is written as $-Y_{ij} = \rho\partial\psi / \partial D_{ij}$, but in the damage evolution law it is replaced by an scalar $-\bar{Y}$ called effective elastic energy density, given by

$$-\bar{Y} = \frac{1}{2}C_{ijkl}\varepsilon_{kl}^e\varepsilon_{ij}^e = \frac{1}{2}\bar{\sigma}_{ij}\varepsilon_{ij}^e = \frac{\bar{\varsigma}^2\bar{R}_v}{2E} \tag{20}$$

where C_{ijkl} are the components of the elasticity tensor $\boldsymbol{C}$ and $\bar{R}_v = (2/3)\left(1+\nu\right) + 3\left(1-2\nu\right)\left(\bar{p}/\bar{\varsigma}\right)^2$ is the effective triaxiality function, with $\bar{\varsigma} = \left(\boldsymbol{HsH}\right)_{eq} = \left[\left(3/2\right)dev\left(\boldsymbol{HsH}\right) : dev\left(\boldsymbol{HsH}\right)\right]^{1/2}$.

According Lemaitre et al. (2000), the anisotropic damage evolution law is a simple extension of the isotropic case, if we considerer the dissipative potential given by Eq. (21), where $\left|\cdot\right|$ applied to a tensor means the absolute value of the principal values, α is the internal variable associated to the isotropic hardening, and f is the von Mises yield function given by Eq. (22):

$$\varphi = f + \varphi^d = f + \left(-\frac{\bar{Y}}{r}\right)^s \left(-Y_{ij}\right)\left|\frac{d\varepsilon^p}{d\alpha}\right|_{ij} \tag{21}$$

$$f = \bar{\varsigma} - \left(\sigma_y + q\right) \tag{22}$$

The plastic flow rule is given by Eq. (23), where the normals $\boldsymbol{N}^X$ and $\boldsymbol{n}^X$ are definide by Eq. (24) (Lemaitre e Desmorat, 2005). The evolution of the hardening variable has the usual format given by Eq. (25), and the accumulated plastic strain rate by Eq. (26).

$$\dot{\boldsymbol{\varepsilon}}^p = \gamma\frac{\partial\varphi}{\partial\boldsymbol{\sigma}} = \gamma\boldsymbol{N}^X \tag{23}$$

$$\boldsymbol{N}^X = dev\left(\boldsymbol{H}\boldsymbol{n}^X\boldsymbol{H}\right), \quad \boldsymbol{n}^X = \frac{3}{2}\frac{\bar{\boldsymbol{s}}}{\bar{\varsigma}} = \sqrt{\frac{3}{2}}\frac{\bar{\boldsymbol{s}}}{\left\|\bar{\boldsymbol{s}}\right\|} \tag{24}$$

$$\dot{\alpha} = \gamma \tag{25}$$

$$\dot{\varepsilon}^{p}_{ac} = \sqrt{\frac{2}{3}\left\|\dot{\boldsymbol{\varepsilon}}^{p}\right\|} = \sqrt{\frac{2}{3}\boldsymbol{N}^{X} : \boldsymbol{N}^{X}}\,\gamma \tag{26}$$

According to present theory, the rate of damage tensor is assumed to follow the directions of plastic strain. The evolution law for damage is defined by

$$\dot{\boldsymbol{D}} = -\gamma \frac{\partial \varphi^{d}}{\partial Y} = \left(\frac{-Y}{r}\right)^{s} \dot{\tilde{\varepsilon}}^{p} \tag{27}$$

with the absolute plastic strain rate defined by $\dot{\tilde{\varepsilon}}^{p} = \sum_{i=1}^{3} \left|\dot{\varepsilon}^{p}_{i}\right| e^{p}_{i} \otimes e^{p}_{i}$, where $\dot{\varepsilon}^{p}_{i}$ are the eigenvalues of the plastic strain rate tensor $\dot{\boldsymbol{\varepsilon}}^{p}$, and $\left\{e^{p}_{i}\right\}$ is an orthornormal basis of eigenvectores of $\dot{\boldsymbol{\varepsilon}}^{p}$.

The damage evolution starts only above a threshold defined in terms of the accumulated plastic strain, i.e. $\dot{\boldsymbol{D}} = 0,$ se $\varepsilon^{p}_{ac} < \varepsilon_{D}$.

According to a physics definition of damage, a mesocrack is initiated when the density of defects in some plane reaches the critical value D_c. For anisotropic damage with principal damage components D_I, this take place when $\max D_I = D_c$ (Lemaitre e Desmorat, 2001).

2.4. Integration and return-mapping algorithm for damage coupled with elastoplasticity model

Next the constitutive model is presented in its implicit incremental form, which leads to the return mapping algorithm. In this case we consider the three-dimensional elastoplasticity model with isotropic hardening coupled with isotropic and orthotropic damage, proposed in Lemaitre e Desmorat (2005).

The numerical algorithm is based on a prediction step which correspond to the elastic trial state and a correction step corresponding to the plastic/damage corrector state.

1) Evaluate the elastic trial test (elastic predictor)

Given $\Delta\boldsymbol{\varepsilon}$ and the state variables at t_n, evaluate the elastic trial test

$$\boldsymbol{\varepsilon}^{e\ teste}_{n+1} = \boldsymbol{\varepsilon}^{e}_{n} + \Delta\boldsymbol{\varepsilon}, \quad \alpha^{teste}_{n+1} = \alpha_n, \quad \varepsilon^{teste}_{ac\ (n+1)} = \varepsilon_{ac\ (n)}, \quad \boldsymbol{D}^{teste}_{n+1} = \boldsymbol{D}_n, \quad \bar{\boldsymbol{\sigma}}^{teste}_{n+1} = \boldsymbol{C}\boldsymbol{\varepsilon}^{e\ teste}_{n+1}$$

2) Check plastic consistency

If $f^{teste}_{n+1} := \bar{\varsigma}^{teste}_{n+1} - \left(\sigma_y + H'\alpha^{teste}_{n+1}\right) \le 0$,

then elastic step, set $\left(\cdot\right)_{n+1} = \left(\cdot\right)^{teste}_{n+1}$ e $\boldsymbol{C}^{ep}_{n+1} = \boldsymbol{C}$,

exit.

Else plastic step go to (3).

3) Return mapping (correction step)

Let the local residual defined as $\left\{R_{loc}\right\} = \left\{R_{\boldsymbol{\varepsilon}^{e}}, R_{\Delta\gamma}, R_{\boldsymbol{D}}\right\}^{T}$. Solve the system for the unknown independent variables $W = \left\{\boldsymbol{\varepsilon}^{e}, \Delta\gamma, \boldsymbol{D}\right\}$.

$$\left\{\begin{array}{l} R_{\boldsymbol{\varepsilon}^e} = \Delta\boldsymbol{\varepsilon}^e - \Delta\boldsymbol{\varepsilon} + \Delta\boldsymbol{\varepsilon}^p = \boldsymbol{\varepsilon}_{n+1}^{e(i)} - \boldsymbol{\varepsilon}_n^e - \Delta\boldsymbol{\varepsilon} + \Delta\gamma_{n+1}^{(i)} N_{n+1}^X \\ R_{\Delta\gamma} = f_{n+1} := \bar{\varsigma}_{n+1}^{(i)} - \left(\sigma_y + H'\left(\alpha_n + \Delta\gamma_{n+1}^{(i)}\right)\right) \\ R_{\boldsymbol{D}} = \left\{\begin{array}{ll} \boldsymbol{D}_{n+1}^{(i)} - \boldsymbol{D}_n - \left(\dfrac{-Y}{r}\right)^s \dfrac{\Delta\gamma_{n+1}^{(i)}}{1-\boldsymbol{D}_{n+1}^{(i)}} & \text{(isotrópico)} \\ \boldsymbol{D}_{n+1}^{(i)} - \boldsymbol{D}_n - \left(\dfrac{-\bar{Y}}{r}\right)^s \tilde{\varepsilon}^p & \text{(ortotrópico)} \end{array}\right. \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right\}$$

where $\Delta\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n$.

The Newton-Raphson iterative scheme may be used again and one has to solve the local iterative problem:

$$\left\{R_{loc\ (n+1)}^i\right\} + \left[\frac{\partial\left\{R_{loc}\right\}}{\partial\Delta W}\right]_{n+1}^{(i)} \left(W_{n+1}^{(i+1)} - W_{n+1}^{(i)}\right) = 0$$

with $\Delta W = \left\{\Delta\boldsymbol{\varepsilon}^e, \Delta\Delta\gamma, \Delta\boldsymbol{D}\right\}$, where the expression for the Jacobian matrix $\left[\boldsymbol{Jac}\right] = \left[\partial\left\{R_{loc}\right\} / \partial\Delta W\right]_{n+1}^{(i)}$ (or any good approximation) is needed for convergence reasons. In this paper one uses the finite differences method to evaluate the Jacobian matrix.

4) Update explicitly the remaining variables

$$\bar{\boldsymbol{\sigma}}_{n+1} = \boldsymbol{C}\boldsymbol{\varepsilon}_{n+1}^e, \qquad \alpha_{n+1} = \alpha_n + \Delta\gamma_{n+1}, \qquad \boldsymbol{\varepsilon}_{n+1}^p = \boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_{n+1}^e$$

$$\varepsilon_{ac\ (n+1)}^p = \varepsilon_{ac\ (n)}^p + \sqrt{\frac{2}{3}\boldsymbol{N}^X : \boldsymbol{N}^X}\,\Delta\gamma_{n+1} \text{ (for orthotropic damage)}$$

$$\varepsilon_{ac\ (n+1)}^p = \varepsilon_{ac\ (n)}^p + \frac{\Delta\gamma_{n+1}}{1-D_{n+1}} \text{ (for isotropic damage)}$$

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{M}_{n+1}^{-1}\bar{\boldsymbol{\sigma}}_{n+1} \text{ (for orthotropic damage)}$$

$$\boldsymbol{\sigma}_{n+1} = \left(1-D_{n+1}\right)\bar{\boldsymbol{\sigma}}_{n+1} \text{ (for isotropic damage)}$$

$$\bar{\varsigma}_{n+1} = \left[\frac{3}{2} dev\left(\boldsymbol{H}_{n+1}\boldsymbol{s}_{n+1}\boldsymbol{H}_{n+1}\right) : dev\left(\boldsymbol{H}_{n+1}\boldsymbol{s}_{n+1}\boldsymbol{H}_{n+1}\right)\right]^{1/2} \text{ (for orthotropic damage)}$$

$$\bar{\varsigma} = \sqrt{\frac{3}{2}\frac{\boldsymbol{s}_{n+1}}{\left(1-D_{n+1}\right)} : \frac{\boldsymbol{s}_{n+1}}{\left(1-D_{n+1}\right)}} \text{ (for isotropic damage)}$$

5) Evaluate the elastoplastic tangent operator:

Let the first column of the inverse of Jacobian matrix $\left[\boldsymbol{Jac}\right]$ at convergence is

$$\begin{bmatrix} [\boldsymbol{Jac}]^{-1}_{\Delta\gamma,\boldsymbol{\varepsilon}^e} \\ [\boldsymbol{Jac}]^{-1}_{\boldsymbol{\varepsilon}^e,\boldsymbol{\varepsilon}^e} \\ [\boldsymbol{Jac}]^{-1}_{\boldsymbol{D},\boldsymbol{\varepsilon}^e} \end{bmatrix}$$

The expression for the elastoplastic tangent operator consistent which the integration algorithm above is (Lemaitre e Desmorat, 2005):

$$\boldsymbol{C}^{ep} = \boldsymbol{M}^{-1} : \boldsymbol{C} : [\boldsymbol{Jac}]^{-1}_{\boldsymbol{\varepsilon}^e,\boldsymbol{\varepsilon}^e} + \bar{\boldsymbol{\sigma}} : \frac{\partial \boldsymbol{M}^{-1}}{\partial \boldsymbol{D}} : [\boldsymbol{Jac}]^{-1}_{\boldsymbol{D},\boldsymbol{\varepsilon}^e}$$

3. NUMERICAL APPLICATION

In this section a numerical test is implemented. The coupling plasticity/isotropic and orthotropic damage is performed through the application of the algorithm detailed in Section 2.4.

The entire computational implementation was done via user code using the MATLAB® 7.6.0. The visualization of the results of the analysis was performed using the GID® 9.0.2 software, which is a system of pre and post-processing of Finite Elements results. In this problem the isoparametric solid elements (hexahedrons) with eight nodes is utilized.

The problem considered is the three-dimensional analysis of a thick-walled cylinder (Fig.1).

The following constraints of displacement are imposed on nodes on the surface $z = 0$:

- Component $z = 0$ on all nodes.
- Component $x = 0$ on nodes along the y-axis.
- Component $y = 0$ on nodes along the x-axis.

Firstly, a prescribed axial displacement is applied at surface $z = 30$ mm of the cylinder, divided in 50 steps. After that, the cylinder is subjected to prescribed outward radial displacement at its inner surface, also over 50 steps. The constitutive model of the material is elastoplastic, with von Mises yield criterion, linear isotropic hardening and isotropic and orthotropic damage, as presented in Section 3.4. The problem data are: E = 210 GPa (elasticity modulus); ν = 0,3 (Poisson ratio); H' = 10,5 GPa (isotropic plastic hardening modulus); σ_y = 620 MPa (yield stress); D_c = 0,40 (critical damage); ε_D = 0,0 (damage threshold); η = 1,0 (parameter of material related to the damage in the isotropic case) ; η = 3,0 (orthotropic case); r = 3,5 and s = 1,0 (material parameters for damage); d_1 = 2 mm (prescribed axial displacement); d_2 = 0,3 mm (prescribed radial displacement).

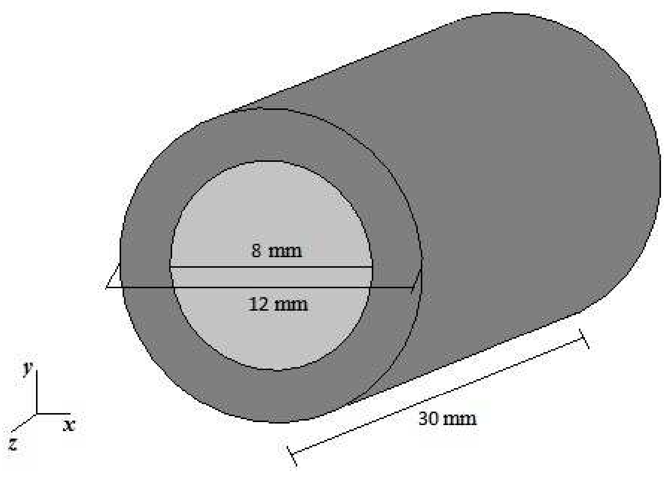

Figure 1. Cylinder dimentions

An mesh consisting of 640 hexahedral elements was used, as shown in Figure 3.

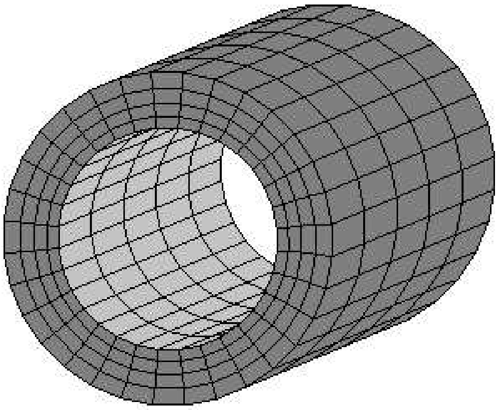

Figure 3. Mesh used in the analysis

3.1. Comparison of results

In this section, some results of the evolution of isotropic and orthotropic damages are compared. Figure 4 shows the damage – displacement variation in *z* direction for the models of isotropic and orthotropic damage for the first prescribed displacement (maximum value in a face element). In the orthotropic case the principal values of damage corresponding to the directions of orthotropy are shown. We can see that in both cases of damage the values of isotropic and orthotropic (first principal value) damage are very close.

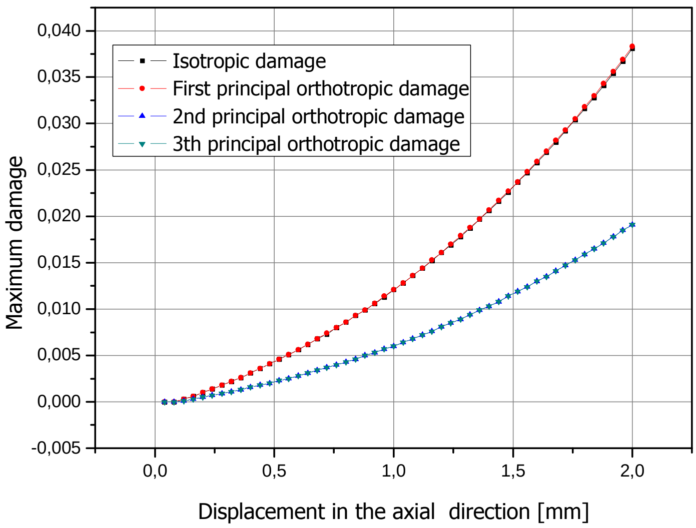

Figure 4. Damage versus displacement in *z* direction [mm]

Figure 5 shows the damage - accumulated plastic strain curve for the models of isotropic and orthotropic damage after the two prescribed displacements. In first prescribed displacement we can see similar results for both isotropic and orthotropic (first principal value) cases, with accumulated plastic strain until 0.06. After the second prescribed displacement we can see that the material response was affected by the change in direction of loading. The values of damage and accumulated plastic strain in the isotropic case are larger than the orthotropic (first principal value) one. In this case, the principal directions were not modified, but a change in the proportion between the principal plastic strains after the first load is verified.

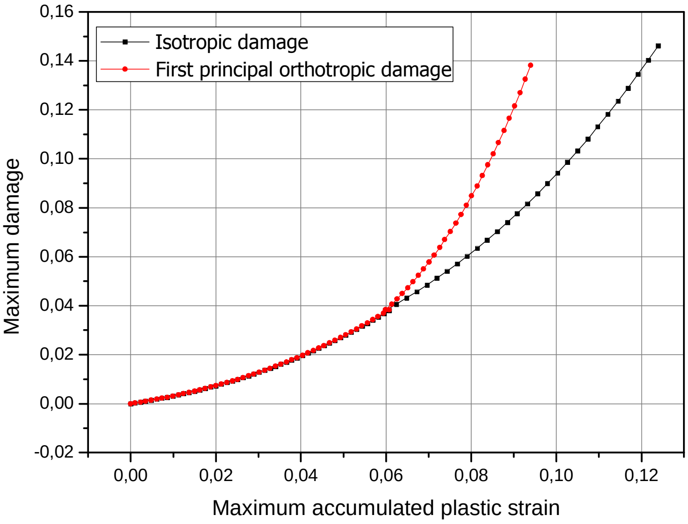

Figure 5. Damage versus accumulated plastic strain

In Figure 6 is shown the behavior of isotropic damage after the two loads.

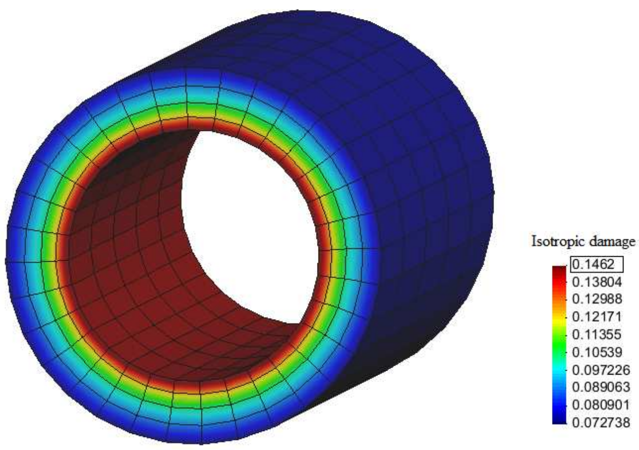

Figure 6. Isotropic damage after the two loads

4. CONCLUSION

The aim of this study was to compare, through numerical simulation the evolution of the isotropic and orthotropic damages in metals coupling theories of plasticity and damage.

The computational implementation adapted to three-dimensional analysis was based on Finite Element Method (FEM) and in a constitutive damage model of Lemaitre that is adequade for metallic materials. This model is applied to the analysis both isotropic and orthotropic beyond the analysis without damage.

Two sequential loads were applied with change of direction. For the orthotropic case each load induced the damage growth in a preferred direction, affecting the response of the material as compared with the isotropic case. We can see that the maximum value of isotropic damage is larger than the orthotropic (first principal value) one.

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