IDENTIFICATION OF ELASTOPLASTIC PARAMETERS UNDER FINITE STRAIN USING A DIGITAL IMAGE CORRELATION METHOD

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Abstract. *Appropriated constitutive models are essential to simulate the mechanical behavior of nonlinear materials. The material parameters of these models are obtained by means of inverse techniques using data acquired from experimental tests and numerical simulation of the specimen. Localized heterogeneous displacement fields are easily found in specimens submitted do finite strains, when contact measurement techniques are no longer appropriated. The optical method Digital Image Correlation (DIC) has been demonstrated great acceptability in the material characterization due its capability to measure the entire displacement field over the specimen. In the identification process, an initial guess of the material parameters are used as input data in the numerical analysis of the discretized specimen. A nonlinear elastoplastic model is used in order to represent the necking phenomenon and the numerical solution of the reaction force and the displacement of points previously defined over the specimen surface are the output data that are used in the cost function of the minimization problem. The experimental displacements of the points used in the numerical simulation are three-dimensionally obtained in the DIC method and included in the cost function. The cost function is computed by the sum of squared differences between the experimental and numeric data that allow updating the material parameters. Uniaxial tensile tests with unloading-reloading are performed in order to obtain the begging of the plastic region. Different experimental-numerical tests are performed in order to evaluate the performance of the proposed technique. (Solid mechanics).*

Keywords: *Digital Image Correlation, Parameter identification, Material properties, Finite element method*

1. INTRODUCTION

The numerical simulation of components is quite useful in modern engineering problems. It gives a important project direction to the designer. Appropriated constitutive models and material parameters are needed to mathematically describe the mechanical behavior of the material. It becomes even more important when the component needs to be simulated under high velocity load or finite strain, like in impact tests.

In order to determine the material parameters of the constitutive model, an inverse problem can be performed. This mixed numerical-experimental procedure has been used in several areas and it is widely applied in parameter identification procedures. The experimental test more used to this scope is the tensile tests due to its simplicity. In Mahnken (1999) a material characterization is presented for an axisymmetric tensile bar of ferritic steel. A sensitivity analysis is performed and the gradient-based descent methods are used for the minimization of a least-square function. Other example is found in Gavrus *et al.* (1996) to characterize a thermal-viscoplastic model for Aluminum using a torsion test.

The presence of large and localized strain in specimens transforms the identification of material parameters in a nontrivial task. Frequently, experimental uniaxial tests provides insufficient or unhelpful information about the mechanical behavior of the material, especially for nonlinear materials. In Frank and Brockman (2001), the authors notice the formation of a traveling neck in glassy polymers materials under tension, differently of the metallic materials that have a localized necking phenomenon. Second the author, this particular behavior masks the real stress-strain curve. In other hand, the identification of the elastic limit (transition between elastic and plastic regions of the material) can be obtained by successive loading and unloading operations in an uniaxial test. These loading/unloading operations are carried out until a permanent deformation is observed (de Souza, 1982).

Since a structural component may presents a triaxial stress state, a complete characterization of the material should be obtained through different experimental tests. In Yoshida (2000) the author performed four types of experimental tests and proposed a viscoplastic constitutive model for metallic materials. However, nonhomogeneous deformation fields obtained in uniaxial tests are capable to add extra information regarding of material behavior (Oomens *et al.*, 1993). The displacement field found in the necking region during a uniaxial test confers a triaxial stress state. Then, this heterogeneous field could be measured and used to correctly identify the constitutive parameters. Measurement techniques, such as clip gauges and extensometers, have some limitations since they need physical contact, mainly in the presence of finite and localized strains. Furthermore, non-contact techniques, like optical techniques, can measure the full-field displacement over the specimen surface.

The optical method of Digital Image Correlation (DIC) was extensively studied by Sutton *et al.* (2000), Lu and Cary (2000) and Pan *et al.* (2009). This optical technique is capable to track predefined points in a sequence of digital images. The position of the points can be obtained acquiring images of the specimen surface during a experimental test. The method has been used to various engineering problems. In Roux *et al.* (2008) the DIC method is presented as an alternative to capture the crack progressing of cyclic tensile tests. Parameter identification procedures using the DIC method have been performed to characterize different constitutive models. Avril *et al.* (2008) uses the optical method in a tensile test for notched steel bars in order to identify the material parameters of an elasto-visco-plastic constitutive model. In Kajberg and Lindkvist (2004) the plastic behavior of thin sheets is analyzed, where the material parameters are adjusted by means of a inverse modeling using finite element analysis. The inverse problem including the optical measurement method also demonstrated good results in different experimental tests, as observed in Sutton *et al.* (2008) and Milani *et al.* (2009). The authors performed a parameters identification of a Johnson-Cook model, respectively, for tensile and compressive tests. In Wang and Cuitiño (2002) and Jin *et al.* (2007), the authors use the DIC method to capture localized and heterogeneous strain fields in order to characterize polymeric foams in compressive tests.

In Meuwissen *et al.* (1998) the author proposes the use of non-standard aluminum specimens, where the force obtained from the testing machine and the displacements measured optically at the clamps region are used as boundary conditions of its numerical simulation. The objective function is constructed only with the displacements measured at the necking region. This study shown the flexibility provided by the use of measurement optical methods associated to numerical simulation of non-standard geometries of specimens. The DIC method is important to the identification of the material parameters, considering that it can provide essential information about the three-dimensional displacement field over the specimen surface (Parker, 2009), i.e., it can capture the formation and propagation of the necking during the tensile test.

The parameter identification using optical methods has been used in the mechanical characterization of biological tissues. It is due to the fact that the optical technique allows to perform tests *in vivo* in different specimen geometries. Gambarotta *et al.* (2005) studied the mechanical characterization *in vivo* of human skin to simulate a reconstructive surgery with the finite element method. In Gundiah *et al.* (2009) is used a neo-Hookean isotropic model to characterize elastin networks obtained from porcine arteries. The experimental data was captured from biaxial test in order to analyze the structural integrity of blood vessels in cycles of pulsatile motion.

In view of the advantages provided by using measurement data obtained from optical methods in the parameter identification procedure, the main objective of the present work is to study a methodology to characterize polymeric materials under finite strains. In this work is analyzed the use of a simple constitutive model (multi-linear elastoplastic model) to represent the mechanical behavior of the polymer PolyVinyl Chloride (PVC).

2. EXPERIMENTAL PROCEDURE

2.1 Specimen definition

The polymers are known by their complex mechanical behavior under finite strains, which provides high levels of heterogeneous strain characterized by increasing stiffness due to internal molecular orientation. The specimens used in this work were manufactured with the glassy polymer PVC. Many studies on the mechanical behavior of glassy polymers were published showing an apparent stress drop in the real stress-strain curve right after yield. In Duan *et al.* (2001) the author show experimental curves for some glassy polymers exhibiting yielding followed by intrinsic strain softening and subsequent orientation hardening. This mechanical behavior is presented in Fig.1. Unlike the widely accepted softening behavior (stress drop), Frank and Brockman (2001) attribute this behavior to the formation of a traveling neck that masks the actual stress response, affirming that the true stress for most polymers in the glassy regime does not actually decrease after yield.

The geometry of the specimen used in the present study is presented in Fig. 2. It was insert a notch at the central region in order to initiate the necking phenomenon at the region of interest.

2.2 DIC method

The DIC method tracks predefined points on the specimen surface in a sequence of digital images obtained by video cameras during an experimental test. The method essentially finds the position of points, called markers, in a sequence of images, correlating the gray values of the neighbor pixels. The correlation is made within small squared areas of $(2M+1)$x$(2M+1)$ pixels around the marker, called subsets, where the geometric center of each subset is the marker (Fig. 3). To evaluate the similarity degree between the reference subset and the deformed subset many correlation criterion was proposed. A wide set of correlating coefficients may be found in the literature (Pan *et al.*, 2009). Two kinds of correlation criterion are mainly used in the DIC technique, the cross-correlation (CC) criterion and the sum-squared difference (SSD) criterion. The position of the deformed subset center is determined when the correlation criterion is a extremum. If a SSD criterion is used, the determination of the marker position is made by means of a minimum problem. In other hand, if a CC criterion is chosen, the determination of the marker position is made by means of a maximization

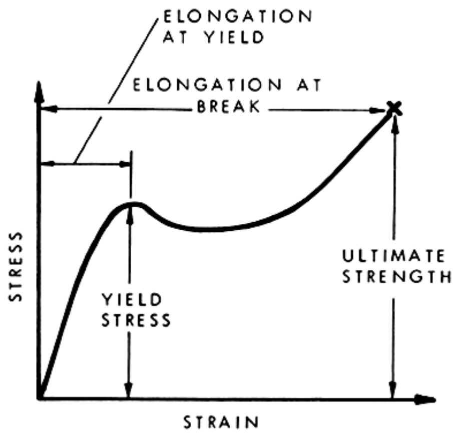

Figure 1. Generalized tensile stress-strain curve for polymeric materials (Blaga, 1973)

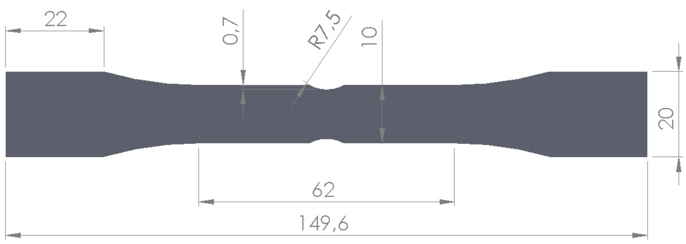

Figure 2. Test sample

problem. In this study was implemented the Zero-Normalized Sum Square Differences (ZNSSD) (Pan *et al.*, 2010):

$$C_{ZNSSD} = \sum_{i=-M}^{M} \sum_{i=-M}^{M} \left[\frac{f(x,y) - f_m}{\sqrt{\sum_{i=-M}^{M} \sum_{i=-M}^{M} (f(x,y) - f_m)^2}} - \frac{g(x',y') - g_m}{\sqrt{\sum_{i=-M}^{M} \sum_{i=-M}^{M} (g(x',y') - g_m)^2}} \right]^2 \quad (1)$$

where $f(x,y)$ is the gray value of the reference image at the pixel position (x,y), $g(x',y')$ is the gray value of the target image at the pixel position (x',y'), f_m and g_m are respectively the means gray value of the reference and deformed subset.

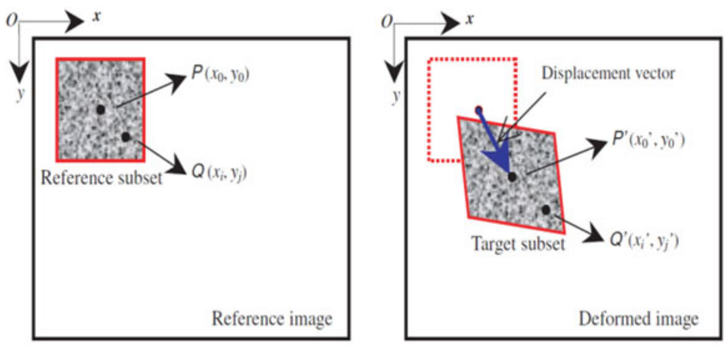

Figure 3. Reference subset and deformed/target subset (Pan *et al.*, 2009).

In order to take into account the shape changing of the subset, a displacement mapping function is used to map the points of the subset area in the reference image to points in the target image. Thus, the coordinates of point $Q(x_i, y_i)$ around the subset center (marker) $P(x_0, y_0)$ is mapped to point $Q'(x'_i, y'_i)$ around the subset center (marker) $P'(x'_0, y'_0)$ according to the displacement mapping function (Fig. 3). Since the necking phenomenon produces complicated deformation states, a second-order function was implemented in this study:

$$\begin{aligned} x' &= x + \bar{u} = x + u + u_x \triangle x + u_y \triangle y + u_{xy} \triangle x \triangle y + u_{xx} \triangle x^2 + u_{yy} \triangle y^2, \\ y' &= y + \bar{v} = y + v + v_x \triangle x + v_y \triangle y + v_{xy} \triangle x \triangle y + v_{xx} \triangle x^2 + v_{yy} \triangle y^2, \end{aligned} \quad (2)$$

where u and v are respectively the $x-$ and $y-$directional displacement components of the reference subset center, u_x, u_y, v_x, v_y are the first-order displacement gradients and u_{xx}, u_{xy}, u_{yy}, v_{xx}, v_{xy}, v_{yy} are the second-order displacement gradients. This mapping function is much more computationally expensive (12 parameters) than a simpler first-order function (6 parameters) which can be obtained by eliminating the last three terms of each expression (Schreier and Sutton, 2002). However, since the image processing is performed off-line (no real-time processing), requirements related to the computational performance may be considered to be of secondary concern. The quadratic terms of the Eq. 2 allow to map the deformation states found in the necking region, and then to obtain the center of a deformed subset.

The DIC algorithm minimizes the ZNSSD correlation coefficient C_{ZNSSD}, solving a set of twelve non-linear functions, where the minimizing arguments, that are the solution to the minimum problem, correspond to the unknown parameters

$$\mathbf{p} = \left[\begin{array}{cccccccccccc} u & v & u_x & v_x & u_y & v_y & u_{xx} & v_{xx} & u_{xy} & v_{xy} & u_{yy} & u_{yy} \end{array} \right]. \tag{3}$$

The set of equations is solved using the iterative Newton-Raphson procedure:

$$\mathbf{p} = \mathbf{p}_0 - \frac{\nabla C_{ZNSSD}(\mathbf{p}_0)}{\nabla\nabla C_{ZNSSD}(\mathbf{p}_0)}. \tag{4}$$

where the vector $\mathbf{p}_0$ is the initial guess of $\mathbf{p}$ and ∇C_{ZNSSD} and $\nabla\nabla C_{ZNSSD}$ are, respectively, the first and second-order derivatives of the correlation coefficient with respect to the unknown parameters $\mathbf{p}$.

This algorithm is highly recommended for practical use due to its higher accuracy and stability and its broader application (Bing *et al.*, 2006). Further details on this DIC algorithm and mapping functions can be found in Pan *et al.* (2009) and Pan *et al.* (2010).

In real problems the marker can be offset a non-integer position (x', y'). The algorithm seek these offsets even with only integer information in the images. To this aim, an interpolation scheme has to be used to reconstruct the gray intensity value $g(x', y')$ at the deformed position. It is known that the spline interpolators can be used successfully to carry out this task. A bicubic spline interpolation is highly recommended by Schreier *et al.* (2000) due to accuracy and convergence of the algorithm, then it is used in this study.

Two modifications proposed in Pan *et al.* (2010) to deal with deficiencies of this method was implemented. The first modification is the implementation of a scanning strategy guided by the correlation coefficient of computed points. Since the algorithm minimizes the ZNSSD correlation coefficient, the initial guess $\mathbf{p}_0$ of the minimizing arguments is quite important. The algorithm computes the solution to the minimum problem $\mathbf{p}$ for a first marker, called seed point. The chosen seed point has to be accurately searched in the deformed image. Subsequently, its solution is used to analyzes the correlation coefficient of its closest neighbors. Once its neighbors have been analyzed, they are inserted into a queue according to the magnitude of its correlation coefficient. Then, the solution $\mathbf{p}$ of the first point of the queue (best match according the correlation coefficient) is computed using as initial guess the deformation parameters used to analyzed it. Once it is computed, it is removed of the queue and its closest neighbors are analyzed and inserted in the queue. This procedure is performed until the queue is not empty, i.e., until all points are computed. This implementation is quite useful when the specimen show high heterogeneous displacement fields in order to ensure the convergence of consecutive points.

The second modification is the use of binary masks to modify the valid subset area in order to allow the computation of the markers displacement located near or at the boundaries of specimen (Fig. 4). It is desiderated when the subsets may contain unwanted or foreign pixels from background image or other regions.

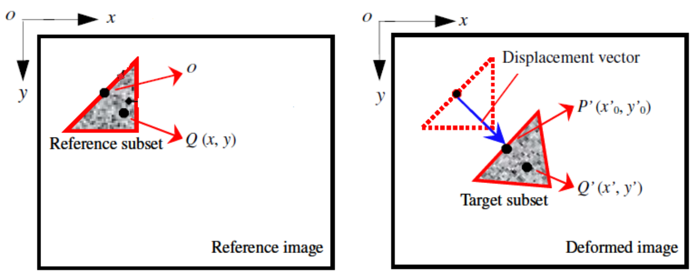

Figure 4. Example of a subset centered at a boundary point (Pan *et al.*, 2010)

In the Fig. 5 Pan *et al.* (2010) compares the results of displacement and ZNCC correlation coefficient of the points located at the boundaries of the region of interest (ROI). It is clearly seen in Figs. 5(a), 5(c) and 5(e) the validity and practicality of the proposed approach, in contrast to the computational results obtained by using the existing standard DIC technique in Figs. 5(b), 5(d) and 5(f).

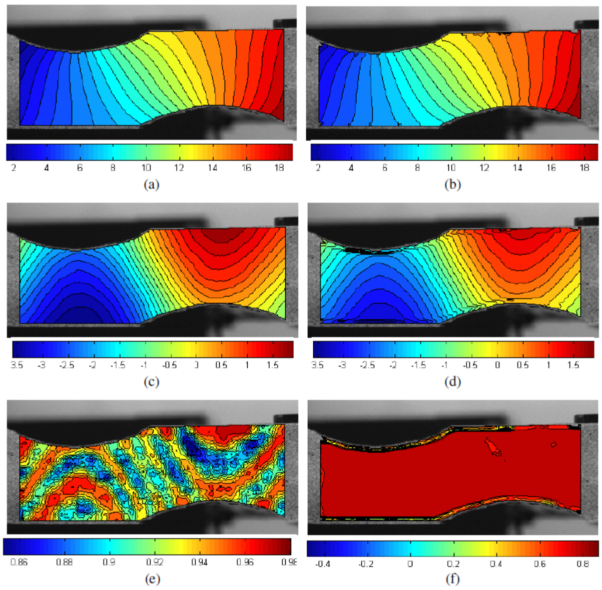

Figure 5. Comparison of the results obtained with the mask technique and existing technique:(a)(b) u-displacement field, (c)(d) v-displacement field, and (e)(f) ZNCC coefficient distributions (Pan *et al.*, 2010)

This optical method can be used to obtain the three-dimensional full-field displacement using two cameras. The same points are seek in the digital images obtained from two cameras appropriately positioned and calibrated. A triangularization routine is used to obtain the displacement in the $x-$, $y-$ and $z-$directions. Figure 6 shows the triangularization of two different instant in time.

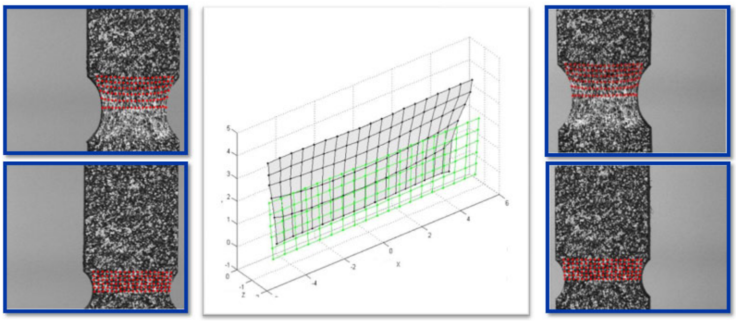

Figure 6. Triangularization procedure of the points of an undeformed and deformed specimen

To properly apply the method, a random speckle pattern is painted with an airbrush on the surface of the specimen, alternating black and white paint. The characteristics and randomness of this pattern has important influence on the accuracy of the correlation. According to Chu *et al.* (1985), it should supply the information needed to correlate the subsets through the test.

2.3 Experimental tests

Two types of tensile tests were performed, both in an universal testing machine equipped with a load cell and a clip-gauge that provide, respectively, the reaction force and longitudinal displacement along time. The clip-gauge was positioned in the necking region with a distance between the clips of $25mm$. A monotonic and a loading/unloading test (cyclic) was conducted with a constant displacement rate of the clamps set to $2mm/min$. The cyclic test was included in this work only to identify the elastic limit of the material. The displacement x time, load x time and load x displacement curves obtained by the load cell and the clip-gauge during both tests are shown in Fig. 7.

The image acquisition system used to obtain the digital images is shown in Fig. 8. The DIC method was used to obtain the curves displacement x time of points shown in Fig. 9.

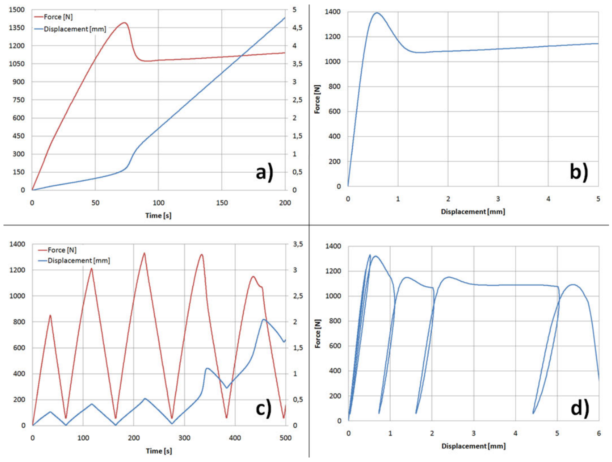

Figure 7. Uniaxial testing machine curves: (a)(b) Monotonic test; (c)(d) Cyclic test

Figure 8. Cameras arrangement during the test

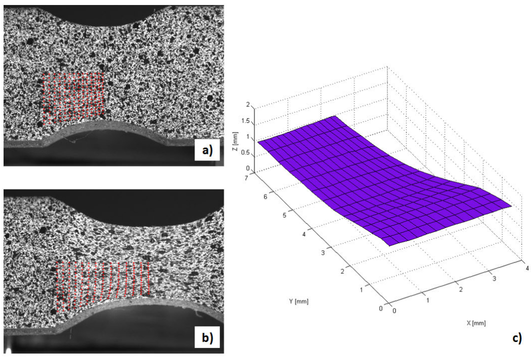

Figure 9. (a) Points on the undeformed specimen; (b) Points on the deformed specimen; (c) Three-dimensional position of the points on the deformed specimen

3. NUMERICAL PROCEDURE

3.1 Geometry

A three-dimensional numerical model of the specimen was generated for each performed test in order to compare exactly the numerical and experimental results. The geometric models were generated from pictures captured by the cameras and measurements made in the specimen before the testing. Only the region between the clips of the clip-gauge was used in the modeling. A symmetry condition in XZ plane was created in both cases. Additionally, vertices were created on the specimen surface with the exactly same coordinates of the markers obtained from DIC method in order to ensure the existence of a node in the same position. Figure 10 shows the geometric models.

3.2 Finite Element Model

The discretization in finite elements was made using the package ANSYS Workbench 12.1 (ANSYS, Inc., 2010). The mesh generated for the monotonic and cyclic tests are shown in Fig. 11. For each test, the longitudinal displacement experimentally obtained from the clip-gauge is applied at one extremes of the discretized specimen, while the other extreme is fixed. Besides, symmetry condition is applied on the XZ plane.

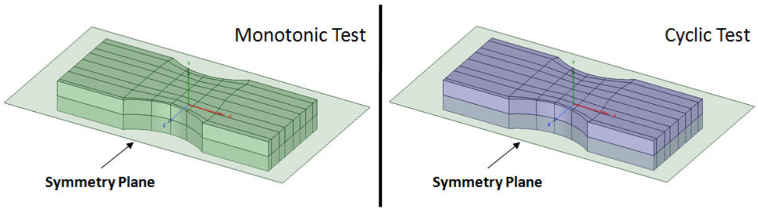

Figure 10. CAD model of the test specimens used for the monotonic and cyclic tests

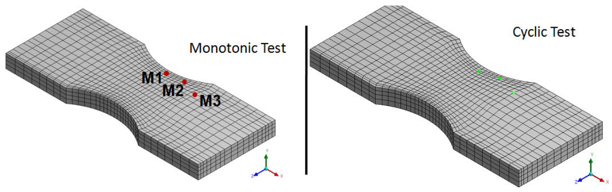

Figure 11. Finite element models for the monotonic and for the cyclic tests

In order to simulate the mechanical behavior of the material, a multi-linear elasto-plastic model presented in Fig. 12 was chosen. This constitutive model was used due to its simplicity and capability to reproduce nonlinear elasto-plastic behaviors, despite of its limitation to reproduce only stress-strain curves with positive tangent slopes. The material model has 2 parameters for the elastic part: elastic modulus E, Poisson's ratio ν; and 6 parameters for the multi-linear plastic part: the yield stress (σ_y), a stress increment (Δ), three tangent modulus (ϕ, H and I) and the begging of the oriented hardening (ε_p). The results of the numerical analysis provide the reaction force x time and displacement x time curves ($x-$, $y-$ and $z-$ directions) for each marker of interest.

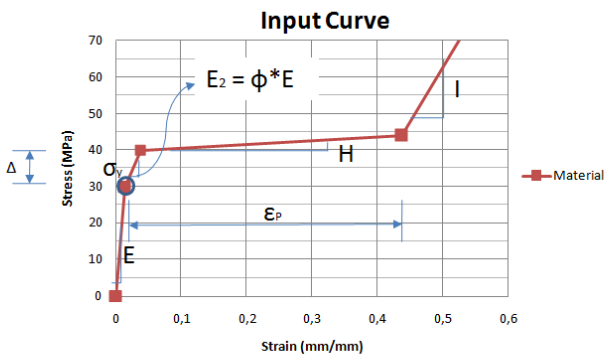

Figure 12. Constitutive model

Since the true stress-strain curve of the PVC may shows nonlinear behavior and negative tangent slopes with a significant softening region (Fig. 1), this material model may not be able to reproduce the mechanical behavior of the specimen. On other hand, if the true stress-strain curve of the PVC actually is in accordance to the assumption of Frank and Brockman (2001), it is expected that the model will be able to achieve reasonable good solutions.

4. OPTIMIZATION PROCESS

The optimization procedure is used to identify a set of material parameters $\mathbf{p}_m$ that minimizes the cost function Ω. The minimizing arguments of the function Ω represent the best approximation between the numerical simulation and the experimental mechanical behavior. This cost function is defined based on the weighted least squares of the difference between experimental and numerical data, including monotonic and cyclic results in both:

$$\Omega = \begin{array}{l} \sum_{i=1}^{N} [w^{F,m} (F^{num,m}(\mathbf{p}_m)_{(i)} - F^{exp,m}_{(i)})^2 + w^{F,c}(F^{num,c}(\mathbf{p}_m)_{(i)} - F^{exp,c}_{(i)})^2] + \\ \qquad + \sum_{i=1}^{N} \sum_{j=1}^{M} [w^{UX^{(j)}} (UX^{num}(\mathbf{p}_m)^{(j)}_{(i)} - UX^{exp}{}^{(j)}_{(i)})^2] \end{array} \qquad (5)$$

where N is number of data points used in the optimization, M is the number of markers analyzed, $w^{F,m}$ is the weight of the force for the monotonic case, $w^{F,c}$ is the weight of the force for the cyclic case, $w^{UX^{(j)}}$ is the weight of the displacement of the $j-$th marker, $UX^{num}{}^{(j)}_{(i)}$ is the $i-$th component of the numerical displacement of the $j-$th marker,

$UX^{exp(j)}_{(i)}$ is the $i-$th component of the experimental displacement of the $j-$th marker, $F^{num,m}{}_{(i)}$, $F^{exp,m}{}_{(i)}$, $F^{num,c}{}_{(i)}$ and $F^{exp,c}{}_{(i)}$ are, respectively, the $i-$th component of the numerical force for the monotonic case, the $i-$th component of the experimental force for the monotonic case, the $i-$th component of the numerical force for the cyclic case and the $i-$th component of the experimental force for the cyclic case and $\mathbf{p}_m$ represent the vector with the material parameters:

$$\mathbf{p}_m = \begin{bmatrix} \mathrm{E} & \nu & \sigma_y & \Delta & \phi & H & I & \varepsilon_p \end{bmatrix}. \quad (6)$$

The schematic representation of the parameter identification is shown in Fig. 13. The numerical analysis computes the reaction force and displacements of some points of interest over the surface of the specimen. This curves and its correspondent experimental curves are introduced in the cost function Ω to be minimized, where this iterative optimization process is performed in the commercial software modeFRONTIER (modeFRONTIER, 2010). In this study only the $x-$displacement of three markers ($M1$, $M2$ and $M3$) of the monotonic test were used for the optimization procedure (see Fig. 11).

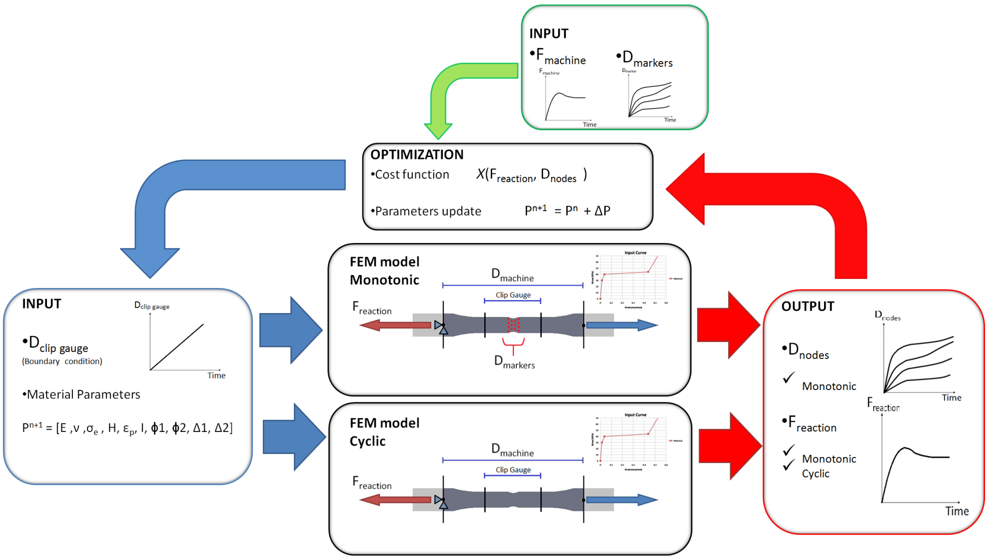

Figure 13. Optimization routine

Since the minimization involves 8 unknown parameters, a genetic optimization was used to obtain the global minimum candidates, and then, these candidates were used as initial guess of a SIMPLEX minimization. This strategy was used to avoid the convergence to local minima points.

5. RESULTS

Different tests were carried out varying the weight of the force for the monotonic case $w^{F,m}$, for the cyclic case $w^{F,c}$ and of the displacements w^{UX} in the expression 5 in order to obtain an adequate fitting of the curves. Should be noted that $w^{F,m}$, $w^{F,c}$ and w^{UX} are used due to the different magnitude of forces (F,m and F,c) and the displacement (UX). When weights are set as $w^{F,m}$ and $w^{F,c} \gg w^{UX}$ good results were achieved for the force but the displacement over the surface shown considerable errors. Conversely, when $w^{F,m}$ and $w^{F,c} \ll w^{UX}$ good results were achieved only for the displacement over the surface of the specimen.

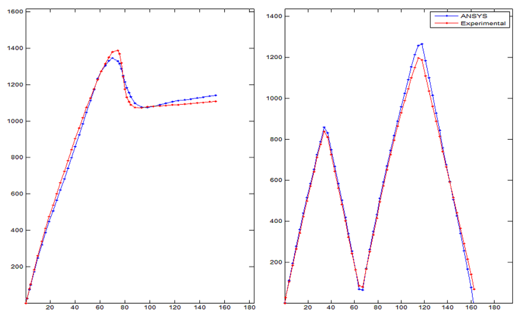

Figure 14. Numerical and experimental curves for the monotonic and cyclic cases

Since this optimization procedure has a multi-objective characteristic, this sensibility analysis defined a region for the minimum of Ω, where the choice of the result is subjective. The material parameters obtained in the parameters identification procedure are $E = 2680$ MPa, $\nu = 0.43$, $\sigma_y = 46.30$ MPa, $\phi = 0.198$, $\Delta = 6.05$ MPa, $H = 0$ MPa, $\varepsilon_p = 0.397$ and $I = 141.29$ MPa. The numerical and experimental curves are shown in Fig. 15

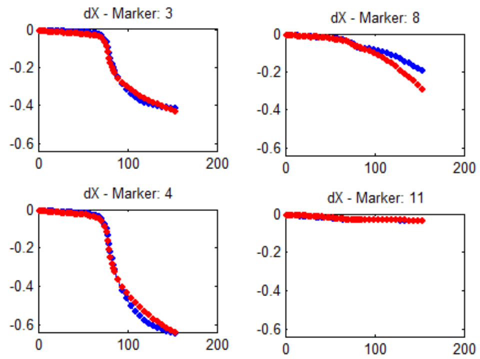

Figure 15. Numerical and experimental curves of four markers

6. CONCLUSION

In this work was investigated the parametric identification of a simple multi-linear constitutive model using experimental data acquired from two mechanical tests and from an optical method.

Despite the highly heterogeneous strain fields involved in the experiment tests, the implemented DIC method was capable to obtain useful information regarding the mechanical behavior of the material when triaxial stress states arise, principally during the traveling neck where the control points were analyzed.

A multilinear elasto-plastic model was chosen to represent the mechanical behavior of the polymer PVC. As expected, the experimental and numerical results show relatives differences (also observed by Meuwissen *et al.* (1998)). It can be attributed to the linear characteristic of the model chosen. It cannot describe nonlinear mechanical behavior or softening phenomenon, where this last is contested by Frank and Brockman (2001) but present in other works (Duan *et al.*, 2001). In order to enhance results and better estimate the mechanical response, nonlinear elastic-plastic or visco-elastoplastic models should be investigated.

A cost function with multi-objective characteristics has used. From the results of the sensitivity analysis of this problem, only the experimental curve force x displacement is not enough to find the material parameters of this model. Different combinations of the parameters can result in a exact match of the numerical and experimental force, but quite different regarding the necking displacements. In addition, since the monotonic experimental data has a smooth transition between the elastic and plastic region, only the data provided by the monotonic test is not enough to find the correct elastic limit. It has to be determined from experimental data provided by a cyclic test.

Despite of the proposed simplifications, like the use of a simple material model and small number of tracking points, it can be concluded that the use of DIC method provided significative information to the characterization of material. In this context, this preliminary study was quite useful and opens a promising field of further improvements with the use of nonlinear models to represent the real mechanical behavior of nonlinear materials.

7. ACKNOWLEDGEMENTS

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9. Responsibility notice

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