CONTRIBUTION TO STUDIES OF THE DIAPHRAGM EFFECT IN DOMES

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***Abstract**: Framed three-dimensional coverings are normally calculated solely considering the structural contribution of members. However, when interconnecting closing plates exist, such elements can effectively contribute with their rigidity to the behavior of the overall structure. Thus, forces that are usually considered as only acting on the structural members of the frame are also shared with the plate elements. It is important to evaluate the behavior of the real structural system, in striving for optimized and cost-effective structures. Therefore, the aim of this study is to evaluate the structural behavior of domes formed by a combination of frame members and plates. The diaphragm effect is considered for domes with triangular-shaped frames. The numerical analysis is performed via finite element analyses using Ansys® software. The results obtained for a dome with a diameter of 20 m and a height of 5 m, using various mesh configurations, are presented and discussed. In particular, it is shown that when the diaphragm effect is taken into account, a significant reduction in values regarding node displacement, forces and moments acting on the frame members can be achieved, and mesh arrangements are recommended when the diaphragm effect is taken into account or ignored.*

***Keywords**: dome, diaphragm, shell, frame*

1. INTRODUCTION

Framed domes allow for light building coverings, which cover large spans, with a favorable geometrical form that enables the structure to withstand forces with small displacements, typically with predominant axial forces (Timoshenko, 1953). Framed domes show evident technical benefits and are very cost-effective structures, given that they allow for significant reduction in the use of materials, with pre-fabricated members that permit their industrialization. Besides these advantages, their structural behavior promotes small nodal displacements (Avram and Anastasescu, 1984; Karlsen *et al.*, 1976). They are widely used in many countries, although this type of structure is unusual in Brazil, and its application still requires disclosure efforts through improvements in design methodologies. There is particular interest in the use of timber in these structures, for economic, environmental and social (Natterer, 1994) reasons.

In the conventional calculation of framed three-dimensional coverings, there is only consideration of the structural contribution of linear elements. However, when interconnecting structural closing plates exist, such elements can effectively contribute, through their rigidity, to the behavior of the overall structure, reducing the loads acting on linear elements, as the forces that are usually considered to be acting solely on the structural members of the frame are also shared by the plate elements. In this case, it is important to evaluate the behavior of the real structural system, striving to achieve optimized and cost-effective structures.

The additional rigidity with which the plate elements contribute to the framed dome is called the “diaphragm effect”, which provides the dome with an overall structural behavior that is very similar to that of a spherical shell.

In this paper, the diaphragm effect is evaluated for several types of triangular-shaped frames, joined internally by secondary beam elements and closed by plywood panels. The goal of the study is to make a comparative evaluation of geometric parameters, forces on the beams, nodal vertical displacements and support reactions, considering the effect of contributions made by plate elements. The numerical analysis is performed via finite element analysis using Ansys® software (Ansys®, 2004), and meshing is achieved by using Gestrut software (Gesualdo, 2010).

The results obtained show that when the diaphragm effect is considered, nodal displacements are greatly reduced, as well as the forces on the beam elements. Ranges of values were defined regarding improved relationships between the radius of the dome and its clear span, taking the diaphragm effect into account or ignoring it, with a view to determining the best mesh arrangement for each situation.

2. DOME MODELING

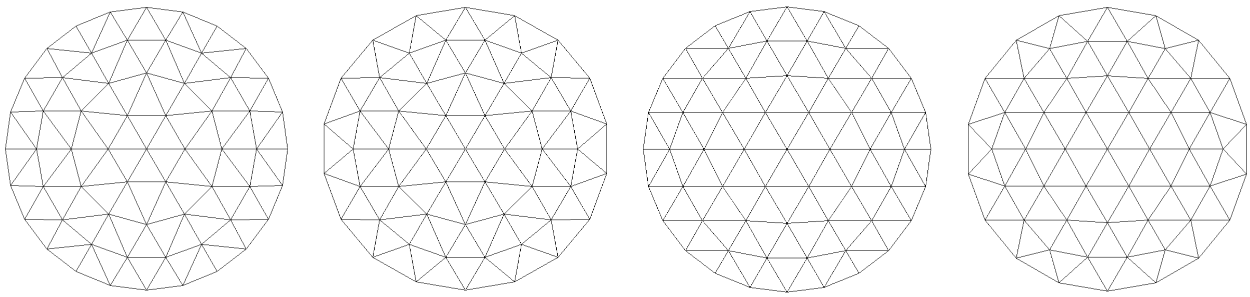
Four mesh arrangements are considered with triangular modulation for the same dome (clear span of 20 m and height of 5 m), with six sectors of the spherical shell divided into four, five or six segments, as illustrated in Fig. 1 (for four divisions of the sectors).

The choice of the number of sectors is based on current dome construction practice, obtained from Arredondo and Holzapfel, 1990; Eberwein, 1989; Makowski, 1984; Matsushita, 1984; Schurwan, 1989; Von Büren, 1985; and WWSI, 2011. For the definition of the number of divisions in the meshes, an attempt is made to have small variation in member length from one configuration to another. All members are considered to have the same cross section.

Variations in elevation above the base of the dome were also introduced for the same mesh arrangement, in order to obtain the best relationships between the radius of the shell and the radius of the covered area.

Figure 1 illustrates four types of arrangements for triangular meshes:

(a) mesh **1A** – constitutes concentric circles, where the bordering arches of the sectors have their ends supported on opposed columns;
(b) mesh **1B** – similar to 1A, but the arches of the sectors have bifurcated ends, supported on two columns;
(c) mesh **2A** – constitutes practically parallel arches, where the bordering arches of the sectors have their ends supported on opposing columns; and
(d) mesh **2B** – similar to 2A, but the arches of the sectors have bifurcated ends, supported on two columns.



(a) mesh **1A** (b) mesh **1B** (c) mesh **2A** (d) mesh **2B**

Figure 1. Mesh arrangements considered, with six sectors and four divisions in each one.
Source: Gestrut (Gesualdo, 2010).

3. ANALYTICAL METHODOLOGY

Gestrut software (Gesualdo, 2010), based on the displacement method (Weaver and Gere, 1980; Przemieniecki, 1968; Rubinstein, 1966) and intended for calculation of framed three-dimensional structures, was implemented in the case of vaults with a reticulated mesh, in several geometric configurations. This computer program allows for analysis of any three-dimensional structural form; however, the domes received special treatment regarding their typically complicated generation of geometric and loading data.

Considering that preparation of data for computational analysis is an important stage in the calculation process, the Gestrut program has one specific pre-processor for input and generation of data. This makes it possible for modeling the meshes in an easy and rapid manner, with visualization of the geometrical characteristics of the structure generated and schematic representation of the mesh, as well the input data.

It should be emphasized that generation of data for the structural system under analysis here is very complex, given that the structure is three-dimensional and each beam has its own different angle of orientation. Using Gestrut (Gesualdo, 2010), the main meshes were generated and their triangular units were divided into smaller portions with secondary horizontal beams, parallel to the main horizontal beams, as exemplified in Fig. 2, for mesh 1A, with four divisions of the individual sectors.

The main and secondary horizontal beams were divided into two or three equal segments. Hence, the triangular plate elements were generated with almost equal dimensions, as illustrated in Fig. 3.

For calculation purposes, besides the geometric characteristics shown, the following parameters were adopted input data for processing by the Gestrut software:

- clear span of 20 m and height of 5 m;
- cross section of beams: 6 cm x 12 cm;
- modulus of elasticity (parallel to grain) (E) = 480 kN/cm^2;
- shear modulus (transverse deformation) (G) = 24 kN/cm^2;
- simple supports (translations are restricted and rotations are free) at the base of the dome;
- continuous ends of beams;
- plywood of 18mm-thickness, E_x = 620 kN/cm^2, E_y = 536 kN/cm^2 e G_{xy} = 94 kN/cm^2, Stamato (2002);
- loads: dead load of structural elements;
- structural elements in Ansys® software: beam element – BEAM 4 and plate element – SHELL 63.

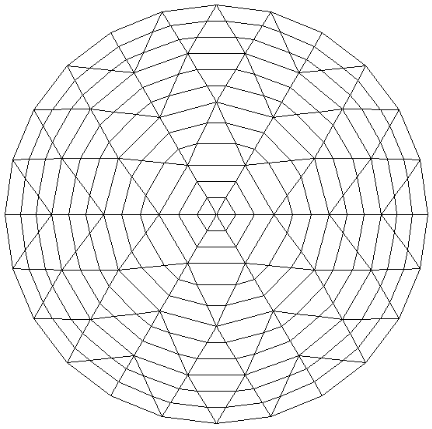


Figure 2. Mesh 1A, with four divisions, with secondary members inserted into the main triangular mesh.
Source: Gestrut (Gesualdo, 2010).

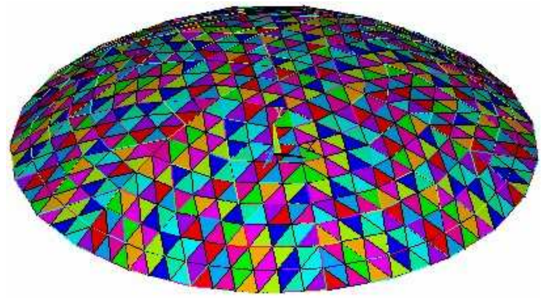


Figure 3. Mesh 1A, with four divisions and triangular plates inserted via the Ansys® program.
Source: Ansys® (Ansys®, 2004).

The modulus of elasticity corresponds to *Pinus*, a reforestation species, Class C30 according to NBR 7190:1997 (ABNT, 1997). Beam cross sections were defined in terms of their slenderness coefficient, in order to work under their maximum capacity.

The structure formed by plate and beam elements was analyzed via the finite element method using Ansys® software, with the code generated by the Gestrut software.

4. RESULTS AND DISCUSSION

Table 1 shows geometric data regarding the meshes evaluated in this study, considering the beam elements of the main triangular modulations, for comparison in terms of number of nodes, members, different lengths of the members, and also minimum and maximum lengths of the members and total volume of wood required.

Based on the data shown in Tab. 1, Tab. 2 presents a summary indicating those meshes that have minimum values for the analyzed geometric variables. It can be seen that mesh B, with fewer nodes, reduces the amount of connectors for joining the ends of the beams, which results in savings in terms of material and labor. Mesh 1B provides for greater reduction of labor, due to less variability in the lengths of the beams, thus facilitating industrialization and the assembly and fastening of parts. Mesh 2B shows a minimum amount of material. Moreover, mesh 2A does not have the advantages of the two previously cited meshes. Of the 4 types of mesh evaluated, it makes a reduction in the amount of material possible due to the shorter lengths of the beams. This means that, for covering the same area, parts can be obtained with smaller cross sections. However, for each design, it should be ascertained whether this reduction is greater than that provided by mesh 2B.

In summary, without analyzing displacement and force values, the meshes that will be able to result in more cost-effective designs are those identified as 2B and 2A.

An assessment was made of the maximum values regarding nodal displacements and forces in the members, with a view to comparative analysis of the diaphragm effect in the overall behavior of the arrangements considered. As an example, vertical nodal displacement values are shown in Tab. 2, where it is possible to observe the contribution made by plate rigidity. The term "p" refers to the structure considering plate rigidity, whereas "np" refers to the structure without consideration of plate rigidity. In the latter case, the Young's modulus of the plate material was considered to be very small in the FE model and only the dead-weight load is accounted for.

Table 1. Geometric data of analyzed meshes

Mesh type		Nº of nodes	Nº of beams	Nº of different bars	Total L(*) [m]	Least L(*) [m]	Greatest L(*) [m]	Volume [m³]
6-4 (6 sectors and 4 divisions)	**1A**	61	156	11	460	2.61	3.58	3.31
	1B	55	144	9	441	2.71	3.54	3.18
	2A	61	156	15	455	2.31	3.30	3.28
	2B	55	144	14	439	2.70	3.47	3.16
6-5 (6 sectors and 5 divisions)	**1A**	91	240	16	569	2.09	2.90	4.10
	1B	85	228	13	553	2.16	2.89	3.98
	2A	91	240	22	563	1.81	2.66	4.05
	2B	85	228	21	549	2.09	2.63	3.95
6-6 (6 sectors and 6 divisions)	**1A**	127	342	22	678	1.74	2.44	4.88
	1B	121	330	18	664	1.79	2.44	4.78
	2A	127	342	31	670	1.48	2.24	4.82
	2B	121	330	30	658	1.70	2.18	4.74

(*) L = beam length

Table 2. Comparative summary of geometric data contained in Tab. 1

Mesh type	Least values of geometric characteristics of mesh			
	Nº of joints and bars	L(*) variation	Individual L(*)	Total L(*) and total volume
1A				
1B				
2A				
2B				

(*) L = beam length

Specifically with regard to vertical nodal displacements, some results and comments follow:

- As expected, consideration of plate rigidity provided significant in reducing nodal displacement values. As the number of divisions of the mesh increases, displacement decreases. This is confirmed by the variations shown in columns 5 and 9 of Tab. 3.
- Comparing displacement values for the same mesh division for all modulation types, for example, 6-41A, 6-41B, 6-42A and 6-42B, it can be seen that configuration 2A-p (taking plate rigidity into account) shows the smallest values.
- Percentage reduction of the displacement values for the same mesh, considering plate rigidity (situation “np” compared to “p”) is virtually the same as that obtained for meshes A and B, the values of which are given in columns 5 and 9 of Tab. 3.
- Considering the contribution of plate rigidity, the nodal displacement values obtained for mesh 2 are smaller than those for mesh 1 (see columns 2 and 6 of Tab. 3). The reverse occurs when rigidity is not considered.
- In all the situations where plate rigidity is considered, meshes of the “A” type showed smaller joint displacement values than their “B” counterparts, as shown by the values in the last column of Tab. 3.
- The mesh with the smallest nodal displacement is mesh 2A, which is the most finely divided (pattern 6-6) and when plate rigidity is considered. By ignoring plate rigidity in analysis of the structure, mesh 6-61A is the one that results in the smallest vertical nodal displacement values.

Table 3. Comparison of maximum vertical nodal displacement values [cm].

	"A" MESHES				"B" MESHES				
6-4 (6 sectors and 4 divisions)	↓ 22.7%	1A np$^{(*)}$	- 3.73	↓ 60.3%	↓ 22.1%	1B np	- 3.86	↓ 60.7%	
		1A p$^{(**)}$	- 1.48			1B p	- 1.52		↑ 2.4%
		2A np	- 3.58	↓ 68.0%		2B np	- 3.63	↓ 67.4%	
		2A p	- 1.15			2B p	- 1.18		↑ 3.2%
6-5 (6 sectors and 5 divisions)	↓ 20.3%	1A np	- 2.13	↓ 63.4%	↓ 19.9%	1B np	- 2.19	↓ 63.6%	
		1A p	- 0.78			1B p	- 0.80		↑ 2.1%
		2A np	- 2.31	↓ 73.1%		2B np	- 2.36	↓ 73.0%	
		2A p	- 0.62			2B p	- 0.64		↑ 2.6%
6-6 (6 sectors and 6 divisions)	↓ 13.6%	1A np	- 1.58	↓ 67.3%	↓ 13.3%	1B np	- 1.61	↓ 67.3%	
		1A p	- 0.52			1B p	- 0.53		↑ 1.7%
		2A np	- 1.82	↓ 75.5%		2B np	- 1.85	↓ 75.4%	
		2A p	- 0.45			2B p	- 0.46		↑ 2.0%

$^{(*)}$ np = plate rigidity ignored $^{(**)}$ p = plate rigidity considered

Although results regarding the values of maximum axial and shear forces and bending moments are not shown herein, it is possible to make the following comments concerning these values:

- Consideration of plate rigidity provided for considerable reduction in the values of axial forces in members and it can be seen that, by increasing the number of divisions of the mesh, such reduction continually diminishes.
- A difference perceived between meshes 1A and 2A is the increase in axial forces when plate rigidity is ignored, and the opposite effect (i.e., decrease of axial forces in the members) when plate rigidity is considered. Thus, considering the diaphragm effect, mesh 2A is more adequate than mesh 1A. On the other hand, comparing meshes 1B and 2B, the inverse occurs, i.e., considering plate rigidity, mesh 1B is found to be more adequate than 2B with respect to axial forces in the members.
- The values of axial forces for "B" meshes are greater than those obtained for similar "A" meshes, when consideration is included of plate rigidity.
- Considering the diaphragm effect, mesh 2A is the most adequate in terms of axial forces and, without such an effect, mesh 1A is the most suitable.
- It has been noticed that, if the number of divisions of the mesh is increased, the values of axial forces are reduced with consideration of plate rigidity. On the other hand, if plate rigidity is not taken into account, an increase in the number of divisions of the mesh leads to a respective increase in the values of axial forces.
- Therefore, as far as axial forces are concerned, the mesh that results in the minimum values is 6-42A (taking plate rigidity into account). If the diaphragm effect is ignored, mesh 6-61A provides the minimum values.
- With respect to bending moments and shear forces, the conclusion regarding the most adequate mesh is the same as that shown above for axial forces.

The influence of the geometric parameter "height/span ratio" (from 10% to 30%, with increments of 5%) was also studied in the case of mesh 6-61A. The following main conclusions can be drawn with respect to this aspect:

- Increased height of the dome results in a decrease in nodal displacement values in all cases (with or without consideration of plate rigidity).
- As shown in Fig. 4, nodal displacement values decrease if plate rigidity is considered. By increasing the central height of the dome, this difference tends to decrease, but it generally stabilizes, indicating that height/span ratios of more than 20% do not result in any considerable reduction in nodal displacement values. The degree of reduction is less significant when the diaphragm effect is considered, making it possible, in this case, to obtain small displacement values for slightly curved domes. This feature is an important element in the design of optimized structures, with savings in terms of structural materials.
- It was found that the conclusions obtained regarding displacement values are also valid for forces, as illustrated in Fig. 4.

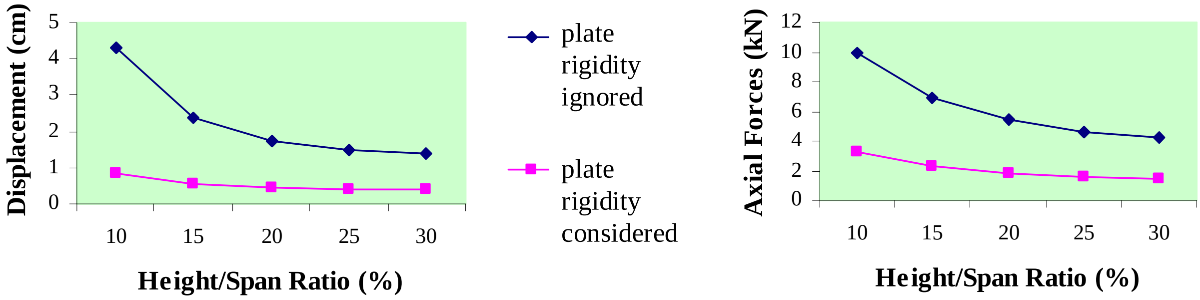


Figure 4. Trends regarding maximum nodal displacements and axial forces with increased height/span ratio.

Table 4 summarizes the characteristics of the various meshes with respect to both geometry (indicated previously in Tab. 3) and the results of structural analyses (displacements, forces and moments), the configurations identified being those that show the minimum values of parameters of concern.

Table 4. Summary of characteristics of the various meshes

Mesh Type	Geometric characteristics				Displacements / Forces / Moments	
	Nº of nodes and members	Variation of L(*)	Individual L(*)	Total of L(*) and volume	Vertical displacement	Forces and bending moment
1A					6-61A np(**)	6-61A np
1B						
2A					6-62A p(***)	6-42A p
2B						

(*) L = member length (**) np = plate rigidity ignored (***) p = plate rigidity considered

It was found that, if the diaphragm effect is considered, meshes 2A and 2B are the most cost-effective. As far as mesh 2A is concerned, the reduced lengths of the members, combined with smaller forces, will make it possible to use members with smaller cross sections, in analyses of both stability (which takes the slenderness of members into account) and strength. It can be seen that, with fewer divisions of the mesh, the action of forces and moments on members decreases, the choice of a larger or smaller degree of division of the mesh being defined by the strength and cost of the plate elements, rather than the volume of the beam elements. In the case of mesh 2B, values regarding forces and displacements are slightly greater than those observed in mesh 2A. As a result, it is necessary to evaluate the cost of connectors and wood, in order to determine which mesh represents the greatest overall savings. It is believed that mesh 2B, with a smaller number of necessary connecting elements and volume of wood, shows greater savings than those achieved by reducing the cross-sectional dimensions of the beams of mesh 2A, in view of the smaller values of forces and the smaller buckling lengths of the beams.

On the other hand, ignoring the structural contribution of plate rigidity values, it can be seen that meshes with the concentric circle configuration of type 1A, with greater divisions of the mesh, confer small values regarding displacements and forces, making it possible to use elements with smaller cross sections, resulting in reduction in the cost of wood. However, in the case of the more divided mesh, there are a larger number of connectors. It becomes necessary to make a comparison between the cost of these connectors and the cost of the wooden parts, which can be sawn lumber or glued laminated (glulam) timber (the cost of which is higher than that of sawn lumber). In summary, if the diaphragm effect is ignored, there is no clear indication of the most cost-effective mesh. Given that values regarding forces in mesh 1B are slightly higher than those observed in mesh 1A, it is believed that its smaller number of joints results in a lower final cost. Moreover, variation in the length of the bars in mesh 1B is also smaller than that observed in mesh 1A.

In summary, in order to choose the best mesh arrangement, based on comparisons between displacements and forces, it is necessary to evaluate the costs of materials used in parts and fastening elements, with the following results:

- taking the diaphragm effect into account: meshes 2A and 2B;
- ignoring the diaphragm effect: meshes 1A and 1B.

Continuing investigation of the structural behavior of the meshes studied, analysis of vertical and horizontal reactions at supports was also performed (in a radial direction). Fig. 5 shows the values of vertical reactions pertaining to mesh sector 6-6 for meshes of the “A” and “B” type. It can be added that the same behavior was observed with respect to radial reaction forces.

On analyzing the curves shown on Fig. 5, it can be immediately seen that the largest support reaction values are obtained for meshes of the “B” type, due to the fact that this type has a smaller number of supports (with one support less per sector, when compared to meshes of the “A” type). In addition, it can also be observed that, taking the diaphragm effect into account, the support reaction values are more evenly distributed than those obtained by ignoring this effect.

It can be seen that type-2 meshes result in improved load distribution, with more uniformity and smaller values at intermediate joints, when compared with those corresponding to type-1 meshes. It was found that type-1 meshes tend to relieve the bordering supports of the sectors and overload intermediate joints.

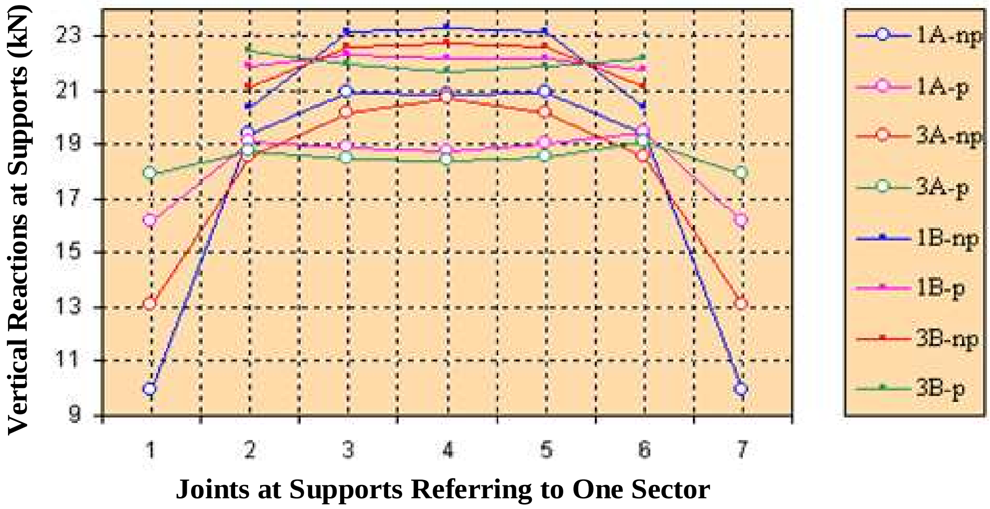


Figure 5. Comparative analysis of vertical reactions.

Therefore, with a view determining whether configuration 2A or 2B represents the most adequate mesh, considering or ignoring plate rigidity, it becomes necessary to carry out more detailed analyses, considering different numbers of supports and evaluating the following aspects:

- larger number of support elements (columns, connectors, foundations) with lower consumption of structural material in the supports; or
- smaller number of support elements, with greater consumption of structural material.

As mentioned above, if the chosen roofing system uses plywood sheets, it becomes interesting to add the rigidity increase provided by the diaphragm effect to the cross-sectional rigidities of the members in computational analysis, especially in the case of large-span coverings, where members tend to be more robust and expensive.

5. CONCLUSIONS

The framed-dome structural system allows for structural arrangement to be defined in a large number of different ways. Several parameters have an influence on structural response, for example, mesh configuration, relationship between height and span, and the use of plates connecting the members. This study provides some guidelines for selecting geometrical parameters, with a view to achieving minimum consumption of structural materials. Even though computer programs are available for modeling and analyzing a range of mesh types, it is up to the structural engineer to perform calculations in order to select the best solution in each individual case, taking into account the aesthetic requirements of architectonic designs.

6. ACKNOWLEDGEMENTS

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