EXPERIMENTAL VALIDATION OF MODAL FILTERS BASED ON ARRAYS OF PIEZOELECTRIC SENSORS

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Abstract. *Modal sensors and actuators working in closed loop enable to observe and control independently specific vibration modes, reducing the apparent dynamical complexity of the system and the necessary energy to control them. Modal sensors may be obtained by a properly designed weighted sum of the output signals of an array of sensors distributed on the host structure. Although several research groups have been interested on techniques for designing and implementing modal filters based on a given array of sensors, the effect of the array topology on the effectiveness of the modal filter has received much less attention. In particular, it is known that some parameters, such as size, shape and location of a sensor, are very important. This work presents an experimental validation of modal filters based on arrays of piezoelectric sensors applied to a free rectangular plate. The topology of the array of piezoelectric sensors bonded to a rectangular plate was optimized in a previous work to improve the effectiveness and frequency range of a set of modal filters showing that using properly located piezoceramic sensors bonded on a rectangular plate, the frequency range could be enlarged by 25% to 50%. Here, an experimental implementation of the modal filters is performed through voltage divider and summing amplifier circuits and used to validate the performance of the modal filters with satisfactory results.*

Keywords: *Modal filters, Piezoelectric materials, Weighted sum circuits*

1. INTRODUCTION

The use of piezoelectric materials (specially piezoceramics) as sensing and actuating elements has been extensively studied due to the possibility of building them as lightweight and compact devices in several geometric configurations, since they are relatively inexpensive and present the necessary electromechanical coupling (Sunar and Rao, 1999). In the last decades, a great research effort has been put on the modeling of the electromechanical coupling of structures with piezoelectric elements (actuators/sensors). In terms of applications, integrated piezoelectric sensors and actuators have been most often applied to the active control of mechanical vibration and noise in structures subjected to several types of excitation, or even self-excited structures, especially for aeronautic and aerospace applications (Ahmadian and DeGuilio, 2001).

On the other hand, the performance of integrated systems applied to active vibration and noise control can be substantially improved by the use of high quality modal filters (Chen and Shen, 1997; Tanaka and Sanada, 2007). In this context, the development of active control strategies with optimal performance using modal sensors and actuators has been the object of intensive research. Modal sensors and actuators working in closed loop enable to observe and control independently specific vibration modes, reducing the apparent dynamical complexity of the system and the necessary energy to control them (Fripp and Atalla, 2001; Preumont et al., 2003; Friswell, 2001). The high performance of modal controllers depends on several parameters. The size, form and effective electromechanical coupling coefficient of a piezoelectric material must be considered to the development of modal sensors and actuators. Although pioneer works have proposed continuous modal sensors and actuators (Lee and Moon, 1990), the evolution of modal filter techniques and its applications to active vibration control indicates several advantages in the use of an array of discrete sensors instead.

Continuous modal sensors are designed to ensure shape coupling between sensing material and elastic strain due to the target vibration modes of the host structure (Friswell, 2001; Lee and Moon, 1990). An array of sensors, on the other hand, is in general composed by small piezoceramic patches and depends on a convenient weighted sum of the sensors signals to achieve an output signal with the properties of a high-performance modal filter (Fripp and Atalla, 2001; Preumont et al., 2003). Several methodologies have been used for the evaluation of the weighting coefficients for the output signals measured by an array of sensors. They can be divided in three groups: target modes output match, optimization techniques and frequency response function (FRF) matrix inversion. Whenever the target mode shapes are known/predicted and their reading in terms of output amplitude in each sensor of the array can be identified, a technique, proposed by Meirovitch and Baruh (1982) and based on the orthogonality of normal modes, considers that the weighting coefficients should match the output of each sensor for the target mode. This technique may be strongly affected by spatial aliasing. The weighting coefficients may also be evaluated using an optimization algorithm to minimize the difference between the weighted response and a desired modal response. Shelley (1991) proposed an on-line adaptation algorithm to estimate the desired modal response and update the weighting coefficients. The third group of methods is based on the inversion of the FRF matrix, which can be either predicted by a numerical model (Chen and Shen, 1997) or experimentally measured (Shelley, 1991), in order to shape the target filtered response.

These techniques may lead to high-performance modal filters, but generally within a limited frequency range. Preumont et al. (2003) have suggested that the frequency range of high-performance filtering depends on the relation between the number of vibration modes to be filtered, in that frequency range, and the number of sensors in the array. They concluded that the number of sensors in the array should be larger than the number of vibration modes to be filtered. Although this is true for an arbitrarily distributed array of sensors, it is possible to show that the location of the sensors, that is the array topology, have a significant effect on the observability of the vibration modes and, thus, on the filtering performance of modal filters derived from it. Therefore, it should be possible to optimize the array topology and, consequently, increase the number of filtered vibration modes, thus the frequency range, for a given number of sensors available.

This work presents the experimental validation of modal filters based on arrays of piezoelectric sensors applied to a free-free rectangular plate. The topology of the array of piezoelectric sensors bonded to a rectangular plate was optimized in a previous work (Pagani Jr. and Trindade, 2009) so that the frequency range of the resulting modal filters is increased for a limited number of sensors (12 sensors). An experimental implementation of the modal filters is performed through a voltage divider circuit and used to validate the performance of the modal filters.

2. DESIGN OF MODAL FILTERS

The design of a modal filter from an array of sensors requires the output signals of each sensor to be weighted and summed such that: i) the responses of the target vibration modes are maximized; and ii) the responses of the undesired vibration modes are minimized. Therefore, it is possible to consider the FRF of an equivalent single degree of freedom system with natural frequency ω_i and damping factor ζ_i, corresponding to the target i-th vibration mode, as the desired FRF of the weighted signal of the modal filter, which can be written as

$$g_i(\omega) = \frac{2\zeta_i\omega_i^2}{\omega_i^2 - \omega^2 + 2j\zeta_i\omega_i\omega}. \tag{1}$$

Whenever the vibration modes are weakly damped and relatively well spaced, the resonance peaks are well defined and, thus, (1) represents a realistic objective for the filtered FRF signal. Let $\mathbf{Y}$ be a matrix with columns that represent the FRFs of the n selected sensors in the array and discretized in a frequency domain $[\omega_1, \ldots, \omega_m]$. Let $\mathbf{G}_i = [g_i(\omega_1), \ldots, g_i(\omega_m)]$ be the vector representing (1) in the discrete frequency domain. The vector of coefficients $\boldsymbol{\alpha}_i$ which equates the filtered output (weighted sum of sensors outputs) to the one defined by $\mathbf{G}_i$ is the solution of

$$\begin{bmatrix} Y_1(\omega_1) & \cdots & Y_n(\omega_1) \\ \vdots & \ddots & \vdots \\ Y_1(\omega_m) & \cdots & Y_n(\omega_m) \end{bmatrix} \begin{bmatrix} \alpha_{i1} \\ \vdots \\ \alpha_{in} \end{bmatrix} = \begin{bmatrix} g_i(\omega_1) \\ \vdots \\ g_i(\omega_m) \end{bmatrix}. \tag{2}$$

In general, the linear system defined by (2) admits only approximate solutions, which will be denoted $\boldsymbol{\alpha}_i^\dagger$. The vector of weighting coefficients $\boldsymbol{\alpha}_i^\dagger$ represents the best solution, in a least squares sense, for the design of a modal filter which isolates the i-th vibration mode response. If several vibration modes are to be considered simultaneously as target modes for the filter design, it is necessary to define $\mathbf{G}$ as the matrix of target FRFs with dimension $m \times p$, where p denotes the number of target modes. Consequently, the approximate solution of (2), $\boldsymbol{\alpha}^\dagger$, is a matrix of dimension $n \times p$, that is one column vector of weighting coefficients for each one of the target modes. This may be written in a compact form as

$$\mathbf{Y}\boldsymbol{\alpha}^\dagger = \mathbf{G}. \tag{3}$$

Actually, $\mathbf{Y}\boldsymbol{\alpha}^\dagger$ approximates $\mathbf{G}^\dagger$, a matrix with columns that are the orthogonal projection of the columns of $\mathbf{G}$ onto the space spanned by the columns of $\mathbf{Y}$. The traditional Moore-Penrose pseudo-inverse solution of (3) for a full column rank $\mathbf{Y}$ matrix (with columns that are linearly independent) may be obtained by pre-multiplying (3) by $\mathbf{Y}^{\mathrm{H}}$,

$$\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\boldsymbol{\alpha}^\dagger = \mathbf{Y}^{\mathrm{H}}\mathbf{G}, \text{ such that } \boldsymbol{\alpha}^\dagger = \left(\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\right)^{-1}\mathbf{Y}^{\mathrm{H}}\mathbf{G}. \tag{4}$$

On the other hand, for a full column rank matrix, the inversion of $\mathbf{Y}^{\mathrm{H}}\mathbf{Y}$ is unnecessary and computationally inefficient, since $\mathbf{Y}$ may be decomposed through QR decomposition, where $\mathbf{Q}$ is an orthonormal matrix and $\mathbf{R}$ is upper triangular, such that $\mathbf{Y} = \mathbf{QR}$ and (4) can be rewritten, after expansion and accounting for $\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{I}$, as

$$\boldsymbol{\alpha}^\dagger = \mathbf{R}^{-1}\mathbf{Q}^{\mathrm{H}}\mathbf{G}. \tag{5}$$

Notice that the inverse of $\mathbf{R}$ does not need to be evaluated, instead the upper triangular linear system, $\mathbf{R}\boldsymbol{\alpha}^{\dagger} = \mathbf{Q}^{\mathrm{H}}\mathbf{G}$, is solved through back substitution, which is computationally more efficient. For all the cases studied in the present work, the solution through QR decomposition was always convenient, since the FRF matrix has had full column rank. If at least two columns of the FRF matrix are linearly dependent, the singular value decomposition (SVD) is the suitable method to approximate the least squares solution. In practice, the truncation of matrix $\mathbf{Y}$ over a given frequency range will affect its QR decomposition and, thus, the approximate solution of the linear system (3). Recent works have shown that there is a value for truncation frequency such that all vibration modes inside a given frequency range are perfectly filtered, except the target ones, whereas resonances larger that the truncation frequency are not filtered (Preumont et al., 2003).

3. NUMERICAL RESULTS FOR THE MODAL FILTERS

In a previous work, the modal filter design technique presented in the previous section was applied to a plate with bonded piezoceramic patches acting as sensors (Pagani Jr. and Trindade, 2009). A free rectangular aluminium plate, of dimensions 320×280×3 mm, was considered as the host vibrating structure. The study aimed at finding a topology of twelve piezoceramic patches to optimize the performance of two modal filters, designed to isolate the first and second vibration modes up to 1000 Hz. Using a genetic algorithm optimization strategy, the twelve piezoceramic patches were chosen between a regular array of thirty-six identical thickness-poled piezoceramic patches, with dimensions 25×25×0.5 mm, bonded to the upper surface of the plate. More details can be found in (Pagani Jr. and Trindade, 2009).

Figure 1 presents the normalized filter output, such that the amplitude at target resonances is unitary, and the corresponding optimal topology, in which the twelve selected sensors are highlighted from the original array of thirty-six sensors. The topology was optimized so that two selected modal filters, one designed to isolate the first vibration mode and the other designed to isolate the second vibration mode, could be effective up to 1000 Hz. Figure 1 shows that effective filtering could be obtained up to 1100 Hz (with unfiltered noise bellow 1% of the resonant response). This means that four additional resonances could filtered compared to an arbitrary topology of twelve sensors. For an arbitrary FRF matrix $\mathbf{Y}$, (5) may yield complex weighting coefficients vectors $\boldsymbol{\alpha}_j$. Since, in practice, it could be much more difficult to implement complex weighting coefficients to the FRF measured by each sensor, the response of the corresponding modal filters evaluated using only the real part of vectors $\boldsymbol{\alpha}_j$ were also considered. Figure 1 shows that the modal filters outputs are not significantly altered by neglecting the imaginary part of weighting coefficients. This indicates that a simple voltage divider circuit (potentiometer) could serve as an analogic weighting of the sensors outputs.

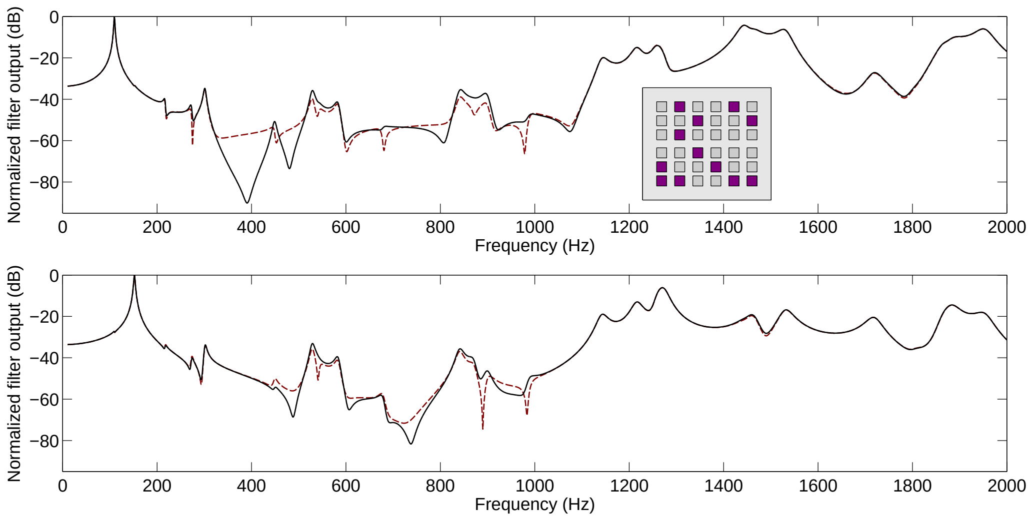

Figure 1. Normalized outputs of the modal filters designed for the isolation of the first and second vibration modes using complex (dashed) and real (solid) weighting coefficients.

4. EXPERIMENTAL RESULTS FOR THE MODAL FILTERS

This section presents preliminary results obtained from experimental tests that were designed to validate the proposed methodology for designing discrete modal filters using QR decomposition of FRF matrices, on one hand, and the optimal topology of twelve sensors obtained from numerical optimization, on the other hand. To this end, twelve transversely-poled PIC151 piezoceramic patches (PI Ceramic) were bonded to one surface of a rectangular aluminium plate using an epoxy-based glue (Araldite) cured at 60°C. The twelve piezoceramic patches are identical, with dimensions 25×25×0.5 mm. Small Copper electrode layers were bonded on the top and bottom surfaces of each piezoceramic patch to enable independent measurement of the electric voltages induced on each patch.

The experimental setup considered for the validation of the modal filters performance predicted by the numerical analysis presented in (Pagani Jr. and Trindade, 2009) is presented in Figure 2. An impact hammer (PCB model 086C03) was considered as the single force excitation applied at the upper-right corner of the rectangular plate, for the evaluation of the FRF measured by each one of the piezoelectric patches, used for the evaluation of the weighting coefficients, and also for the evaluation of the filtered FRF measured by the modal filter. A spectral analyser (LMS SCADAS Mobile running on LMS Test.Lab software) was used for the acquisition of the voltage outputs at each piezoceramic patch and the force input measured at the impact hammer and for the evaluation of the FRF between them.

Then, the optimal weighting coefficients vector $\boldsymbol{\alpha}_j$ for a modal filter designed to isolate the response of j-th vibration mode is evaluated using the QR decomposition of the experimental FRF measured by the twelve piezoelectric patches according to (5). The practical implementation of the weighted sum is obtained using the electric circuit shown in Figure 3, composed of one voltage divider for each sensor and a summing amplifier to sum the weighted sensor signals. For that, the real part of the weighting coefficients, normalized to the maximum weight of 0.35 allowed by the designed voltage divider circuit, is evaluated and implemented in the weighted sum circuit board by adjusting the potentiometers. In this preliminary setup, a digital weighted sum was considered alternatively for some piezoelectric patches and when the weighting coefficients were too small and, thus, difficult to implement using the voltage divider circuit. In this case, the digital weighted sum was obtained with the help of a dSPACE control board (dSPACE DS1104).

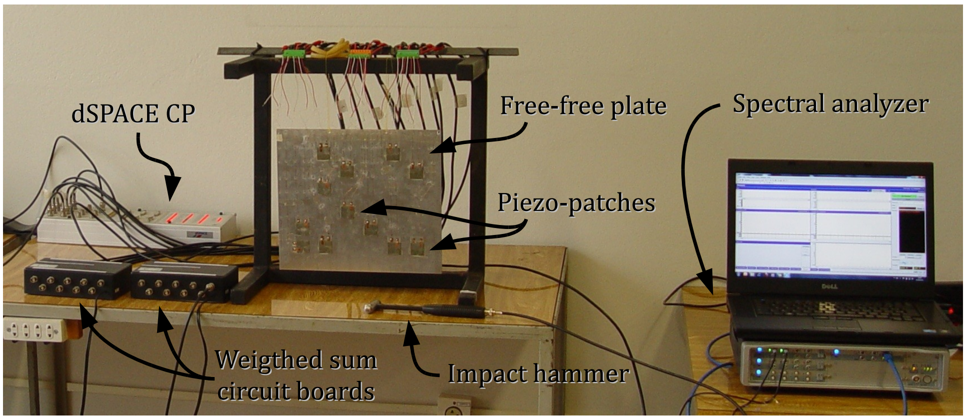

Figure 2. Experimental setup: plate with twelve bonded piezoceramic patches.

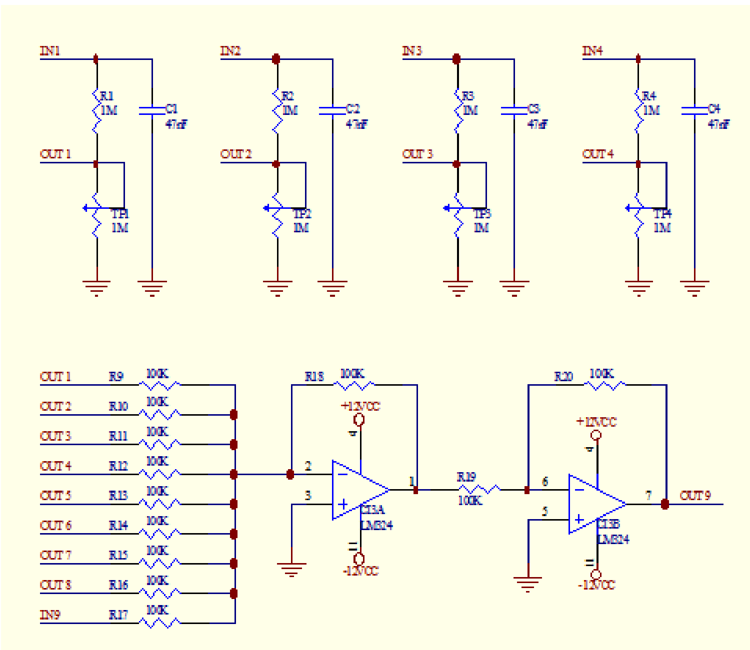

Figure 3. Electric circuit designed for weighting and summing sensor signals.

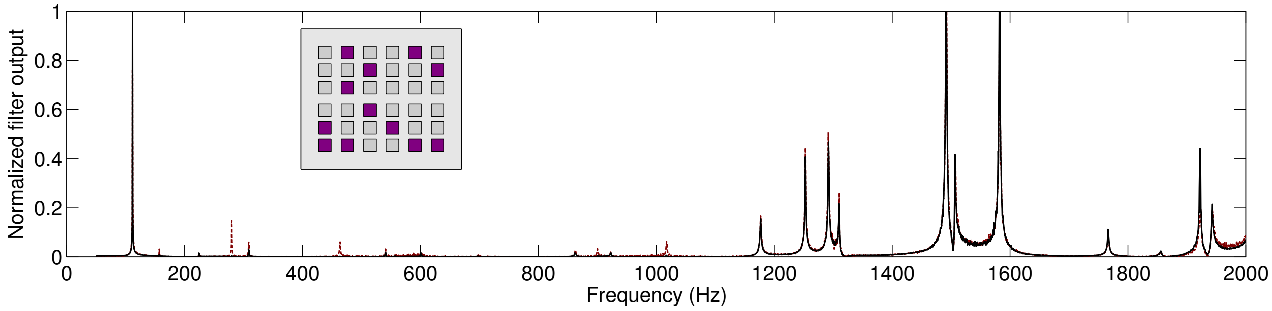

Figure 4. Experimental normalized output of the modal filter designed for the isolation of the first vibration mode using ideal (black solid) and implemented (red dashed) weighting coefficients.

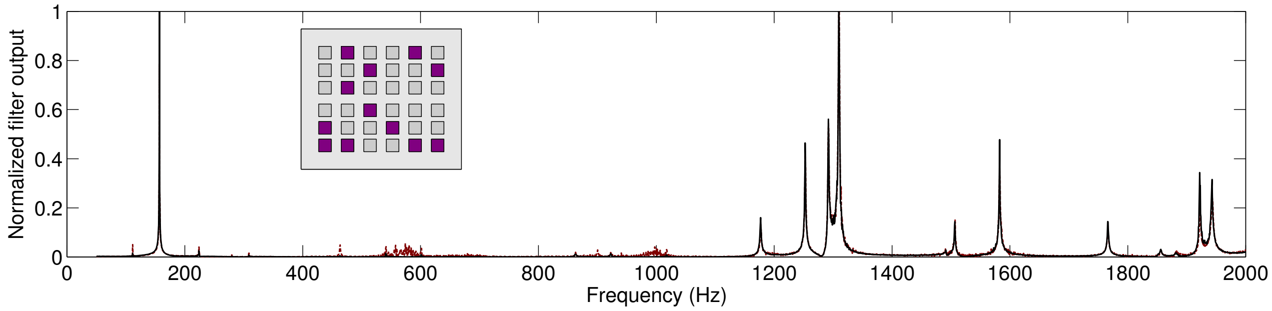

Figure 5. Experimental normalized output of the modal filter designed for the isolation of the second vibration mode using ideal (black solid) and implemented (red dashed) weighting coefficients.

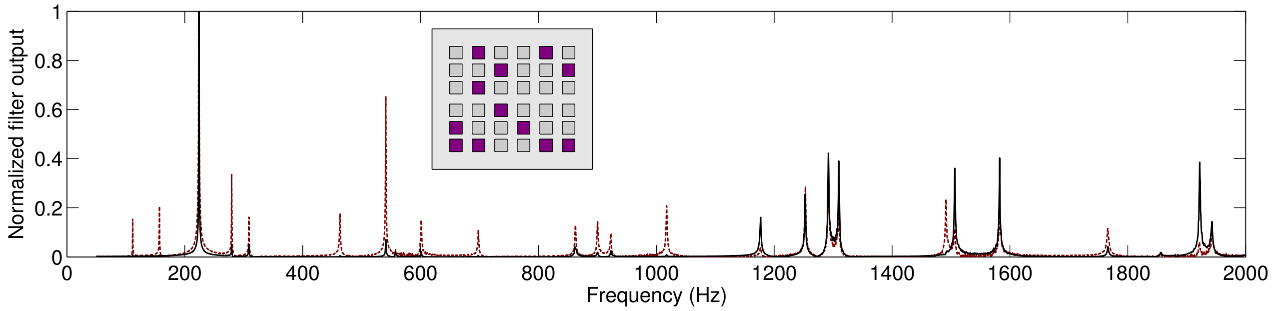

Figure 6. Experimental normalized output of the modal filter designed for the isolation of the third vibration mode using ideal (black solid) and implemented (red dashed) weighting coefficients.

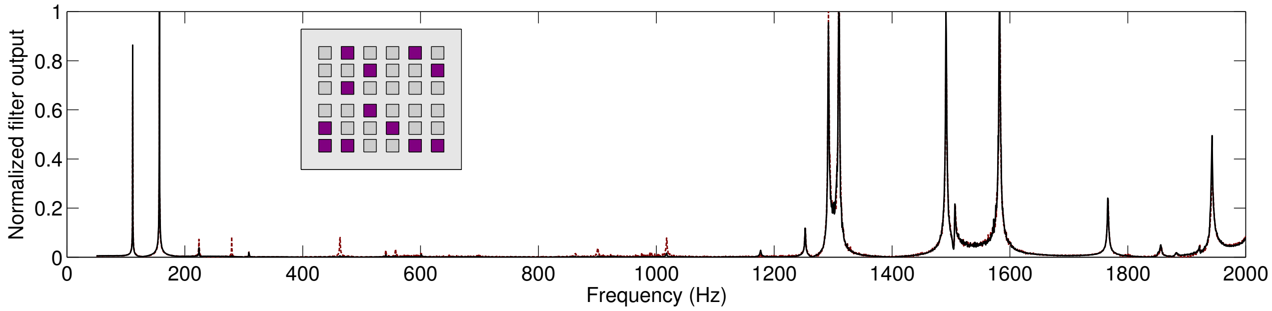

Figure 7. Experimental normalized output of the modal filter designed for the isolation of the first and second vibration modes simultaneously using ideal (black solid) and implemented (red dashed) weighting coefficients.

After adjustment of the weighting coefficients, the filtered output signal, corresponding to the weighted sum of the twelve FRF measurements, is then fed back to the spectral analyser. Hence, it should be possible to evaluate the FRF between the impact force and the weighted voltage sum. Figures 4, 5 and 6 show the FRFs evaluated for a set of modal filters. These modal filters and, thus, their corresponding weighting coefficients vectors, were designed to isolate the responses of the first, second and third vibration modes, respectively. Notice that the proposed topology of piezoelectric patches was designed to optimize the performance of the modal filters that isolate the response of the first and second vibration modes. Therefore, satisfactory results should only be expected for the first and second filters outputs. Indeed, both Figures 4 and 5 present quite satisfactory results for the isolation of the first and second vibration modes responses, although it can be noticed that the modal filters using real (implemented) weighting coefficients (red curve) lead to higher error levels than the ones numerically evaluated using the ideal weighting coefficients (black curve). In particular for the first modal filter (Figure 4), while the maximum amplitude in the frequency range of interest could be around 3% (at the fifth resonance) of the first resonance amplitude for ideal weighting coefficients, the maximum amplitude of implemented modal filter reaches 15% (at the fourth resonance) of the first resonance amplitude. For the second modal filter (Figure 5), the maximum amplitude reaches 5% of the second resonance amplitude.

On the other hand, the output of the modal filter designed to isolate the response of the third vibration mode was not satisfactory (Figure 6). This could be due to both inadequate design of the topology and implementation of weighting coefficients. However, since the maximum amplitude of the modal filter using ideal weighting coefficients is around 7% of the third resonance amplitude, it could be guessed that better performances should be obtained for this modal filter although the topology was not designed with that objective.

Another interesting result is obtained by combining the first and second modal filters to design a modal filter that isolates simultaneously the responses of the first and second vibration modes. This can be obtained using the sum of the corresponding weighting coefficients vectors. The resulting weighting coefficients vector was rescaled between 0 and 0.35 and implemented in the weighted sum circuit board. The output of this modal filter can be observed in Figure 7. The filtering quality is satisfactory with maximum amplitude around 8% of second resonance amplitude (while the ideal one should be around 3%).

5. ANALYSIS OF WEIGHTING COEFFICIENTS UNCERTAINTIES

From the observations of the previous section, it seems that the difficulty in implementing the weighting coefficients with the weighted sum circuit board is the main responsible for the differences between the ideal and real filter outputs. Therefore, this section presents an approach for analysing the effect of random uncertainties in the weighting coefficients on the filters outputs. In order to build a stochastic model for the weighting coefficients, a Gaussian probability density function is assumed for each weighting coefficient α_{jk} ($k = 1, \ldots, 12$), for which the mean values are based on the optimal ones designed to isolate the j-th vibration mode response and the standard deviations are estimated from experiments, such that

$$p(\alpha_{jk}) = \frac{1}{\sqrt{2\pi}\sigma_\alpha} \exp\left\{-\frac{1}{2\sigma_\alpha^2}\left(\alpha_{jk} - \bar{\alpha}_{jk}\right)^2\right\}, \tag{6}$$

and where $\bar{\alpha}_{jk}$ are the real part of the weighting coefficients, normalized to the maximum weight of 0.35 allowed by the voltage divider circuit used for the measurements. σ_α is an estimation of the standard deviation based on experiments. Since the level of precision in the manual setup is much more dependent on the sensibility of each potentiometer, the measurement technique for setup verification and the user's experience, than the nominal value of the weighting coefficient, the standard deviation σ_α was considered to be constant for all weighting coefficients. Based on laboratory experiments, the value of σ_α was set to 0.01.

Based on these assumptions, N random realizations were generated for each weighting coefficient with MATLAB function *normrnd* and, then, combined to form N random realizations of the vector of weighting coefficients $\boldsymbol{\alpha}_j(\theta_i)$. Each realization $\boldsymbol{\alpha}_j(\theta_i)$ was then used to evaluate a realization of the filter output $\tilde{\mathbf{G}}_j(\theta_i) = \mathbf{Y}\boldsymbol{\alpha}_j(\theta_i)$. The mean-square convergence analysis with respect to the independent realizations $\tilde{\mathbf{G}}_j(\theta_i)$ was carried out considering the function

$$conv(n_s) = \frac{1}{n_s}\sum_{i=1}^{n_s} \|\tilde{\mathbf{G}}_j(\theta_i) - \tilde{\mathbf{G}}_j^N\|^2, \tag{7}$$

where n_s is the number of simulations, or the number of sets of weighting coefficients considered, and $\tilde{\mathbf{G}}_j^N$ is the response calculated using the corresponding nominal (ideal) model. It was observed that around 2000 simulations were enough to assure convergence. Despite that, the statistical analyses presented in the following sections consider all $N = 4000$ simulations performed.

The statistical analyses of the FRF amplitudes were performed using their 4000 realizations at each frequency to calculate the corresponding mean values and 95% confidence intervals. The 95% confidence intervals were evaluated using the 2.5% and 97.5% percentiles of the realizations of FRF amplitudes at each frequency. More details on the stochastic modeling methodology used here can be found in (Cataldo et al., 2009; Soize, 2001; Santos and Trindade, 2009).

The experimental (real) normalized outputs of the modal filters compared to ideal ones and its 95% confidence intervals are presented in Figures 8, 9, 11 and 10. They correspond, respectively, to the modal filters designed to isolate the first,

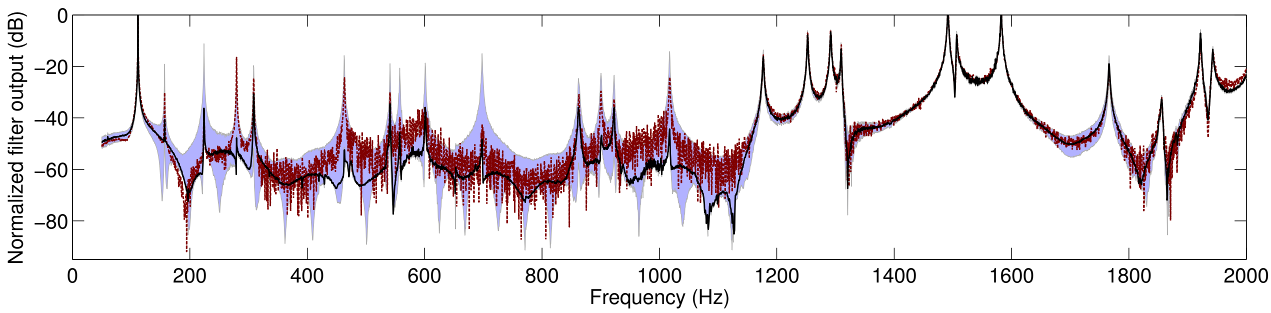

Figure 8. Experimental normalized output of the modal filter designed for the isolation of the second vibration mode using ideal (black solid) and implemented (red dashed) weighting coefficients and its confidence interval for uncertain weighting coefficients with $\sigma_\alpha = 0.01$ (magenta filled).

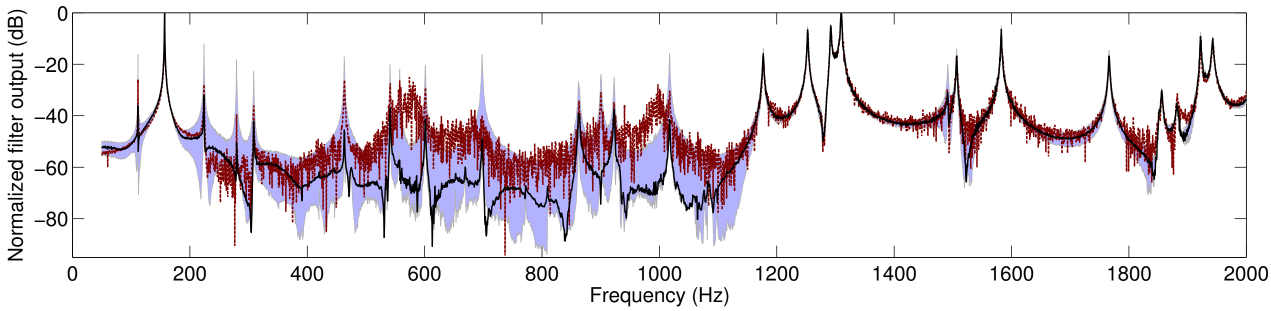

Figure 9. Experimental normalized output of the modal filter designed for the isolation of the second vibration mode using ideal (black solid) and implemented (red dashed) weighting coefficients and its confidence interval for uncertain weighting coefficients with $\sigma_\alpha = 0.01$ (magenta filled).

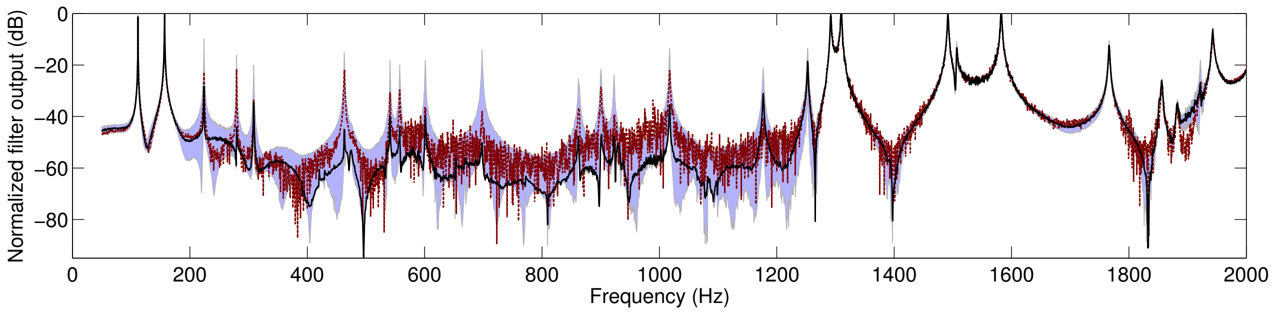

Figure 10. Experimental normalized output of the modal filter designed for the isolation of the first and second vibration modes simultaneously using ideal (black solid) and implemented (red dashed) weighting coefficients and its confidence interval for uncertain weighting coefficients with $\sigma_\alpha = 0.01$ (magenta filled).

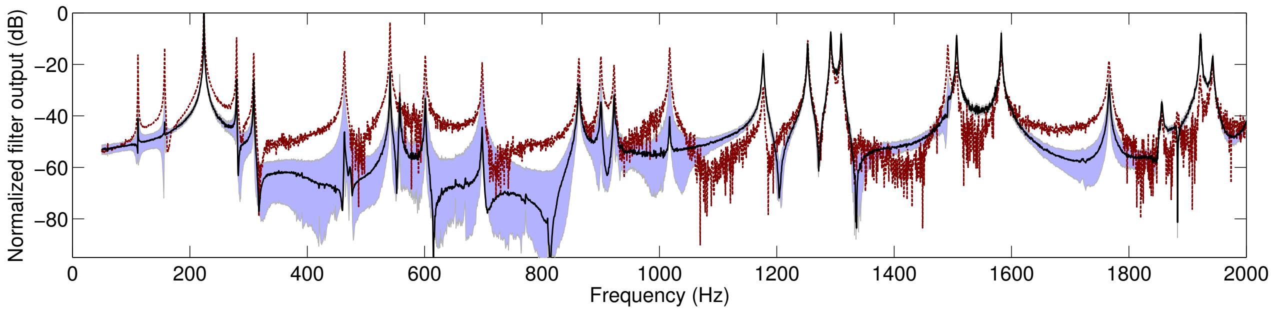

Figure 11. Experimental normalized output of the modal filter designed for the isolation of the third vibration mode using ideal (black solid) and implemented (red dashed) weighting coefficients and its confidence interval for uncertain weighting coefficients with $\sigma_\alpha = 0.01$ (magenta filled).

second, third and first and second vibration modes simultaneously. From Figures 8 and 9, it could be concluded that the variability of the weighting coefficients almost completely explains the experimental filter outputs (since they are inside the confidence intervals). A similar result is observed for the modal filter designed to isolate the first and second vibration modes response simultaneously (Figure 10). On the other hand, this is not true for the modal filter designed to isolate the response of the third vibration mode, for which the output is presented in Figure 11. In this case, the filter output does not coincide with the 95% confidence interval and, thus, the difference between real and ideal modal filter output could be mainly due to another reason (besides weighting coefficients variability). This effect should be further investigated in future works.

6. CONCLUSIONS AND FUTURE WORKS

This work has presented an experimental validation of modal filters based on arrays of piezoelectric sensors applied to a free rectangular plate. The topology of the array of piezoelectric sensors bonded to a rectangular plate was optimized to improve the effectiveness and frequency range of a set of modal filters. An experimental implementation of the modal filters was performed using voltage divider and summing amplifier circuits and used to validate the performance of the modal filters. Preliminary experimental results indicate that the chosen array of piezoceramic sensors yields indeed high quality modal filters up to 1100 Hz, thus validating the designed sensors topology and weighting coefficients design methodology. These results are being analysed further to evaluate the filtering performance for other frequency ranges and target vibration modes. Other strategies for the weighted sum implementation are being analyzed.

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