Review in Optimized Spatial and Time Discretization Scehemes for Aeroacoustic Issues

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Abstract. *This work presents some of fundamental characteristics of the issues faced in the field of aeroacoustics computation like characteristics of dissipation and dispersion, specially about the acoustic propagation phenomenon. The first part brings two analysis of some discretization methods proposed in the literature. These analyses are carried out on the sense of testing the numerical methods of time discretization and the methods of spatial discretization checking the numerical error value and the range of convergence of each scheme. Among the analyzed methods, the work gives highlights to the optimized methods proposed by recent researches because of its low dispersion and low dissipation properties reached due its building on the Fourier space, which preserves the important characteristics on an wave propagation. Since the scheme of time and spatial discretization give a good approach, they can be employed on another propagation case test whose governing equation are the so called Linearized Euler Equations which model in a better way the propagation phenomenon because they transports the three wave modes: the vorticity, the acoustic and the entropy modes. This is the second part which brings an one-dimensional wave propagation test case and an two-dimensional one. At this part the aim is not only test the methods performance but also achieve a good solve to the problems. Thus, a good set of boundary conditions is fundamental, mainly on aeroacoustics, because the outgoing waves easily lead to numerical errors due the numerical reflection and bad treatment. So that, enough attention is payed to the boundary condition equations also proposed in the literature. In these case tests the boundary conditions needs an boundary region where the boundary equations are employed and where asymmetric schemes takes place, an not more the symmetric schemes like on the whole domain. Therefore this work reaffirmed the better performance of the optimized numerical methods not only the spatial but also the time discretization in the aeroacoustics propagation, specially because they have the property of dispersion and dissipation properties.*

Keywords: *Aeroaoucstics, Acoustic propagation, optimized schemes, Linearized Euler Equations, boundary conditions, asymmetric schemes, low dispersion and low dissipation.*

1. INTRODUCTION

The noise generate from fluid flow is a new challenge for airplane e automobile industry among others whose the main aim is predicting the sources and the propagation of acoustic waves. This is what encourages the development on the research field called Aeroacoustics.

The wave propagation is the main focus of this work in the sense of take into account the relevant characteristics of the phenomenon in order to modeling it in a reasonable way, but mainly apply appropriate discretization schemes to solve numerically the problem.

Recent researches in the aeroacoustics have given special attention to optimized numerical methods. The first ones being studied were the spatial discretization numerical methods; although since the wave propagation is a time dependent phenomenon (TAM, 2001), so that it's also necessary to optimize the time discretization methods. Moreover, the boundary conditions require appropriate treatment because they must support outgoing disturbances of acoustic, entropy and vorticity, and a simple set of values to the field variables or its derivative is not enough. The boundaries have to be treated with more specific equations that simulate a region on the space where the acoustic pulses decayed substantially.

The aeroacoustics has even more specialties to motivate several works, but these are enough reason to this work.

Thus, the aim in this work is to investigate some discretization numerical methods and apply them on a typical acoustic propagation problem modeled by the Linearized Euler Equations (LEE) – these will be presented later – with an appropriate set of boundary conditions.

2. METODOLOGY

2.1. Spatial Discretization

The classical discretization methods like finite difference are appropriate approach to the Computational Fluid Dynamics, but to the aeroacoustics, just assure good precision with more points on stencil is not enough. It is necessary to ensure the properties of the propagating wave because the frequency or the wave number is sensible and important information to aeroacoustics.

The building of the standard numerical methods comes from a Taylor series truncated. The coefficients of the optimized numerical methods are chosen in order to minimize the error between the numerical wave number and the real wave number which is possible when one compares the Fourier transform of the approach in a finite difference method and the Fourier transform of the real derivative.

$$\frac{\partial f}{\partial x}(x)=\frac{1}{\Delta x}\sum_{j=-N}^{M}a_j f(x+j\Delta x). \tag{1}$$

$$i\alpha\tilde{f}=\left(\frac{1}{\Delta x}\sum_{j=-N}^{M}a_j e^{i\alpha j\Delta x}\right)\tilde{f}. \tag{2}$$

By comparison one can write:

$$\alpha^*=\frac{-i}{\Delta x}\sum_{j=-N}^{M}a_j e^{i\alpha j\Delta x}. \tag{3}$$

That is the wave number of the Fourier transform of the Finite Difference scheme. In order to ensure the integrity of the Fourier transform of the scheme, the error between the real wave number must be minimized on the Eq. (3) proposed by Tam and Webb (1993).

$$Error=\int_{(\alpha\Delta x)_L}^{(\alpha\Delta x)_U}\left|\alpha\Delta x-\alpha^*\Delta x\right|^2 d(\alpha\Delta x). \tag{4}$$

On the Equation (1), if $M = N$ the scheme built is a symmetric scheme or a Centered Finite Difference scheme. The symmetric optimized scheme has no imaginary parts and do not generate numerical errors or spurious waves. This inconvenient problem occurs when $M \neq N$ and the scheme is asymmetric. Because of this, the asymmetric schemes are not suppose to be widely employed over large spaces. But Tam and Webb (1993) consider negligible the effect if these asymmetric schemes are applied just over the boundary region which occupies a reduced part of the domain.

The analyzed schemes were the family of optimized schemes proposed by Bogey and Bailly (2004) with 9($M=N=4$), 11($M=N=5$) and 13($M=N=6$) points on stencil and the scheme proposed by Tam and Webb (1993) on the method called Dispersion Relation Preserving (or DRP). Besides these, will be analyzed the standard schemes: Centered Finite Difference scheme of second order, fourth and sixth order of precision in order to compare the performance.

2.2. Time Discretization

The aeroacoustics issues are, by definition, time dependent problems, so that, the time discretization methods also must be optimized like the schemes proposed by Hu et. al. (1996) and Berland et. al. (2006) called Low Dispersion and Low Dissipation Runge-Kutta Algorithm (or LDDRK) and Fourth order and 6-stage to non-linear operators Runge-Kutta Algorithm (or RK46-NL) respectively. These and the classic time discretization methods called from Euler and second order Runge-Kutta and Fourth order Runge-Kutta were the analyzed methods.

The building of the optimized methods for time discretization is done in the same way like shown on preview section. But for better explanation Silva (2008) or the cited authors may be consulted.

2.3. Boundary Conditions

The external problems of the aeroacoustics lead to an issue not so crucial to the perturbations found on the Fluid Dynamics, but, since the acoustics perturbations decay in a slower way, for the aeroacoustics, would be necessary large computational domain to solve the problems satisfactorily. These are the problems defined on a infinite or semi-infinite space. So that, this is the main reason of the boundary conditions must be treated in a specific way.

This is particular important because of the dispersive nature of discretized equations can leads to spurious numerical reflections, and then to a poor solution. And more, using the one-sided differences for derivatives near the boundary can leads to serious inaccuracy like indicate Rowley and Colonius (1998).

Two main approaches have been used in the boundary conditions for the continuous Linearized Euler Equations (LEE). One of them consists of asymptotic expansions of the discretized equations with the distance approaches infinity. The other one involves the decomposition of the solution in a linear region into Fourier/Laplace modes like Rowley and Colonius (1998) explain.

So, to simulate such problems, it is necessary to impose the called *radiation* and *outflow* boundary conditions at the edges of the computational domain.

The first case test intend to measure the performance of each scheme, but the second one apply boundary conditions adequate to the LEE governing problem like proposed by Tam and Webb (1993).

3. NUMERICAL PROCEDURE

3.1. Test Case – Convection Equation

This problem consists of a one-dimensional acoustic pulse propagating over a space during a certain time and not reaching the boundary which in this case will not properly treat.

The governing equation is the Convection Equation:

$$\frac{\partial u}{\partial t} + M\frac{\partial u}{\partial x} = 0 \tag{5}$$

The propagation velocity 'u' varies on time 't' and space 'x'. And 'M' is the Mach number, the ratio between the mean velocity and the sound velocity on the mean flow. That takes the value 0.5.

$$M = \frac{u_\infty}{c_\infty} \tag{6}$$

This is a good test case because of its known analytical solution.

$$u(x,t) = 0{,}1\exp\left[-\ln(2)\left(\frac{x-20{,}0-Mt}{b}\right)^2\right]. \tag{7}$$

On the equation (7) the amplitude, 0.1, and the width, b = 3.0, of the pulse, which starts from an initial condition (eq. 8) and propagates in the positive direction of 'x' axis.

$$u(x,t) = 0{,}1\exp\left[-\ln(2)\left(\frac{x-20{,}0}{b}\right)^2\right]. \tag{8}$$

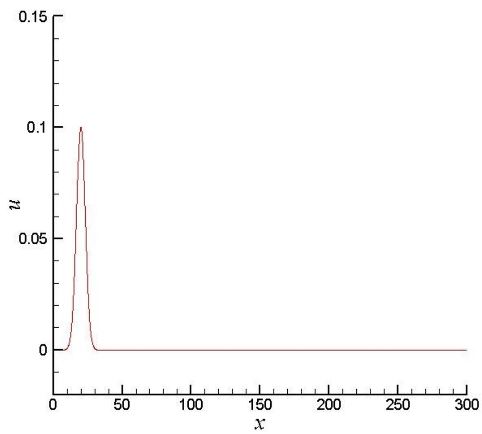

Figure 1. Initial condition and length of the domain of the test case.

This test case is vastly simulated with each scheme of time and spatial discretization. The first group of simulations was carried out in order to test the performance of the time discretization schemes. For this, the better spatial discretization scheme analyzed is employed: Optimized Scheme with 13 points proposed by Bogey and Bailly (2004). To this end, the value of mesh unit (Δx) takes the value of 0.3, and the pulse propagates over the space of $733\Delta x$. The analysis of the range of convergence is done varying the value of the coefficient CFL:

$$CFL = M\,\frac{\Delta t}{\Delta x}\,. \tag{9}$$

To mesurement of the performance of the time discretization method is used the follow error equation.

$$Error = \sqrt{\frac{\sum_{i=1}^{nx}\left(u_{analy} - u_{num}\right)^2}{nx}}\,. \tag{10}$$

The number of points inside the mesh of the domain is expressed on the equation by '*nx*' and the indices 'analy' and 'num' indicate the variable of the analytical solution and the numerical solution respectively.

The second group of simulations is dedicated to analyze the performance of the spatial discretization schemes. Like on the first, this series of simulation employ the better time discretization scheme – and it is the RK46-NL proposed by – in order to observe the errors from the spatial ones. The time step (Δt) and the mesh unit remain constant with the values of 0,05 and 1,0 respectively. With these parameters, the pulse propagates over a space of $360\Delta x$. The solution of each spatial discretization scheme is presented on section 4.

3.2. Test Case – The Two-dimensional Linearized Euler Equation

The second test case consists of two-dimensional acoustic pulse propagation in the radial direction with the speed of sound and simultaneously, all the pattern of the wave propagation undergoes convection on the direction of the mean flow.

The governing equation employed on this problem is the system of the two-dimensional LEE with no source term on dimensionless form:

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0 \tag{11}$$

And U, E and F are given by:

$$U = \begin{bmatrix} \rho \\ u \\ v \\ p \end{bmatrix}, \quad E = \begin{bmatrix} M_x\rho + u \\ M_x u + p \\ M_x v \\ M_x p + u \end{bmatrix} \quad e \quad F = \begin{bmatrix} M_y\rho + v \\ M_y u \\ M_y v + p \\ M_y p + v \end{bmatrix}, \tag{12}$$

where the constants *Mx* and *My* are the Mach number on the *x* and *y* directions respectively. The variables ρ, u, v and p

are the density, the velocity on the *x* direction and *v* on the *y* direction and the pressure respectively.

The computational domain used on the problem is defined by the intervals *x = [-100,0; 100,0]* and *y = [-100,0; 100,0]*. With the follow initial condition:

$$
\begin{aligned}
&p(x,y,t=0)=\exp\left\{\left(-\ln 2,0\right)\left[\frac{\left(x-x_c\right)^2+\left(y-y_c\right)^2}{b^2}\right]\right\} \\
&\rho(x,y,t=0)=u(x,y,t=0)=v(x,y,t=0)=0,0
\end{aligned} \tag{13}
$$

The pair *(xc,yc) = (0,0)* is the pair coordinates of the initial position of the pulse. And the analytical solution is known for this case.

$$
p(x,y,t)=\frac{1}{2\alpha_1}\int_0^{\infty} e^{\frac{\xi^2}{4\alpha_1}}\cos(\xi t)J_0(\xi\eta)\xi\,d\xi. \tag{14}
$$

With:

$$
\alpha_1=\frac{\ln 2,0}{b^2},\quad \eta=\left[\left(x-x_c-M_x t\right)^2+\left(y-y_c-M_y t\right)^2\right]^{1/2}
$$

The boundary conditions applied on the restrict region on the edge of the domain with three rows are the following equations achieved from the asymptotic expansion solution of the Linearized Euler Equations discretized.

The outflow boundary equations (top, bottom and right edges) like proposed by Zhang et. al. (2004) are:

$$
\begin{aligned}
&\frac{\partial\rho}{\partial t}=-M_x\frac{\partial\rho}{\partial x}+\frac{\partial p}{\partial t}+M_x\frac{\partial p}{\partial x}, \\
&\frac{\partial u}{\partial t}=-M_x\frac{\partial u}{\partial x}-\frac{\partial p}{\partial x}, \\
&\frac{\partial v}{\partial t}=-M_x\frac{\partial v}{\partial x}-\frac{\partial p}{\partial y} \\
&\frac{\partial p}{\partial t}=-V(\theta)\left(\cos\theta\frac{\partial p}{\partial x}+sen\theta\frac{\partial p}{\partial y}+\frac{p}{2r}\right),
\end{aligned} \tag{15}
$$

And the inflow boundary conditions (left edge) are given by the eq. (16).

$$
\begin{aligned}
&\frac{\partial\rho}{\partial t}=-V(\theta)\left(\cos\theta\frac{\partial\rho}{\partial x}+sen\theta\frac{\partial\rho}{\partial y}+\frac{\rho}{2r}\right), \\
&\frac{\partial u}{\partial t}=-V(\theta)\left(\cos\theta\frac{\partial u}{\partial x}+sen\theta\frac{\partial u}{\partial y}+\frac{u}{2r}\right), \\
&\frac{\partial v}{\partial t}=-V(\theta)\left(\cos\theta\frac{\partial v}{\partial x}+sen\theta\frac{\partial v}{\partial y}+\frac{v}{2r}\right), \\
&\frac{\partial p}{\partial t}=-V(\theta)\left(\cos\theta\frac{\partial p}{\partial x}+sen\theta\frac{\partial p}{\partial y}+\frac{p}{2r}\right).
\end{aligned} \tag{16}
$$

After analyzing the optimized spatial and time discretization schemes and verifying its superior performance, the employment of these schemes is similar to the proposal of Tam and Webb (1993) and more like Silva (2008).

The time discretization scheme is chosen the RK46-NL proposed by Berland et. al. (2006), but the spatial discretization schemes employed are all the optimized analyzed schemes. As soon as the interest point approaches the edge, the fewer-point scheme is employed. For example, when the interest point is the fifth point from the boundary (four points between the interest point and the boundary) the 9-point scheme proposed by Bogey and Bailly (2004) and not the 13-point one, applied inside the domain region.

Furthermore, on the boundary region, the asymmetric schemes (TAM, 2004) are employed because of fewer points from the edge until the interest point. And this can be accomplished with no significant injury to the solution like consider Tam and Webb (1993).

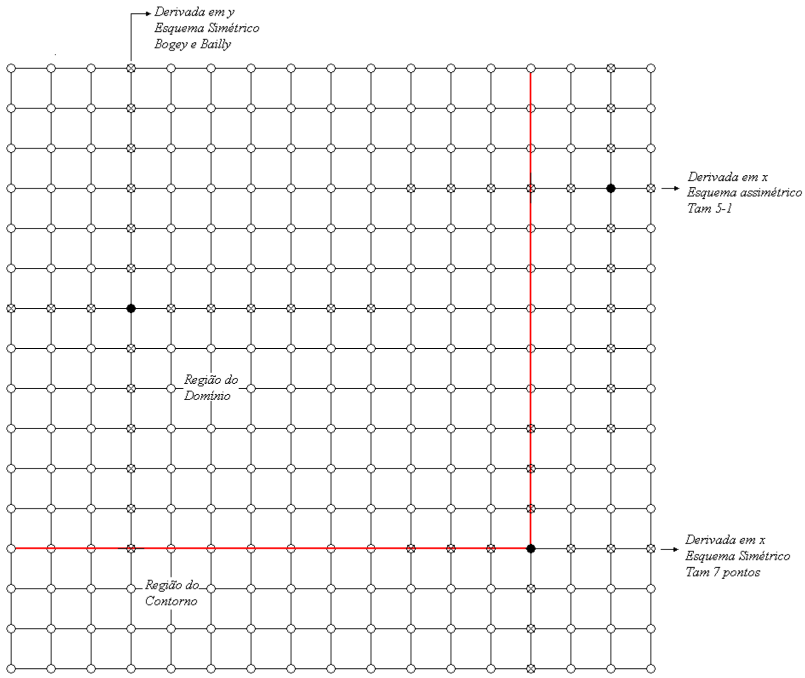

Figure 2. Sketch about the dynamic employment of the spatial discretization schemes.

4. RESULTS

4.1. Test case – Preliminary Analyses

The analysis of the time discretization schemes gave the follow comparison chart.

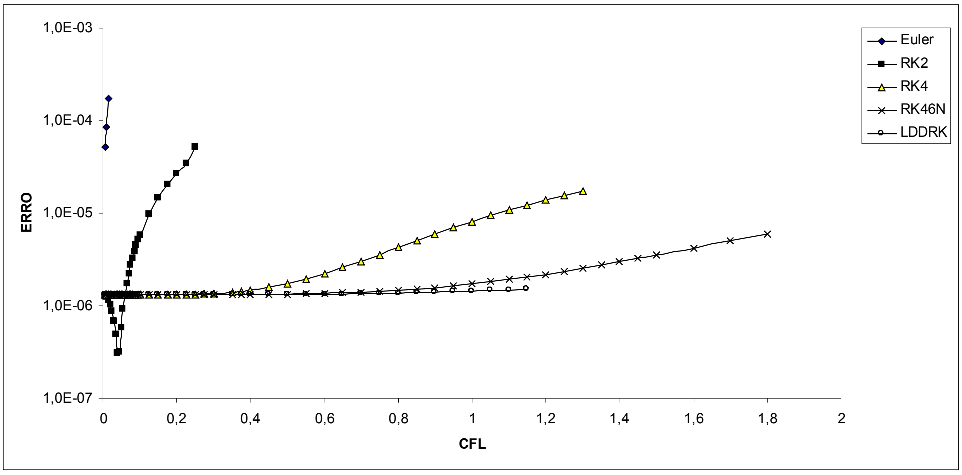

Figure 3. Ranges of convergence for the time discretization scheme.

The range of convergence of the optimized methods is wider the classical ones. Three schemes deserves highlights: RK4, RK46-NL and the LDDRK. For CFL < 0.7 the RK46-NL and LDDRK have almost the same performance. But on an overview one can note that the scheme RK46-NL has the wider range of convergence and with a good performance.

Next are the results of the spatial discretization scheme analysis.

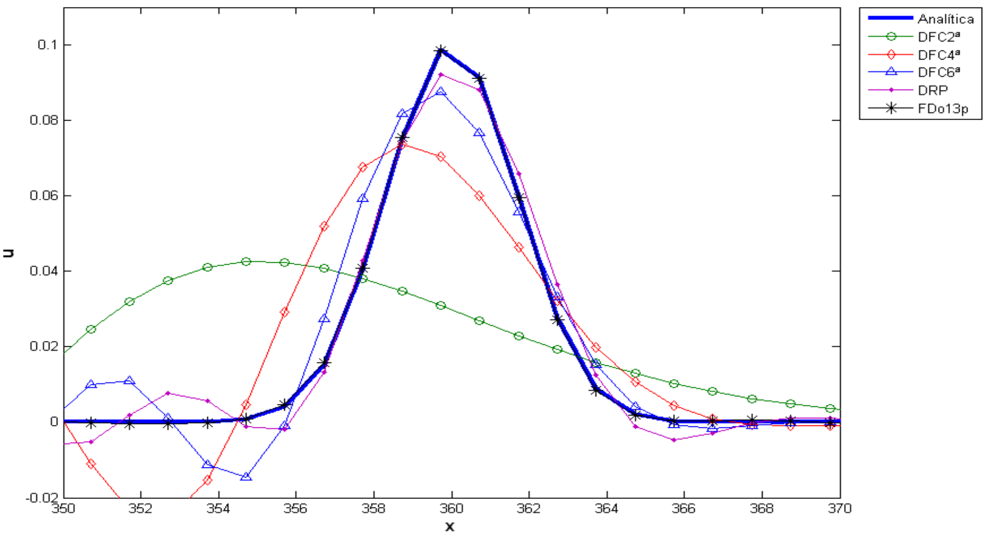

Figure 4. Solution from each spatial discretization schemes**.**

Once again one can note that the optimized schemes leads to better solutions with less dissipation – the amplitude of the pulse remains almost the same – and less dispersion which refers to the reduction on the amplitude of the wave train propagating before the main wave.

4.2. Test case – The Two-Dimensional Propagation

With the better schemes of discretization, the two-dimensional pulse propagation can be carried out with reasonable solution. The simulation was done with three values to the mesh unit, with the domain containing 64x64, 128x128 and 240x240 points on the mesh.

The sample of the field was extracted in two times, at 1600Δt and at 1900Δt.

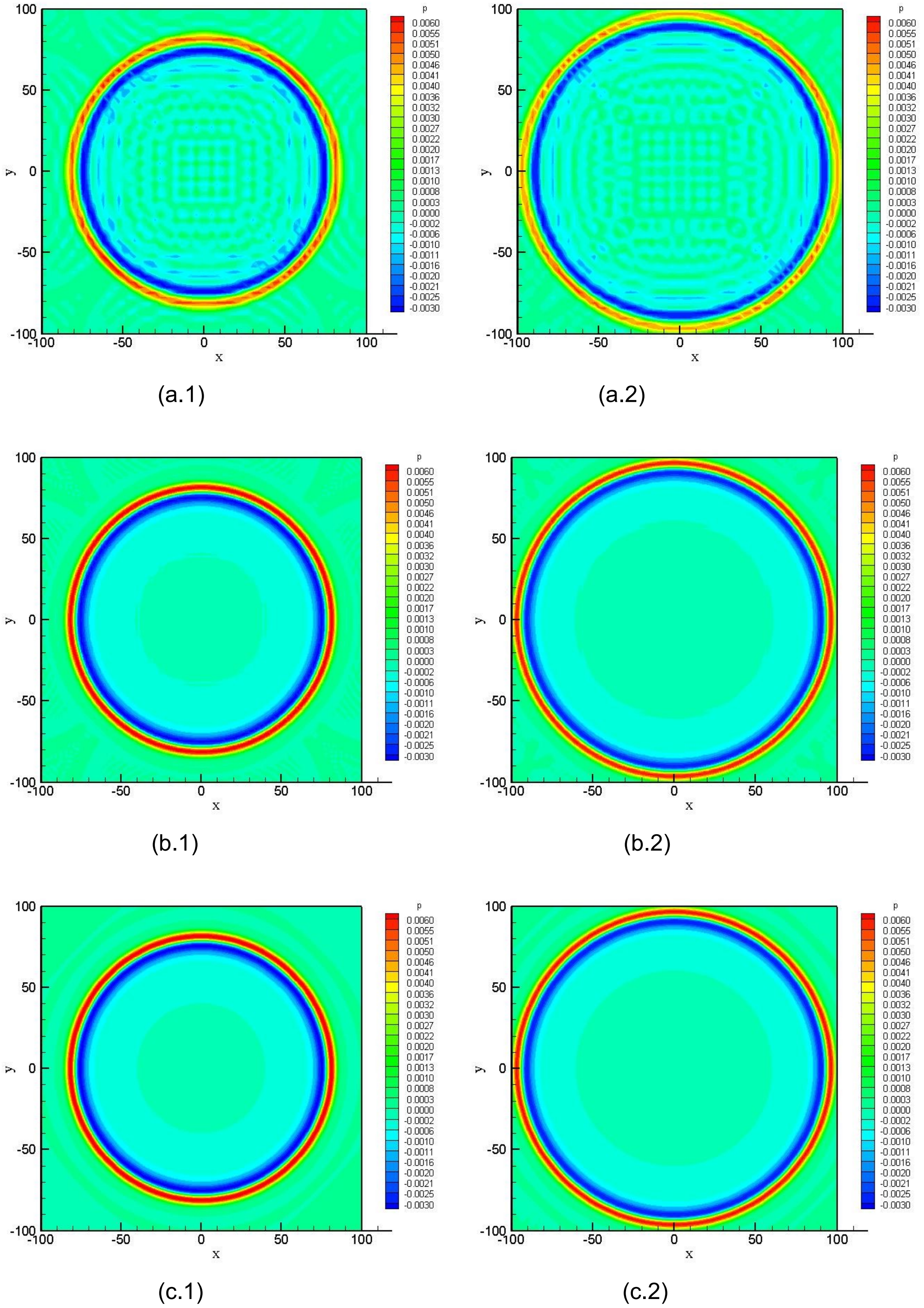

Figure 5. Graphics with the acoustic field at t=1600Δ*t* (1) and at 1900Δ*t* (2), to the mesh 64x64 (a), 128x128 (b) and 240x240 (c).

Even applying the optimized schemes, with high mesh unit the solution presents some numerical noise which is exclusively a numerical phenomenon. But for mesh with low value, the solution is well produced. Despite this, some other important features must be approached, but this work is not facing it.

5. CONCLUSION

The optimized discretization methods and appropriate treatment to the boundary conditions are especially important for trustful solutions on the aeroacoustics problems.

On this case, explicit formulation was used, so the possibility of a high CFL coefficient is a substantial gain with the employ of the optimized schemes here treated, specially for a complex future calculation involving fluid dynamics field as well.

Some subjects must be studied to the better comprehension of the phenomenon of the propagation like selective damping, reflexive boundary conditions and the presence of a flow in the mean.

6. ACKNOWLEDGEMENTS

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