DESIGN OF REGULATORS WITH OPTIMAL INITIAL CONDITIONS COMPENSATION USING LMIs

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Abstract. *Typically control systems are designed aiming at the specification of parameters of the controller that is usually described by a differential equation. In most cases, the controller is artificially constructed and you can also update their initial conditions. In the design of optimal quadratic regulators to update initial conditions of the optimal controller can significantly improve the performance of the controlled system. In this work is also considered other constraints on the controller design, for example, restrictions on output and entry and also restrictions on the decay rate. A design procedure formulated in the context of linear matrix inequalities (LMIs) to update the initial conditions in PI controllers considering also other constraints is presented. The applications of the proposed method for controlling an inverted pendulum and the control of a chemical reaction prove its effectiveness.*

Keywords: *quadratic regulator, initial conditions, IVC, optimal control, LMIs.*

1. INTRODUCTION

In the design of automatic control systems, the goal is to obtain a control law that supplies the inputs of a process, so that the system has an acceptable dynamic performance. In the vast control literature, there are several results on pole allocation in control system design. However, the correct location of the zeros, for instance of a transfer function, can also be indispensable to obtain a good transient response. It is possible to allocate zeros of transfer functions, by using available dynamic feedback structures (Kienitz and Grubel, 2000). In many aerospace control systems, which request high precision, the design of optimal control has been considered a very important subject. For instance, a method that has been very much used is optimal control based on the minimization of quadratic performance indexes. In Ogata (1997), it was observed that the initial condition of a controlled system influences the quadratic performance index.

Many servo control schemes in mechatronics systems, such as hard disk drives, must meet the specifications of both fast movement and precise positioning on a known reference. To meet this requirement, one servo structure for fast access and other for precise positioning are designed. Then, the control is switched between these two servo structures. This type of servo system is called Variable Structure Control (VSC). Each servo mode can be optimally designed by the minimization of its desired performance index. Therefore, the remaining problem is how to switch from one mode to other. In Yamaguchi *et al*., (1996), is proposed the method called Initial Value Compensation (IVC) to improve the performance of the transient response after switching. This method was also used with the intention of reducing the stabilization time of the controlled system (Johansson, 2000; Hirose *et al.,* 2011). In these references, the design goal was to minimize a quadratic performance index (denominated IVC I), with the reference signals equal to zero. The plants and controllers in these researches are discrete-time systems. In Teixeira *et al*., 2002, it is considered a controlled system, consisting of the plant and one or more dynamic controllers, continuous in the time, with a step reference signal, and it is shown analytically that the initial conditions in the controller can be modified, improving the transient response of the system, according to a quadratic index. A modification of the initial conditions in the controller can be interpreted as a change in the positioning of the zeros of the system. In Teixeira *et al*., 2006, the authors present an alternative method for optimum compensation of the initial conditions on the controller, in the case considered the integral type, based on LMIs. This work is also considered other constraints on the controller design, for example, restrictions on exit and entry and restrictions on the decay rate. A LMI-based design procedure to update the initial conditions in PI controllers considering also other constraints is presented. This article is organized as follows. In the next section is revised the first method proposed in Teixeira *et al.* (2006) for the optimal specification of initial conditions on the controller using LMIs, the design of optimal quadratic regulators to update the initial conditions on the controller. In Section 3, we approach the update on the controller when considering further restrictions on the controller design, such as decay rate and restrictions on exit and entry. In Section 4, we apply the methods presented in the control of an inverted pendulum and the control of a chemical reaction. Section 5 presents conclusions.

2. STATEMENT OF THE PROBLEM

Given the system shown in Figure 1, will design both a matrix $\hat{\boldsymbol{K}} = \begin{bmatrix} -\boldsymbol{K} & K_I \end{bmatrix}$ and the optimal initial condition, ξ_{lo}, the controller in the case considered the integral type:

$$u(t) = -\boldsymbol{K}\boldsymbol{x}_p(t) + K_I\xi(t) . \tag{1}$$

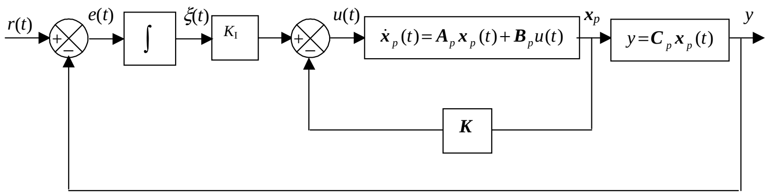

Figure 1 - Closed-loop System.

The system is described in state variables as:

$$\dot{x}(t) = A_p x_p(t) + B_p u(t) , \quad (2)$$

$$y(t) = C_p x_p(t), \quad (3)$$

$$e(t) = r(t) - y(t), \quad (4)$$

$$\dot{\xi}(t) = e(t) = r(t) - C_p x_p(t), \quad (5)$$

being that $x_p(t)\,\Re^n$ is the state vector, $u(t) \in \Re$ is the input vector and given in (1), $e(t) \in \Re$, $e(t)$ is tracking error the vector error and r (t) is the reference, $A_p \in \Re^{nxn}$, $B_p \in \Re^{nx1}$ and $C_p \in \Re^{1xn}$ are constant matrices.

From equations (2)-(5) can describe the dynamics of the system by:

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} A_p & \mathbf{0} \\ -C_p & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} B_p \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r(t) , \quad (6)$$

being that $\mathbf{0}$ denotes a vector with all elements null and size. $n \times 1$.

Defining

$$x_e(t) = x_p(t) - x_p(\infty); \quad \xi_e(t) = \xi(t) - \xi(\infty); \quad u_e(t) = u(t) - u(\infty) . \quad (7)$$

Now, defining the vector-error of size (n +1) by

$$e(t) = \left[x_e(t)^T \quad \xi_e(t) \right]^T . \quad (8)$$

Thus, the dynamics of vector-error is described by:

$$\dot{e}(t) = A e(t) + B u_e(t) , \quad (9)$$

being:

$$A = \begin{bmatrix} A_p & \mathbf{0} \\ -C_p & 0 \end{bmatrix}; B = \begin{bmatrix} B_p \\ \mathbf{0} \end{bmatrix}; \quad u_e(t) = -\hat{K}\, e(t); \quad \hat{K} = \left[K \quad -K_I \right] \quad (10)$$

By replacing (9) in (8):

$$\dot{e}(t) = (A - B\hat{K})\, e(t) \quad (11)$$

2.1 Analysis of the Lyapunov Stability

In this case, the study of method of Lyapunov to analyze the stability of the closed-loop system (11) is accomplished through the study of the following LMIs:

$$P(A - B\hat{K}) + (A - B\hat{K})^T P < 0, \quad P > 0, \quad (12)$$

Thus, of (12) doing a manipulation, multiplying both sides of (12) per P^{-1}, and defining $X = P^{-1}$ and $M = \hat{K} P^{-1} = \hat{K} X$ we have:

$$A X + X A^T - M^T B^T - BM < 0, X > 0 , \quad (13)$$

being $X = X^T$, which now being are LMIs. If these LMIs are feasible, ie presenting at least one solution X and M, then the controller gain is given by $\hat{K} = M X^{-1}$. In the design of optimal control is desired to minimize a performance index. Tanaka and Wang (2001), design an optimal fuzzy controller for nonlinear systems by solving an optimization problem that minimizes the upper bound of a quadratic performance index. Then, this idea is applied to design optimal control for linear systems.

2.2 Performance Index

The gain matrix of state feedback controller is obtained by $\hat{K} = \left[K \quad -K_I \right]$, in order to minimize the upper limit of the index:

$$J_1(u_e) = \int_0^{+\infty} (\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}\,\boldsymbol{e}(t) + u_e^{\mathrm{T}}(t) R\, u_e(t))\, dt \tag{14}$$

being that $\boldsymbol{Q}$ is a real symmetric positive definite matrix and $\boldsymbol{R}$ is a real matrix symmetric positive definite or $\boldsymbol{R} = 0$. The following theorem provides an upper bound to J_1.

Theorem 1: The system (8) - (9) can be stabilized by controller (10), if there is a symmetric positive definite matrix satisfying:

$$\boldsymbol{A}\,\boldsymbol{X} + \boldsymbol{X}\,\boldsymbol{A}^{\mathrm{T}} - \boldsymbol{M}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} - \boldsymbol{B}\boldsymbol{M} + \begin{bmatrix} \boldsymbol{X} & -\boldsymbol{M}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{0} & R \end{bmatrix} \begin{bmatrix} \boldsymbol{X} \\ -\boldsymbol{M} \end{bmatrix} < 0, \tag{15}$$

$$\boldsymbol{X} > 0. \tag{16}$$

Moreover, the performance index satisfies

$$J_1(u_e) < \boldsymbol{e}^{\mathrm{T}}(0)\,\boldsymbol{P}\,\boldsymbol{e}(0), \tag{17}$$

being $\boldsymbol{P} = \boldsymbol{X}^{-1}$ e $\hat{\boldsymbol{K}} = \boldsymbol{M}\,\boldsymbol{P}$.
Proof: See Teixeira *et al.* (2006).

Inequality (15) can be transformed into LMI. The Schur complement (Boyd et al., 1994) converts a class of nonlinear inequalities in linear matrix inequalities. Following is presented a controller design "sub-optimal" based on LMIs, which stabilizes the system and minimizes the upper bound of performance index based on the result of Theorem 1.

Theorem 2: Given matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ of system (9)-(10), and the initial condition $\boldsymbol{e}(0)$, then $\boldsymbol{X} = \boldsymbol{X}^{\mathrm{T}}$ and $\boldsymbol{M}$ matrices that allow determining the feedback gain that stabilizes the system and minimizes the upper bound of performance index J_1 can be obtained by solving the following LMIs:

$$\underset{\boldsymbol{X},\boldsymbol{M}}{\text{minimize}}\ \lambda$$

subject to

$$\begin{bmatrix} \lambda & \boldsymbol{e}^{\mathrm{T}}(0) \\ \boldsymbol{e}(0) & \boldsymbol{X} \end{bmatrix} > 0, \tag{18}$$

$$\boldsymbol{X} > 0, \tag{19}$$

$$\begin{bmatrix} \boldsymbol{A}\,\boldsymbol{X} + \boldsymbol{X}\,\boldsymbol{A}^{\mathrm{T}} - \boldsymbol{M}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} & \boldsymbol{X}\sqrt{\boldsymbol{Q}} & -\boldsymbol{M}^{T}\sqrt{R} \\ \sqrt{\boldsymbol{Q}}\boldsymbol{X} & -\boldsymbol{I} & \boldsymbol{0} \\ -\sqrt{R}\boldsymbol{M} & \boldsymbol{0} & -1 \end{bmatrix} < 0 \tag{20}$$

Of the solution of LMIs, the feedback gain can be obtained by the expression: $\hat{\boldsymbol{K}} = \boldsymbol{M}\boldsymbol{X}^{-1}$.

Then the performance index satisfies $J_1(u_e) < \boldsymbol{e}^{\mathrm{T}}(0)\,\boldsymbol{P}\,\boldsymbol{e}(0) < \lambda$, with $\boldsymbol{X} = \boldsymbol{P}^{-1}$.
Proof: See Teixeira *et al.* (2006).

Remark 1: From the definition in (7) e (8):

$$e(t) = \begin{bmatrix} \boldsymbol{x}_p(t) - \boldsymbol{x}_p(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix}, \quad e(0) = \begin{bmatrix} \boldsymbol{x}_p(0) - \boldsymbol{x}_p(\infty) \\ \xi(0) - \xi(\infty) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_0 \\ \xi_l \end{bmatrix}. \tag{21}$$

Remark 2: If the plant (2)-(3) has no transmission zeros at the origin, then (more detail, see Teixeira *et al.*, 2006), $\boldsymbol{x}_0$ is known and it is not a function of $\hat{\boldsymbol{K}}$ or ξ_l. Therefore ξ_l can be arbitrary chosen and offers a new degree of freedom in the design of the controller and the value can be chosen conveniently. So is the choice of a new degree of freedom in controller design. The problem of optimal controller design, to update the initial conditions of the controller, using LMIs defined below.

2.3 Specification of the Optimal Compensation of the Initial Conditions of the Controller

Consider the following problem:

Problem 1: Consider the system described by (9)-(10). Suppose that the reference input $r(t)$ (a step function with value equal to r_0) is applied in $t=0$: $r(t)=\begin{cases} r_0, & \text{to} \quad t\geq 0, \\ 0, & \text{to} \quad t<0, \end{cases}$
where r_0 is a known real constant. Determine ξ_{lo}, the compensation in $\xi_l(t)$ in $t=0$, such that there exists $\boldsymbol{P}=\boldsymbol{P}^T>0$, for the solution of the following LMIs:

(i) $\boldsymbol{P}(\boldsymbol{A}-\boldsymbol{B}\hat{\boldsymbol{K}})+(\boldsymbol{A}-\boldsymbol{B}\hat{\boldsymbol{K}})^{\mathrm{T}}\boldsymbol{P} < 0$;

(ii) the upper bound, $\boldsymbol{e}^{\mathrm{T}}(0)\boldsymbol{P}\boldsymbol{e}(0)$, of the performance function below is minimized:

$$J_1(u_e)=\int_0^{+\infty}\left(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{e}+u_e^{\mathrm{T}}R\,u_e\right)dt\,,$$

where $\boldsymbol{Q}$ is a symmetric and real positive definite (or semi positive definite) matrix and $\boldsymbol{R}$ is a real constant, and $R\geq 0$.

The solution proposed is shown in Theorem 3 below:

Theorem 3: Given the matrices and the system (9)-(10), and initial condition $\boldsymbol{e}_p(0)$, the Problem 1 has solution when the following LMIs are feasible:

$$\underset{\boldsymbol{X}_{11},\boldsymbol{X}_{12},\boldsymbol{X}_{22},\boldsymbol{M}_1,\boldsymbol{M}_2,\xi_l}{\textit{minimize}}\ \lambda$$

subject to

$$\begin{bmatrix}\boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}}\\ \boldsymbol{X}_{12} & \boldsymbol{X}_{22}\end{bmatrix}>0, \tag{22}$$

$$\begin{bmatrix}\lambda & \boldsymbol{e}_p^{\mathrm{T}}(0) & \xi_l\\ \boldsymbol{e}_p(0) & \boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}}\\ \xi_l & \boldsymbol{X}_{12} & \boldsymbol{X}_{22}\end{bmatrix}>0, \tag{23}$$

$$\begin{bmatrix}\boldsymbol{U}_{11} & \boldsymbol{U}_{12} & \boldsymbol{V}_{11} & \boldsymbol{V}_{12} & -\boldsymbol{M}_1^{\mathrm{T}}\sqrt{\boldsymbol{R}}\\ \boldsymbol{U}_{12}^{\mathrm{T}} & -\boldsymbol{X}_{12}\boldsymbol{C}_p^{\mathrm{T}}-\boldsymbol{C}_p\boldsymbol{X}_{12}^{\mathrm{T}} & \boldsymbol{V}_{21} & \boldsymbol{V}_{22} & -\boldsymbol{M}_2^{\mathrm{T}}\sqrt{\boldsymbol{R}}\\ \boldsymbol{V}_{11}^{\mathrm{T}} & \boldsymbol{V}_{12}^{T} & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{V}_{12}^{\mathrm{T}} & \boldsymbol{V}_{22}^{T} & \boldsymbol{0} & -1 & 0\\ -\sqrt{\boldsymbol{R}}\boldsymbol{M}_1 & -\sqrt{\boldsymbol{R}}\boldsymbol{M}_2 & \boldsymbol{0} & 0 & -1\end{bmatrix}<0 \tag{24}$$

where:

$\boldsymbol{V}_{11}=\boldsymbol{X}_{11}\boldsymbol{Q}_{11}+\boldsymbol{X}_{12}^{\mathrm{T}}\boldsymbol{Q}_{12}$, $\boldsymbol{V}_{12}=\boldsymbol{X}_{11}\boldsymbol{Q}_{12}^{\mathrm{T}}+\boldsymbol{X}_{12}^{\mathrm{T}}\boldsymbol{Q}_{22}$; $\boldsymbol{V}_{21}=\boldsymbol{X}_{12}\boldsymbol{Q}_{11}+\boldsymbol{X}_{22}\boldsymbol{Q}_{12}$; $\boldsymbol{V}_{22}=\boldsymbol{X}_{12}\boldsymbol{Q}_{12}^{\mathrm{T}}+\boldsymbol{X}_{22}\boldsymbol{Q}_{22}$, $\boldsymbol{M}=[\boldsymbol{M}_1 \quad \boldsymbol{M}_2]$

$\boldsymbol{U}_{11}=\boldsymbol{X}_{11}\boldsymbol{A}_p^{\mathrm{T}}+\boldsymbol{A}_p\boldsymbol{X}_{11}-\boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{B}_p-\boldsymbol{B}_p\boldsymbol{M}$; $\boldsymbol{U}_{12}=\boldsymbol{A}_p\boldsymbol{X}_{12}^{\mathrm{T}}-\boldsymbol{X}_{11}\boldsymbol{C}_p^{\mathrm{T}}-\boldsymbol{B}_p\boldsymbol{M}$; $\boldsymbol{X}=\begin{bmatrix}\boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}}\\ \boldsymbol{X}_{12} & \boldsymbol{X}_{22}\end{bmatrix}$, $\sqrt{\boldsymbol{Q}}=\begin{bmatrix}\boldsymbol{Q}_{11} & \boldsymbol{Q}_{12}^{\mathrm{T}}\\ \boldsymbol{Q}_{12} & \boldsymbol{Q}_{22}\end{bmatrix}$

$$\boldsymbol{e}_p(0)=\boldsymbol{x}_p(0)-\boldsymbol{x}_p(\infty)=\boldsymbol{x}_p(0)-[\boldsymbol{I}_n \quad \boldsymbol{0}]\begin{bmatrix}\boldsymbol{A}_p & \boldsymbol{B}_p\\ -\boldsymbol{C}_p & 0\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{0}\\ r_0\end{bmatrix}. \tag{25}$$

From the solution of these LMIs, the controller gain is obtained by: $\hat{\boldsymbol{K}}=\boldsymbol{M}\boldsymbol{X}^{-1}$. Furthermore, the optimal compensation of the initial condition in the controller, ξ_{lo}, is given by $\xi_{lo}=\xi_l+\xi(\infty)-\xi(0^-)$, where $\xi(0^-)$ is the initial condition of the controller before the compensation with:

$$\boldsymbol{e}(\infty)=\left[\boldsymbol{x}_p^{\mathrm{T}}(\infty) \quad \xi(\infty)\right]^{\mathrm{T}}=-\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}}r_0\,,\ \hat{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A}_p-\boldsymbol{B}_p\boldsymbol{K} & \boldsymbol{B}_pK_I\\ -\boldsymbol{C}_p & 0\end{bmatrix};\ \hat{\boldsymbol{B}}=\begin{bmatrix}\boldsymbol{0}\\ 1\end{bmatrix} \tag{26}$$

Proof: The LMIs (22)-(24) are obtained from the (18)-(20) rewritten:

$$\boldsymbol{X}=\begin{bmatrix}\boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}}\\ \boldsymbol{X}_{12} & \boldsymbol{X}_{22}\end{bmatrix},\ \boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}_p & \boldsymbol{0}\\ -\boldsymbol{C}_p & 0\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}\boldsymbol{B}_p\\ 0\end{bmatrix},\ \boldsymbol{C}=[\boldsymbol{C}_p \quad 0],\ \boldsymbol{e}(0)=\begin{bmatrix}\boldsymbol{e}_p(0)\\ \xi_l\end{bmatrix}\ \text{e}\ \boldsymbol{M}=[\boldsymbol{M}_1 \quad \boldsymbol{M}_2]. \tag{27}$$

The compensation value follows from the fact:

$$\xi_l=\xi(0)-\xi(\infty), \tag{28}$$

Define $\xi(0^-)$, the value of the initial condition of the controller before the compensation, then from (43c) the compensation value is given by:

$$\xi_{lo} = (\textit{optimal initial value}) - (\textit{initial value before the compensation}), \text{ that is },$$

$$\xi_{lo} = \xi_l + \xi(\infty) - \xi(0^-), \tag{29}$$

where $\xi(\infty)$ is obtained from $\boldsymbol{e}(\infty) = \begin{bmatrix} \boldsymbol{x}_p{}^{\mathrm{T}}(\infty) & \xi(\infty) \end{bmatrix}^{\mathrm{T}} = -\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}}r_0$.

3. Compensation of Initial Conditions on Controller with Others Indexes Performance

In this method, the following indices of performance are considered beyond the stability : the speed of response and restriction of input and output.

3.1 Restriction on Input

Assume that the initial condition of the plant is known. The restriction $\|u(t)\| < \mu$ is imposed on whole time if the LMIs :

$$\begin{bmatrix} 1 & \boldsymbol{x}^{\mathrm{T}}(0) \\ \boldsymbol{x}(0) & \boldsymbol{X} \end{bmatrix} \geq 0 \tag{30}$$

and

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{M} \\ \boldsymbol{M} & \mu^2 \boldsymbol{I} \end{bmatrix} \geq 0 \tag{31}$$

are satisfied, (see (Boyd et al., 1994)), with $\boldsymbol{X} = \boldsymbol{P}^{-1}$ e $\boldsymbol{M} = \hat{\boldsymbol{K}}\boldsymbol{X}$.

3.2 Restriction on Output

Assume that the initial condition of the plant is known. The restriction $\|y(t)\| < \lambda$ is imposed on whole time if the LMIs (30) and:

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{XC} \\ \boldsymbol{C}X & \lambda^2 \boldsymbol{I} \end{bmatrix} \geq 0, \tag{32}$$

are satisfied, (see (Boyd et al., 1994)), with $\boldsymbol{X} = \boldsymbol{P}^{-1}$.

3.3 Decay Rate

Consider a candidate Lyapunov function $V(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}$ and that $\dot{V}(x(t)) < 0$, for all $\boldsymbol{x} \neq \boldsymbol{0}$. The decay rate $\gamma > 0$, is obtained if the condition $\dot{V}(\boldsymbol{x}(t)) \leq -2\gamma V(\boldsymbol{x}(t))$ (see (Boyd et al., 1994)), is satisfied for any trajectory which is equivalent to:

$$\boldsymbol{AX} + \boldsymbol{XA}^{\mathrm{T}} - \boldsymbol{M}^{\mathrm{T}}\boldsymbol{B}^{T} - \boldsymbol{BM} + 2\gamma\boldsymbol{X} < 0. \tag{33}$$

The speed of response is related to the decay rate, ie with the largest Lyapunov exponent. A problem of stable controller design with constraint on input, output and rate of decay can be defined respectively by:

i) the restriction of input: Find $\boldsymbol{X}$, satisfying, (13), (30) and (31);
ii) the restiction of output: Find $\boldsymbol{X}$, satisfying, (13), (30) and (32);
iii) decate rate γ : Find, $\boldsymbol{X}$, satisfying $\boldsymbol{X} > 0$ and (33).

The problem of controller design with compensation the initial conditions of the controller, which simultaneously considers the constraints of input, output and decay rate is described below.

3.4 Specification Optimal of Initial Conditions in the Controller with others Indexes Performances

Problem 2: Consider the system described by (9)-(10). Suppose that the reference input $r(t)$ (a step function with value equal to r_0) is applied in $t = 0$: $r(t) = \begin{cases} r_0, & \text{to} \quad t \geq 0, \\ 0, & \text{to} \quad t < 0, \end{cases}$

where r_0 is a known real constant. Determine ξ_{lo}, the compensation in $\xi_l(t)$ in $t = 0$, so that the system is stable, $\|u_e(t)\| < \mu$, $\|y(t)\| < \lambda$ and the speed of the response decay γ, where μ , λ e γ are positive real constants. The solution proposed is presented in the following theorem:

Theorem 4: Given the matrices and the system (9)-(10), and initial condition $\boldsymbol{e}_p(0)$, the Problem 2 has solution when the following LMIs are feasible:

$$\underset{\boldsymbol{X}_{11},\boldsymbol{X}_{12},\boldsymbol{X}_{22},\boldsymbol{M}_1,\boldsymbol{M}_2,\xi_l}{\textit{maximize}}\ \gamma$$

subject to

$$\begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}} \\ \boldsymbol{X}_{12} & \boldsymbol{X}_{22} \end{bmatrix} > 0, \tag{34}$$

$$\begin{bmatrix} \boldsymbol{U}_{11} & \boldsymbol{U}_{12}^{\mathrm{T}} \\ \boldsymbol{U}_{12} & \boldsymbol{U}_{22} \end{bmatrix} > 0, \tag{35}$$

$$\begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}} & \boldsymbol{X}_{12}\boldsymbol{C}_p \\ \boldsymbol{X}_{12} & \boldsymbol{X}_{22} & \boldsymbol{X}_{11}\boldsymbol{C}_p^{\mathrm{T}} \\ \boldsymbol{C}_p^{\mathrm{T}}\boldsymbol{X}_{12}^{\mathrm{T}} & \boldsymbol{C}_p\boldsymbol{X}_{11} & \lambda^2 \end{bmatrix} > 0, \tag{36}$$

$$\begin{bmatrix} 1 & \boldsymbol{e}_p^{\mathrm{T}}(0) & \xi_l \\ \boldsymbol{e}_p(0) & \boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}} \\ \xi_l & \boldsymbol{X}_{12} & \boldsymbol{X}_{22} \end{bmatrix} > 0, \tag{37}$$

$$\begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}} & \boldsymbol{M}_1^{\mathrm{T}} \\ \boldsymbol{X}_{12} & \boldsymbol{X}_{22} & \boldsymbol{M}_2^{\mathrm{T}} \\ \boldsymbol{M}_1 & \boldsymbol{M}_2 & \mu^2 \end{bmatrix} > 0, \tag{38}$$

where:

$$\boldsymbol{U}_{11} = \boldsymbol{X}_{11}\boldsymbol{A}_p^{\mathrm{T}} + \boldsymbol{A}_p\boldsymbol{X}_{11} - \boldsymbol{M}_1^{\mathrm{T}}\boldsymbol{B}_p^{\mathrm{T}} - \boldsymbol{B}_p\boldsymbol{M}_1 + 2\gamma\boldsymbol{X}_{11}, \boldsymbol{U}_{12} = \boldsymbol{X}_{12}\boldsymbol{A}_p^{\mathrm{T}} - \boldsymbol{C}_p\boldsymbol{X}_{11} - \boldsymbol{M}_2^{\mathrm{T}}\boldsymbol{B}_p + 2\gamma\boldsymbol{X}_{12}$$

$$\boldsymbol{U}_{22} = -\boldsymbol{X}_{12}\boldsymbol{C}_p^{\mathrm{T}} - \boldsymbol{C}_p\ \boldsymbol{X}_{12}^{\mathrm{T}} + 2\gamma\boldsymbol{X}_{22},\quad \boldsymbol{M} = [\boldsymbol{M}_1 \quad \boldsymbol{M}_2];\ \hat{\boldsymbol{K}} = \boldsymbol{M}\boldsymbol{X}^{-1};\ \boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_{11} & \boldsymbol{X}_{12}^{\mathrm{T}} \\ \boldsymbol{X}_{12} & \boldsymbol{X}_{22} \end{bmatrix}. \tag{39}$$

Proof : analogous to the proof of theorem 3. From the solution of these LMIs, the controller gain is obtained by: $\hat{\boldsymbol{K}} = \boldsymbol{M}\boldsymbol{X}^{-1}$. Furthermore, the optimal compensation of the initial condition in the controller, ξ_{lo}, is given by $\xi_{lo} = \xi_l + \xi(\infty) - \xi(0^-)$, where $\xi(0^-)$ is the initial condition of the controller before the compensation .

4. APPLICATIONS EXEMPLES

4.1 Control of an Inverted Pendulum with Optimal Compensation on the Initial Conditions of the Controller

It is considered, as in Teixeira et alli (2002) the system inverted pendulum, the described by equations following (Ogata,1997):

$$\ddot{\theta}(t) = 20.601\ \theta(t) - u(t), \tag{40}$$

$$\ddot{y}(t) = 0.5u(t)\ -0.405\,\theta(t). \tag{41}$$

To control the position of the cart system with zero error for a step type input is made retroactively to the position signal (indicating the position of the cart) for the entry, and an integrator is inserted in the path of action ahead as shown in Figure 2.

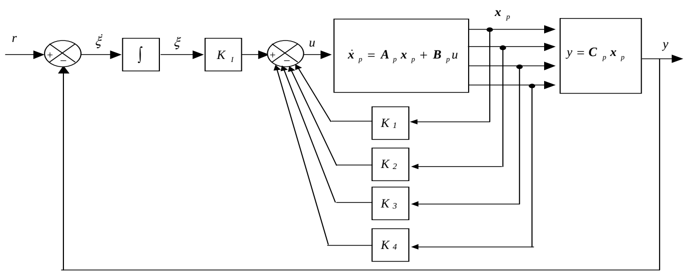

Figure 2: System of the control inverted pendulum.

Considering the definition of state variables as: $x_1 = \theta$; $x_2 = \dot{\theta}$; $x_3 = y$; $x_4 = \dot{y}$. Then, based on equations (40)-(41), and considering the position of the cart as the system output, are obtained the following equations

$$\dot{\boldsymbol{x}}_p(t) = \boldsymbol{A}_p\, \boldsymbol{x}_p(t) + \boldsymbol{B}_p\, u(t)\,, \tag{42}$$

$$y(t) = \boldsymbol{C}_p\, \boldsymbol{x}_p(t), \tag{43}$$

$$u(t) = -\boldsymbol{K}\boldsymbol{x}_p(t) + K_I\xi(t),\ \boldsymbol{K} = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix}, \tag{44}$$

$$\dot{\xi}(t) = r(t) - y(t) = r(t) - \boldsymbol{C}_p\boldsymbol{x}_p(t)\,, \tag{45}$$

where:

$$\boldsymbol{A}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4095 & 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{B}_p = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -0.5 \end{bmatrix},\ \boldsymbol{C}_p = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.$$

The performance index is described by (14) with $\boldsymbol{Q}$=diag*{100,1,1,1,0.01}* and *R=0.01*. The design problem of controller with optimal compensation of the initial conditions, using LMIs is: minimize λ and find $\boldsymbol{X}_{11} = \boldsymbol{X}_{11}^{\mathrm{T}}$, $\boldsymbol{X}_{12}$, $\boldsymbol{X}_{22} = \boldsymbol{X}_{22}^{\mathrm{T}}$, $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, ξ_I satisfying (22), (23) e (24), being:

$$\boldsymbol{x}(0^-) = \begin{bmatrix} \boldsymbol{x}_p(0)^{\mathrm{T}} & \xi(0^-) \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \quad \text{and} \quad \boldsymbol{e}_p(0) = \boldsymbol{x}_p(\boldsymbol{0}) - \boldsymbol{x}_p(\infty) = \begin{bmatrix} 0 & 0 & -1 & 0 \end{bmatrix}.$$

From the solution obtained with the software LMISol, one has λ, ξ_I and the matrices $\boldsymbol{X}_{11}$, $\boldsymbol{X}_{12}$, $\boldsymbol{X}_{22}$, $\boldsymbol{M}_1$, $\boldsymbol{M}_2$ that solve the LMIs . The solution is given:

$$\hat{\boldsymbol{K}} = \boldsymbol{M}\boldsymbol{X}^{-1} = \begin{bmatrix} -123.40 & -20.67 & -11.44 & -18.00 & 1.29 \end{bmatrix},\ \xi_I = -2.2597 \quad , \quad \lambda = 15.7488\,.$$

Therefore, $\boldsymbol{K} = \begin{bmatrix} -123.40 & -20.67 & -11.44 & -18.01 \end{bmatrix}, K_I = -1.2971$. So,

$$\hat{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_p - \boldsymbol{B}_p\boldsymbol{K} & \boldsymbol{B}_p K_I \\ -\boldsymbol{C}_p & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -102.8034 & -20.6700 & -11.442 & -18.0065 & 1.2971 \\ 0 & 0 & 0 & 1 & 0 \\ 61.2117 & 10.3350 & 5.7221 & 9.0033 & -0.6486 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix},\quad \hat{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix}, \tag{46}$$

$$e(\infty) = -\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}}r_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 8.8228 \end{bmatrix}^{\mathrm{T}} \text{ e } \ \xi(\infty) = 8.8228\,.$$

Since, $\xi(0^-) = 0$, we obtained from (29), the optimal compensation initial conditions of the controller as:

$$\xi_{Io} = \xi_I + \xi(\infty) - \xi(0^-) = 6.5631\,.$$

The theorem 3 solved Problem 1, it providing simultaneously the gain $\hat{\boldsymbol{K}}$, “sub-optimal”, $\hat{\boldsymbol{K}} = [\boldsymbol{K}\ \ -\mathrm{K_I}]$ and ξ_I obtain the optimal compensation of the controller , ξ_{Io}, $\xi_{Io} = \xi_I + \xi(\infty) - \xi(0^-)$. Note that this optimal value depends of the $\hat{\boldsymbol{K}}$ because, $\xi(\infty) = -\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}}r_0$ with matrices $\hat{\boldsymbol{A}}$ e $\hat{\boldsymbol{B}}$ of the equation (46). Fig. 3 presents the output *y(t)* and the control law *u(t)* of the feedback system with the initial conditions compensation on the controller, when a unit step input is applied. The performance index was obtained: J_1=2.0984.

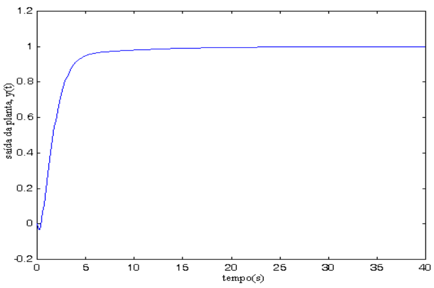

Figure 3: Unit step response for the inverted pendulum system with optimum compensation of the initial conditions.

4.2 Control of an Inverted Pendulum with Compensation for Initial Conditions in the Controller with Restriction in the Input, Output and in the Rate of Decay

We considered the problem of inverted pendulum described by equations (42) to (45). The gains of the controller were designed so that: *Maximize* γ and find the symmetric matrices $\boldsymbol{X}_{11}$, $\boldsymbol{X}_{12}$, $\boldsymbol{X}_{22}$ and matrices $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, ξ_l satisfying (34), (35), (36), (37), (38), with $\mu = 14$, $\lambda = 3$ e $\boldsymbol{x}_p = [0 \quad 0 \quad 0 \quad 0]^T$.

When $\gamma = 1.1 = \gamma_{\max}$, we obtain with software LMISol, the matrices that verify the LMIs above :

$$\boldsymbol{X} = \begin{bmatrix} 0.6329 & -1.6757 & 0.5320 & -1.5064 & 0.1874 \\ -1.6757 & 7.5055 & 0.3587 & -0.4847 & 0.2493 \\ 0.5320 & 0.3587 & 8.3880 & -9.2799 & 6.1487 \\ -1.5064 & -0.4847 & -9.2799 & 15.1079 & -5.2064 \\ 0.1874 & 0.2493 & 6.1487 & -5.2064 & 5.5277 \end{bmatrix}, \ \boldsymbol{M} = [-1.7676 \quad 4.9742 \quad -113922 \quad 35.0689 \quad -4.6893] .$$

Therefore, are obtained $\hat{\boldsymbol{K}} = \boldsymbol{M}\boldsymbol{X}^{-1} = [-61.3069 \quad -13.2682 \quad -10.6459 \quad -8.9509 \ 5.2393]$, $\xi_l = -1.5497$. Figure 4 presents the output *y(t)* and the control law *u(t)* of the feedback system, to a unit step input. It can be verified that $\|y(t)\| = 1.004 < \lambda$ e $\|u(t)\| = 2.6569 < \mu$ **.**

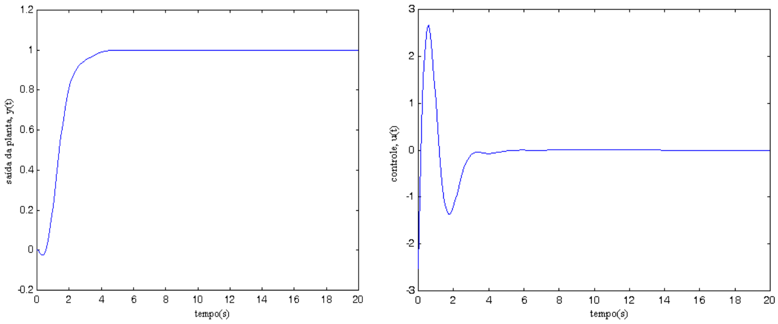

Figure 4: Unit step response and control law for inverted pendulum system with compensation of the initial conditions on the controller, restrictions on output and input and maximum rate of decay.

4.3 Optimal Compensation of Initial Conditions in a Reactor Controller with Agitation (RA)

Consider that in an (RA) with a liquid phase isothermal and multicomponent chemicals react according to the nonlinear dynamic model described by (Scarratt et al., 2000):

$$\dot{x}_1(t) = 1 - (1 + D_{a_1}) x_1(t) + D_{a_2} x_2^2(t) , \tag{47}$$

$$\dot{x}_2(t) = D_{a_1} x_1(t) - x_2(t) - (D_{a_2} + D_{a_3}) x_2^2(t) + u(t) \tag{48}$$

$$\dot{x}_3(t) = D_{a_3} x_2^2(t) - x_3(t) , \tag{49}$$

$$y(t) = x_3(t) , \tag{50}$$

where $x_i(t) > 0$, $i = 1,2,3$ to $t \geq 0$ and represented :

$x_1(t)$: normalized concentration C_A / C_{AF} of the specie A,
$x_2(t)$: normalized concentration C_B / C_{AF} of the specie B,
$x_3(t)$: normalized concentration C_C / C_{AF} of the specie C,
C_{AF} : steady regime of the specie A (mol.m^{-1}), $u(t)$: control signal,
and parameters : $r_0 = 0.7737$, $D_{a_1} = 3.0$, $D_{a_2} = 0.5$, $D_{a_3} = 1.0$. (51)

The system (47)-(50) has the following equilibrium point (x_e, u_e):

$$x_e = [0.3467 \quad 0.8796 \quad 0.7737], \ u_e(t) = 1. \tag{52}$$

Want to design a controller so that the output reaches and remains in the value r_0 = 0.7737 and that:

$$0.25 \leq x_1(t) \leq 0.4 ; \quad 0.8 \leq x_2(t) \leq 1.1 ; \quad 0.5 \leq x_3(t) \leq 0.8 . \tag{53}$$

Then the system (47)-(50) will be described in other coordinates to that the linearized system has no transmission zeros at the origin. Set

$$\tilde{x}_2(t) = x_2^2(t) , \tag{54}$$

$$u_N(t) = u(t) - x_2(t) . \tag{55}$$

Now,

$$\dot{\tilde{x}}_2(t)=2x_2(t)\dot{x}_2(t)=2x_2(t)\left[D_{a_1}x_1(t)-x_2(t)-\left(D_{a_2}+D_{a_3}\right)x_2^2(t)+u(t)\right]=2x_2(t)\left[D_{a_1}x_1(t)-\left(D_{a_2}+D_{a_3}\right)\tilde{x}_2(t)+u_N(t)\right] \quad (56)$$

Therefore from (54) – (55), the system (47) – (50) can be represented by:

$$\dot{x}_1(t)=1-\left(1+D_{a_1}\right)x_1(t)+D_{a_2}\tilde{x}_2(t), \quad (57)$$

$$\dot{\tilde{x}}_2(t)=2D_{a_1}x_1(t)\sqrt{\tilde{x}_2(t)}-2x_2(t)\left(D_{a_2}+D_{a_3}\right)\tilde{x}_2(t)+2\sqrt{\tilde{x}_2(t)}\,u_N(t), \quad (58)$$

$$\dot{x}_3(t)=D_{a_3}\tilde{x}_2(t)-x_3(t), \quad (59)$$

$$y(t)=x_3(t). \quad (60)$$

Considering,

$$\Delta x_1(t)=x_1(t)-0.25 \quad \text{e} \quad \Delta u_N(t)=D_{a_1}0.25+u_N(t) \quad (61)$$

We obtain from (57):

$$\Delta\dot{x}_1(t)=-\left(1+D_{a_1}\right)\Delta x_1(t)+D_{a_2}\tilde{x}_2(t). \quad (62)$$

Finally the nonlinear system (47) - (50) in the variables $\Delta x_1(t)$, $\tilde{x}_2(t)$ e $x_3(t)$ is described as:

$$\Delta\dot{x}_1(t)=-\left(1+D_{a_1}\right)\Delta x_1(t)+D_{a_2}\tilde{x}_2(t) \quad (63)$$

$$\dot{\tilde{x}}_2(t)=2D_{a_1}\sqrt{\tilde{x}_2(t)}\,\Delta x_1(t)-2\sqrt{\tilde{x}_2(t)}\left(D_{a_2}+D_{a_3}\right)\tilde{x}_2(t)+2\sqrt{\tilde{x}_2(t)}\,\Delta u_N(t), \quad (64)$$

$$\dot{x}_3(t)=D_{a_3}\tilde{x}_2(t)-x_3(t), \quad (65)$$

$$y(t)=x_3(t). \quad (66)$$

Therefore of (54)-(60) and considering the values of the parameters adopted D_{a_1}, D_{a_2} e D_{a_3} in (51):

$$\begin{bmatrix}\Delta\dot{x}_1(t)\\ \dot{\tilde{x}}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}-4 & 0.5 & 0\\ 6\sqrt{\tilde{x}_2(t)} & -3\sqrt{\tilde{x}_2(t)} & 0\\ 0 & 1 & -1\end{bmatrix}\begin{bmatrix}\Delta x_1(t)\\ \tilde{x}_2(t)\\ x_3(t)\end{bmatrix}+\begin{bmatrix}0\\ 2\sqrt{\tilde{x}_2(t)}\\ 0\end{bmatrix}\Delta u_N(t). \quad (67)$$

From the equations (54)–(55), (61), (52) and defining $\Delta_p\boldsymbol{x}(t)=\left[\Delta x_1(t) \quad \tilde{x}_2(t) \quad x_3(t)\right]$, we obtain that equilibrium point of nonlinear system (67), $\left(\Delta_p x(t),\Delta u_N\right)=\left(\Delta_p\boldsymbol{x}_e,\Delta u_e\right)$, is

$$\Delta_p\boldsymbol{x}_e=\left[0.096 \;\; 0.7737 \;\; 0.7737\right], \qquad \Delta u_e=0.8704. \quad (68)$$

From (68), (54), (55) e (60):

$$0\le\Delta x_1(t)\le 0.15\,;\;\; 0.64\le\tilde{x}_2(t)\le 1.21\,;\; 0.5\le x_3(t)\le 0.8 \quad (69)$$

Locally, the non-linear system (67) can be approximated by truncation of the representation by the Taylor series expansion around the equilibrium point:

$$\Delta_p\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_p\Delta_p\boldsymbol{x}(t)+\boldsymbol{B}_p\Delta u_N \quad \text{being} \quad \boldsymbol{A}_p=\begin{bmatrix}-4 & 0.5 & 0\\ 5.2776 & -2.6413 & 0\\ 0 & 1 & -1\end{bmatrix}; \quad \boldsymbol{B}_p=\left[0 \quad 1.75 \quad 0\right]^{\mathrm{T}}.$$

To design the controller so that the output of the system remains in the value r_0 = 0.7737 is considered the augmented system:

$$\begin{cases}\Delta_p\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_p\Delta_p\boldsymbol{x}(t)+\boldsymbol{B}_p\Delta u_N\\ \dot{\xi}(t)=r(t)-\boldsymbol{C}_p\Delta_p\boldsymbol{x}(t)\end{cases} \quad (70)$$

The performance index of the system is described in (14) being: $\boldsymbol{Q}$=diag*{100,1,0,0.01}* and *R=0.01*. The design of the controller to compensation the initial conditions, using LMIs, is: minimize λ and find $\boldsymbol{X}_{11}=\boldsymbol{X}_{11}^{\mathrm{T}}$, $\boldsymbol{X}_{12}$, $\boldsymbol{X}_{22}=\boldsymbol{X}_{22}^{\mathrm{T}}$, $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, ξ_l satisfying (22), (23) e (24), with:

$$\boldsymbol{x}(0^-)=\left[\boldsymbol{x}_p(0)^{\mathrm{T}} \quad \xi(0^-)\right]^{\mathrm{T}}=\left[0 \quad 0.64 \quad 0.5 \quad 0\right]^{\mathrm{T}},\; \boldsymbol{e}_p(0)=\boldsymbol{x}_p(\boldsymbol{0})-\boldsymbol{x}_p(\infty)=\left[-0.0967 \quad -0.1337 \quad -0.2737\right]^{\mathrm{T}}.$$

The *software LMISol* provided λ, ξ_l and the matrices $\boldsymbol{X}_{11}$, $\boldsymbol{X}_{12}$, $\boldsymbol{X}_{22}$, $\boldsymbol{M}_1$, $\boldsymbol{M}_2$ that solve the LMIs above . The solution obtained was: $\hat{\boldsymbol{K}}=\boldsymbol{M}\boldsymbol{X}^{-1}=\left[13.5763 \quad 1.5615 \quad 1.4874 \quad -0.4202\right]$; $\xi_l=-0.2336$, b, and so

$$\hat{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_p - \boldsymbol{B}_p \boldsymbol{K} & \boldsymbol{B} K_I \\ -\boldsymbol{C}_p & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0.50 & 0 & 0 \\ -18.6518 & -5.3727 & -2.6029 & 0.7354 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \hat{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix},$$

$$\boldsymbol{e}(\infty) = -\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}} r_0 = [0.0967 \quad 0.7737 \quad 0.7737 \quad 10.8446]^{\mathrm{T}}, \ \xi(\infty) = 10.8446.$$

Therefore, considering $\xi(0^-) = 0$, from (29), the optimum compensation of the initial conditions of the controller is : $\xi_{Io} = \xi_I + \xi(\infty) - \xi(0^-) = 10.6110$. Figure 5 presents the output y(t) and the control law u(t) of the feedback system, to a input step of amplitude $r_0 = 0.7737$, with compensation of the initial condition, considering $\boldsymbol{x}_p(0) = [0 \ \ 0.64 \ \ 0.5]^{\mathrm{T}}$.

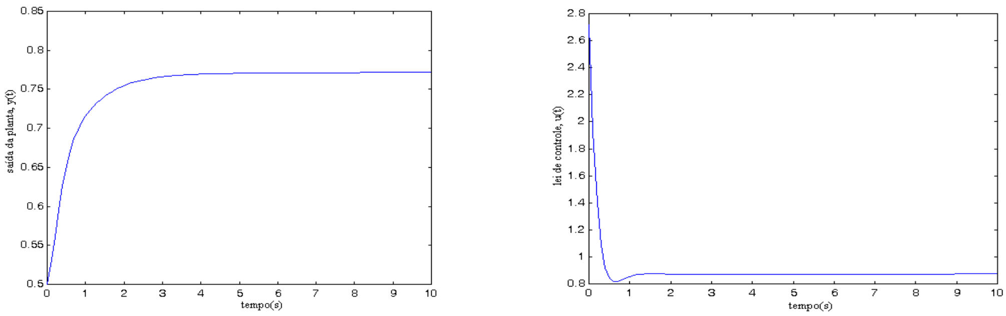

Figure 5: Step response of amplitude r_0=0.7737 , and of the control signal of the system (RA) with optimum compensation of the initial condition , considering $x_p(0)=[0 \ \ 0.64 \ \ 0.5]^{\mathrm{T}}$.

5. CONCLUSIONS

This article presents methods for optimal updating of the initial conditions on the controller, in the case considered the integral type, based on LMIs. The first method is the design of optimal quadratic regulators, in which the controller is designed to minimize the performance index. This is an alternative method (Teixeira *et al.*, 2006) to update the initial conditions because it takes an approximate optimal value of the update. The method finds the exact value was presented in Teixeira *et al.*, (2002). The second method is presented when we consider further restrictions on the project, for example, restrictions on output and input constraints, rate of decay. To our knowledge, the optimal updating of the initial conditions with the use of LMIs, while other restrictions are considered had not been made. This upgrade method using LMIs, provides a way to study the update of the initial conditions for nonlinear systems.

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