HIDROKINETIC TURBINE DESIGN ADAPTED TO LOW SPEEDS

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Abstract.*An alternative source of renewable energy is the using ofhorizontal-axis hydrokinetic turbines, because in contrast to the hydroelectric dam, the hydrokinetic turbine does not require the construction of barrages, thus avoiding the flooding of large areas, using the kinetic energy of flowing rivers instead of potential energy. This work aims to design a hydrokinetic turbine with a diffuser from an extension of the Bussel model using a new correction for the high values of the axial induction factor on the rotor plane.*

Keywords:*Hydrokinetic Turbine, Renewable Energy,Bussel Model.*

1. INTRODUCTION

The Amazon area has the greatest world hydrological potential and there is a big part of it in the Brazilian territory and it has the Amazon as its main river. It is the second biggest river in extension on the world and it is the biggest river of the world in water flow, it is believed that several locations in Amazon have a good potential to the deployment of hydrokinetic turbines, even though/although the average speeds are usually very low to the installation (Brazil-Junior et al., 2006), which shows the need of the development of efficient models for the design of horizontal-axis hydrokinetic turbines.

Thus, this paper presents a new mathematical model that extends the Bussel model (1999) to the project of hydrokinetic turbine with diffusers. In the proposed model it is described a new mathematical relation to the correction of high values of the axial induction factor, besides, it presents an approach to calculate the power coefficient as a function of the rotor geometry.

The results are compared to the Glauert classic model (1935), where it is possible to verify an extrapolation of the Betz limit (1926) due to the diffuser influence.

2. MATHEMATICAL MODEL FOR HYDROKINETIC TURBINE DESIGN WITH DIFFUSER

The Betz limit (1926) may be exceeded when a horizontal-axis turbine is placed in a diffuser, since the flow inside the diffuser shows an increase in the speed ratio in the rotor plane due to the suction pressure caused in the output diffuser (Rodrigues, 2007, Hansen et al. 2000; Bussel, 1999). Figure 1 illustrates the flow through a hydrokinetic turbine with diffuser conical.

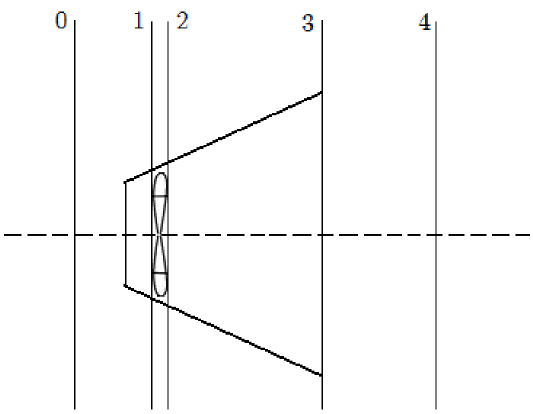


Figure 1: Simplified diagram of the speeds in the rotor plane, inside the diffuser and outside the diffuser.

A formulation that relates the total pressure ptot and the speeds at the 0 and 3 (Fig. 1), considering the diffuser without rotor, it is established through the energy conservation equation to a non viscous flow.

$$p_{tot} = p_0 + \frac{1}{2}\rho V_0^2 = p_3 + \frac{1}{2}\rho V_3^2 \qquad (1)$$

Where p0, p3, V0 and V3 are the pressures and speeds that correspond to 0 and 3 flow positions, respectively, and ρ is the fluid density. Using the continuity equation, it is possible to establish a relation between the flow speed at a position 1 and the flow speed at the diffuser exit, at position 3.

$$V_1 = nV_3 \qquad (2)$$

Where n is the relation between the areas of the diffuser exit and the area at position 1. The total pressure, therefore, can be obtained by doing:

$$p_{tot} = p_1 + \frac{1}{2}\rho\left(nV_3\right)^2 \qquad (3)$$

Bussell (1999) proposed the need to take into account the pressure loss that exists due to the diffuser presence. Thus, Bussel (1999) introduced a parameter, γ, which considers the pressure loss, given by the relation between speeds V3 and V0.

$$\gamma = \frac{V_3}{V_0} \qquad (4)$$

The parameter γ can be written as a function of pressure loss at the diffuser exit $\tilde{C}_p$ from the energy balance given by Eq. (1), which results in:

$$\gamma = \sqrt{1 - \tilde{C}_p} \qquad (5)$$

Where:

$$\tilde{C}_p = \frac{p_3 - p_0}{\frac{1}{2}\rho V_0^2} \qquad (6)$$

Therefore, after the substitution Eq. (4) in Eq. (3), we have:

$$p_{tot} = p_1 + \frac{1}{2}\rho\left(n\gamma V_0\right)^2 \qquad (7)$$

Connecting Eq. (1) and Eq. (7), we have:

$$p_1 = p_0 + \frac{1}{2}\rho\left(1 - n^2\gamma^2\right)V_o^2 \qquad (8)$$

It will be applied the continuity equation between the positions along the flow (Fig. 1) to determine the diffuser speeds. Therefore, from Eq. (2) and Eq. (4), we have:

$$V_1 = n\gamma V_0 \qquad (9)$$

Equation (9) shows that the flow speed without the turbine, at position 1, considers the effects caused by the diffuser geometry and the pressure loss, respectively, through the parameters n and γ. Bussell (1999) proposed that in position 4 of the flow, to the turbine with diffuser, the speed is given by the relation described in the Glauert model (1935), considering the mat formed after the diffuser, where it no longer exists the influence of geometry diffuser. Thus, the flow velocity V4 is given by:

$$V_4 = \left(1 - 2a\right)V_0 \qquad (10)$$

At position 3, the speed is given by:

$$V_3 = g(1 - a)V_0 \tag{11}$$

Equation (11) considers the pressure loss g at the exit of the diffuser and interference caused by the turbine through the term (1 - a), since the flow loses kinetic energy caused by the energy conversion imposed by the rotor shaft.

Using Eq. (2) and Eq. (11) it is possible to establish a relation to the rotor plane speed under the influence of diffuser.

$$V_1 = V_2 = ng(1 - a)V_0 \tag{12}$$

Bussel (1999) considers that entry diffuser speed has no influence in pressure loss. Therefore, the speed V can be given by:

$$V = n(1 - a)V_0 \tag{13}$$

It only has influence of the diffuser cross section geometry. It is used the energy equation between positions 3 and 4 to determine the pressures of the turbine with diffuser.

$$p_3 + \frac{1}{2}rV_3^2 = p_4 + \frac{1}{2}rV_4^2 \tag{14}$$

Using Eq. (10), where pressure p4= p0, we have:

$$p_3 = p_0 + \frac{1}{2}r\left[(1 - 2a)^2 - g^2(1 - a)^2\right]V_0^2 \tag{15}$$

Then the pressure p2 is determined from:

$$p_2 + \frac{1}{2}rV_2^2 = p_4 + \frac{1}{2}rV_4^2 \tag{16}$$

Using Eq. (10) and Eq. (12), we have:

$$p_2 = p_0 + \frac{1}{2}r\left[(1 - 2a)^2 - n^2g^2(1 - a)^2\right]V_0^2 \tag{17}$$

The pressure p1 is determined from:

$$p_1 + \frac{1}{2}rV_1^2 = p_0 + \frac{1}{2}rV_0^2 \tag{18}$$

It results in:

$$p_1 = p_0 + \frac{1}{2}r\left[1 - n^2g^2(1 - a)^2\right]V_0^2 \tag{19}$$

Therefore, the pressure difference on the rotor is:

$$p_1 - p_2 = \frac{1}{2}r\left[4a(1 - a)\right]V_0^2 \tag{20}$$

This way, the thrust coefficient is given by:

$$C_E = \frac{(p_1 - p_2)A}{\frac{1}{2}\rho A V_0^2} = 4a(1-a) \tag{21}$$

2.1. Correction to a High Values

The main limitation of the Bussel model (1999) is not to establish a relation between the calculation of the power coefficient and the geometry of turbine with diffuser, besides it does not provide a correction for the high values of axial induction factor. Note that Eq. (21) corresponds to the same formulation obtained in the actuator disk theory (Alves, 1997). Therefore, it is used Eq. (22) to establish a correction for the a high values, the equation was obtained from Hansen work (2008).

$$C_E = \begin{cases} 4a(1-a)F & ; a \le \frac{1}{3} \\ 4a\left[1 - \frac{a}{4}(5-3a)\right]F & ; a > \frac{1}{3} \end{cases} \tag{22}$$

Once it is known the rotor plane speeds and the speeds triangle of Fig. 2, it is possible to determine a formulation for the calculation of thrust coefficient on the turbine, which relates the rotor geometry and the parameters of pressure loss and the diffuser geometry, once the speed V1 in the rotor plane is known.

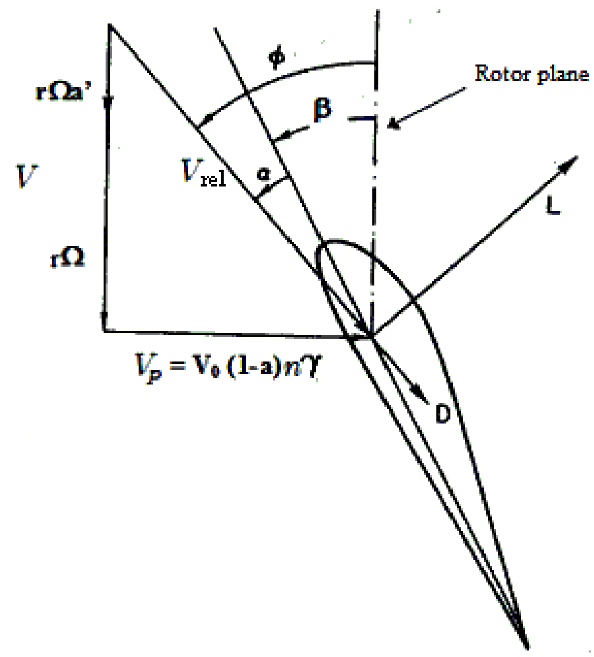


Figure2:Triangle speedand force onthe blade section.

$$C_E = \left(\frac{V_1}{V_0}\right)^2 \frac{\sigma C_n}{sen^2 \phi} = n^2 \gamma^2 (1-a)^2 \frac{\sigma C_n}{sen^2 \phi} \tag{23}$$

Where σ is the strength in a blade section and Cn is the coefficient of the force that is normal to the rotor plane. If we match Eq. (22) and Eq. (23), we have an equation system to the calculation of axial induction factor with correction to the values a > ac. The parameter ac is 0.2, and it can be verified in Spera work (2009).

$$a = \begin{cases} \dfrac{1}{1+K}; & a \le a_c \\ \dfrac{1}{2}\left\{2 + K(1-2a_c) - \sqrt{\left[K(1-2a_c)+2\right]^2 + 4\left(Ka_c^2 - 1\right)}\right\}; & a > a_c \end{cases} \qquad (24)$$

Where

$$K = \frac{4F sen^2\phi}{\nu^2\gamma^2\sigma C_n} \qquad (25)$$

The flow angle ϕ is easily obtained from the speeds triangle in Fig. 2:

$$\tan\phi = \frac{\nu\gamma(1-a)V_0}{(1+a')\Omega r} \qquad (26)$$

Where Ω is the turbine angular speed. It was applied the moment conservation equation and the mass equation to calculate the tangential induction factor a’, as it is described in Hansen work, 2008, it results in:

$$a' = \frac{\nu\gamma(1-a)V_0\sigma C_t}{4\Omega r F sen^2\phi} \qquad (27)$$

Where Ct is the coefficient of the force that is tangential to the rotor plane. It can be noticed that both the flow angle and the tangential induction factor depend on the diffuser and the pressure loss, which can be seen in Eq. (26) and Eq. (27). The power coefficient is obtained from the analysis about the fact that control volume cancels each blade section, where the power is given by:

$$dP = 4\pi r^3\rho\Omega^2 V_1 a' dr \qquad (28)$$

If we substitute Eq. (12) into Eq. (28), we have:

$$dP = 4\pi r^3\rho\Omega^2\nu\gamma(1-a)V_0 a' dr \qquad (29)$$

Finally, the power coefficient.

$$Cp = \frac{P}{\frac{1}{2}\rho A V_0^3} = \frac{\int 4\pi r^3\rho\Omega^2\nu\gamma(1-a)V_0 a' dr}{\frac{1}{2}\rho A V_0^3} = \frac{8}{X^2}\int_0^X \nu\gamma a' F(1-aF)x^3 dx \qquad (30)$$

Where

$$X = \frac{\Omega R}{V_0} \qquad (31)$$

and

$$x = \frac{\Omega r}{V_0} \qquad (32)$$

The turbine power is determined from:

$$P = \frac{1}{2} r A Cp V_0^3 \quad (33)$$

The power coefficient, locally, can be determined from:

$$Cp_{local} = \frac{(p_1 - p_2)V_1}{\frac{1}{2} r V_0^3} = n g 4a(1 - a)^2 \quad (34)$$

The iterative procedure for the calculation of induction factors considers the known parameters n, g , r, c(r), β(r), CL(α), CD(α) and V0, and it is described below:

Step	Procedure
1	Initial values are assigned to a e a', in this paper a = 1/ 3 and a ' = 0;
2	Calculate the value of ϕ with Eq. (26);
3	CL and CP are determined from $a = f - b$;
4	Calculate a and a' with Eq. (24) and Eq. (26);
5	Verify if there is convergence to a e a' values, if the tolerance is not achieved, you must return to step 2.

3. RESULTS AND DISCUSSIONS

Figure 3 shows a comparison between the speed rate along the flow that goes through a turbine with diffuser and without diffuser, using the proposed model to the speeds calculation. It can be noticed that the diffuser effect causes an increase in speed rate, achieving values around 1.6 between positions 1-2 of the diffuser. The curve that corresponds to the free-flow rotor decreases to 0.66 when in crosses the turbine, it indicates that the speed in the plane of rotor with diffuser increases 2.4 times, compared to the speed in a turbine without diffuser. This fact shows that the suction region caused at the diffuser exit increases the mass flow at rotor plane, overcoming the Betz limit (1926), which is 59.26%.

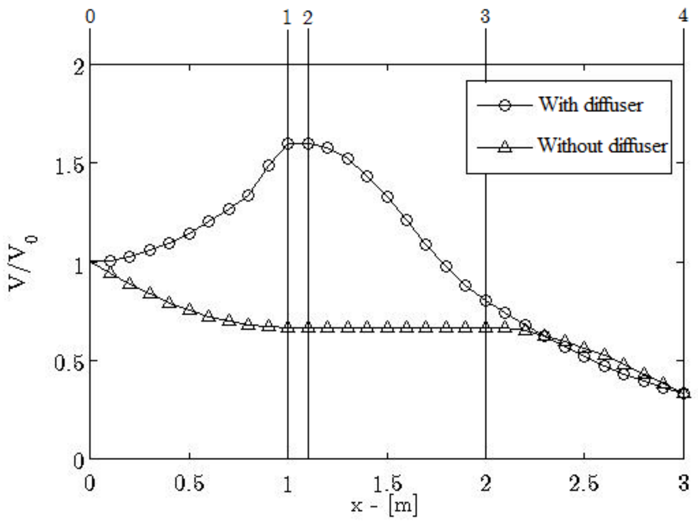


Figure 3: Speed rate along the flow to a turbine with diffuser and without diffuser.

It is noteworthy that the speeds were calculated only in the positions shown in Fig. 3. In the intervals among positions 0-1, 2-3 and 3-4 the data was interpolated in this work. The proposed model calculates speeds rate values in positions 1 and 2, where the turbine must be positioned, since in these positions are the highest flow speeds inside the diffuser. These results are similar to those obtained by Abe and Ohya (2004) with wind turbines positioned in flanged diffusers, where the speed rate in turbine plane is 1.5. Other studies with diffusers can also be verified in Abe et al. (2005) and Ohya and Karasudani (2010) works. It was considered an axial induction factor to a great turbine, a = 1/3, an areas ratio n = 2 and a pressure loss factor γ = 1.2.

Figure 4 shows the local power coefficient, Eq. (34), related to the thrust coefficient, Eq. (21). It is feasible that the presence of the diffuser causes a significant increase in the turbine power coefficient. The power coefficient to a turbine without diffuser is determined from Eq. (34), when n =1 and γ= 1.

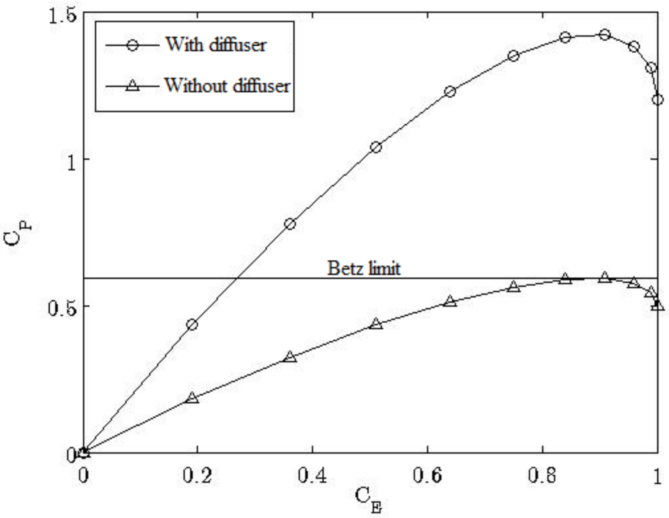


Figure 4: Local power coefficient related to the thrust coefficient.

There is a big difficulty in finding experimental data in the literature, as well as other models that relate the calculation of power coefficient with the turbine geometry that has clear information about the project parameters (rope distribution, torsion angle and lift and drag coefficients) so then it is possible to evaluate the proposed model. However, in this paper it will be done an application of the model to a hydrokinetic turbine with the follow characteristics:

Rotor Diameter = 0.60 m; Cube Diameter = 0.15 m;
Diffuser Exit Diameter = 1.0 m;
Blades Number = 4;
Water Density = 997 Kg/m3 at 25 oC;
Turbine Spins = 60, 80, 100 e 120 rpm;
Hydrodynamic Profile = NACA 0012;

In this paper it was considered the coefficients of lift and drag of the NACA 0012 profile (Abbot and Doenhoff, 1959) for a Reynolds number 3,106. The rotor blade geometry was estimated and it is shown in Fig. 5.

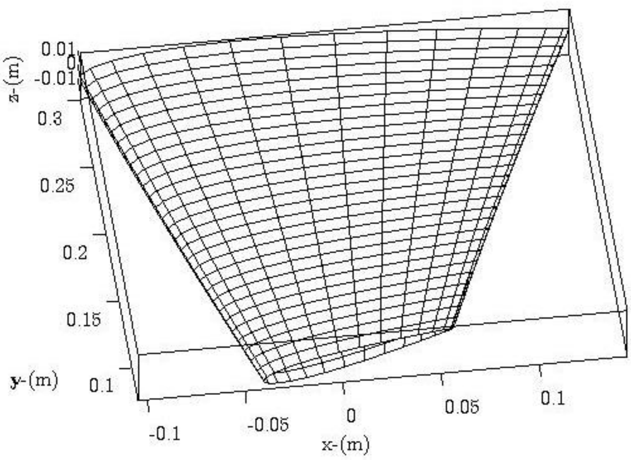
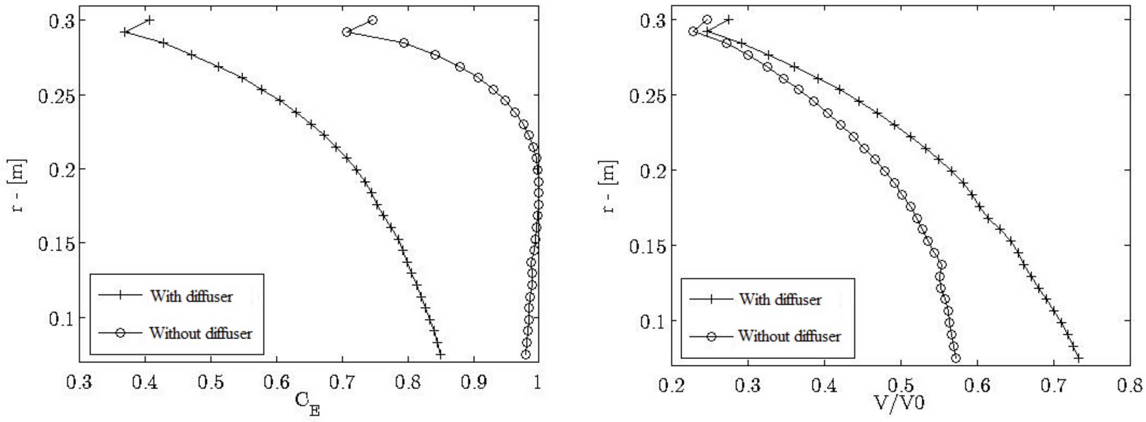


Figure 5: 3D blade used in the project.

The results are compared with the classical Glauert model (1935) without diffuser. Figure 6a shows the behavior of the local thrust, given by Eq. (23) on the rotor with and without diffuser. It is observed that in the case of the turbine with a diffuser, the thrust is lower when compared with the rotor without diffuser. This effect is justified because of the decrease of movement inside the diffuser. This has been shown by Hansen et al. (2000) through a study using CFD in a turbine with keel diffuser with NACA 0015 profile. It is noteworthy that the effect of movement inside the diffuser is harmful to the turbine performance.

However, the circulation on the diffuser exit improves the turbine efficiency, as shown in Gaden (2007), where the main effect of movement at the diffuser exit is the increasing of the mass flux in the rotor plane, with a consequent increase in the flow rate inside the diffuser.

Figure 6 shows that the ratio of the rotor blade speed tends to be bigger in the presence of a diffuser. It evidences the effect of the increasing in speed rate in suction area that happens in diffuser exit.

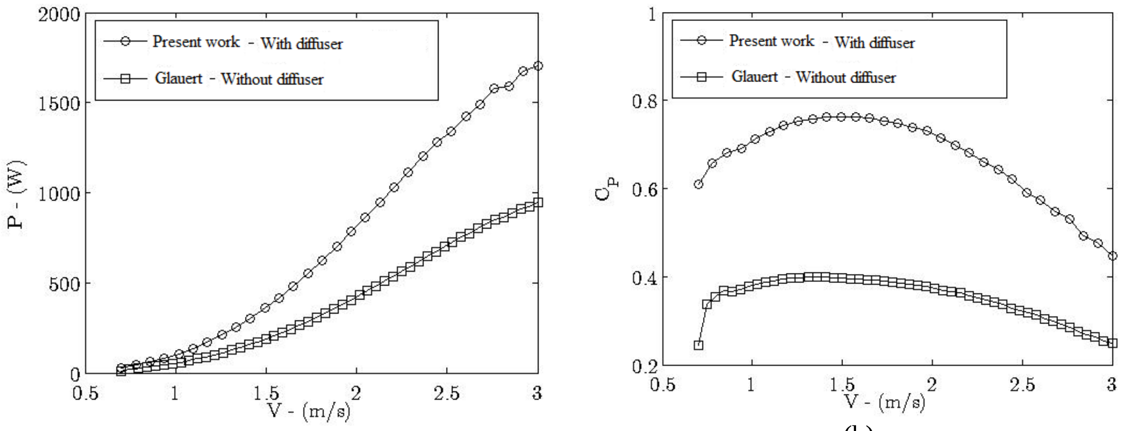


(a) (b)

Figure 6: (a) Thrust coefficient profile on the rotor; (b) Speed rate on rotor blade.

Figure 7 shows power variation developed by the turbine as a function of the flow axial speed. In this case, the power calculated to the turbine with diffuser is approximately 1800 W to the constant rotation of 100 rpm and the flow speed is 3 m/s, while in the case there is no diffuser, using the classic Glauert model (1935), the power is about 1000 W in the same conditions of speed and rotation axis.

Figure 7b shows that the highest power coefficient for a constant rotation of 100 rpm, is 77% at a speed of 1.5 m/s. In the case without diffuser, the maximum power coefficient is 40% at a speed of 1.3 m/s.

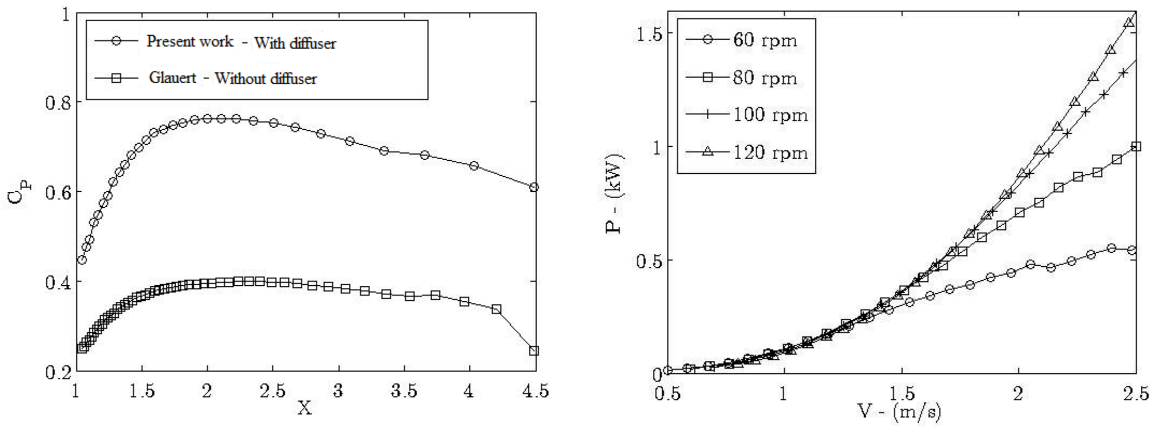


(a) (b)

Figura 7. (a) Power curve as a function of fluid speed; (b) Power coefficient curve related to fluid speed.

Figure 8 presents the curve of the power coefficient related to X, where the maximum Cp (77%) corresponds to X = 2, it shows that Betz limit (Betz, 1926) is exceeded due to the pressure loss in the diffuser and the consequent turbine speed flow increase.

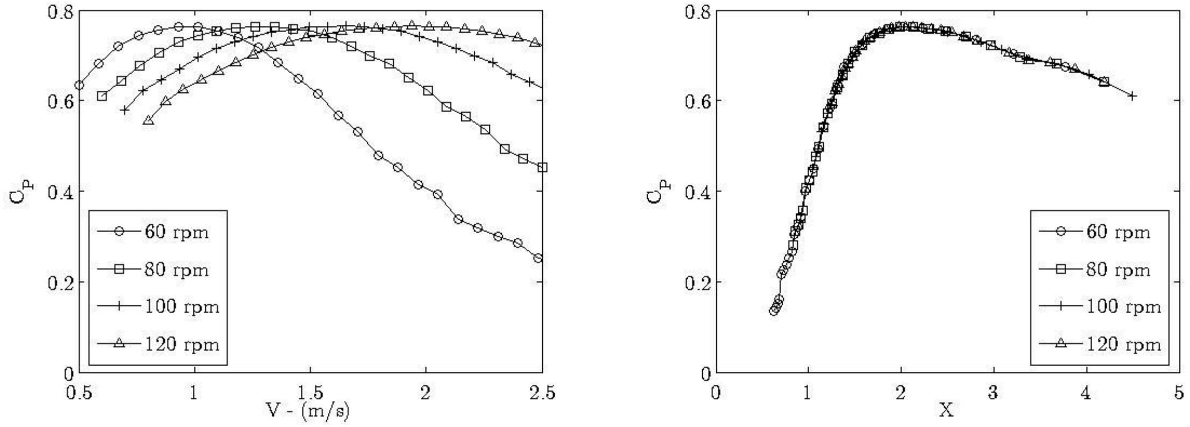
Figure 8b shows the power turbine curves on the diffuser influence to the rotations of 60, 80, 100 and 120 rpm, which is a considerable improvement in energy generation, compared with the classical model.



(a) (b)

Figure 8: (a) Power coefficient curve related to X; (b) Power coefficient curve as a function of fluid speed.

Figure 9a presents the power coefficient related to flow speed to different rotations. It is noticed that to velocities of 1, 1.5 and 2 m/s, the power coefficient is 77%. Figure 9b shows that the curves presented the same behavior, where the machine operates with maximum power coefficient at different intervals of flow speeds, keeping the trend of the power coefficient curve, even though the machine rotation is variable. The maximum efficiency obtained is 77%.



(a) (b)

Figura 9: (a) Power coefficient curve related to flow speed; (b) Power coefficient curve related to X.

4. CONCLUSION

The mathematic model shown is an alternative approach to hydrokinetic turbine design with diffusers based on the BEM model (Hansen, 2008), which provides corrections of Prandtl (as it is described by Hansen, 2008) and Glauert (1935) that are modified for the case of diffusers, considering the turbine geometry influence to determine the power coefficient. However, it is necessary to take in account a few limitations of the model as the estimative of the effect of the pressure loss at the diffuser, that it is still being done in a arbitraly way, and the development of the studies to make possible establish a relation between γ and the diffuser geometry. Another difficulty that was founded is the comparison to experimental data, because they are very scarce in literature, once it is through the experimental data that it is possible to assess the parameter which calculates the pressure loss effect.

5. ACKNOWLEDGEMENTS

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