A CONTINUUM DAMAGE MODEL FOR FAILURE ANALYSIS OF BONDED JOINTS IN COMPOSITE MATERIALS

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Abstract. *In this paper it is proposed a phenomenological model, developed within the framework of continuum damage mechanics, to perform the failure analysis of a family of adhesive single lap joints in composite materials. The study is conceived for adhesives with brittle-elastic behaviour and highly resistant adherends. It is shown that, under certain conditions, the rupture forces (in the case of monotonic loading) and lifetimes (in the case of cyclic loading) of two joints with different overlap lengths can be correlated through a shape factor. Hence, only a few tests performed in a reference joint are necessary for assessing the structural integrity of any similar joint with arbitrary overlap length. Results from experimental static and fatigue testing of joints with carbon/epoxy laminates bonded with epoxy adhesive different bonding areas were compared with model prediction showing a good correlation.*

Keywords*: B. Strength; C. Analytical modeling; Damage mechanics; E. Joints/joining*

1. INTRODUCTION

The present study is concerned with the failure analysis of composite adhesive single lap joints. The single lap joint is the most studied type of adhesive joint in literature, however, strength prediction of such joints is still a controversial issue as it involves a lot of factors that are difficult to quantify. In general, tensile and fatigue tests are often used to assess information about the mechanical strength of a given adhesive/adherend system.

Adhesive bonding offers a simple and efficient way of joining structural components without weakening them with holes or welding. Nevertheless, the exact determination of the failure behaviour of a bonded joint with arbitrary geometry is still one of the most severe limitations for the practical use of this technique.

The design of adhesively bonded structural joints is usually considered complicated due the presence of singularities at the ends of the joints and the lack of suitable failure criteria (for a general review, see Kinloch (1987), Minford (1993), Baldan, (2004a), (2004b), Silva et al (2009a), (2009b). Many studies about the effect of different variables - material, geometry, surface treatment and environment – on the static and fatigue strength of single lap joints have been developed. In particular, some of these studies were concerned with the structural integrity and damage assessment of composite-to-composite joint structures (Ferreira et al (2002), Aymerich et al (2006), Quaresimin and Ricotta (2006a), (2006b), (2006c), Meneghetti et al (2010), Goyal et al (2008), Ascione (2009a), (2009b), Shenoy et al (2009), Aymerich et al (2006), for instance). Nevertheless, the complex nature of bonded composite joints failure is still a challenge to designers since the combined effect of micro and macro mechanisms involved is extremely complex (adhesives and adherend materials, geometry, etc.).

The present paper is concerned with a simple but consistent phenomenological framework to perform a failure analysis of an arbitrary composite adhesive single lap joint. The approach is global and the local concepts of stress and strain are not used because the joint is taken as a system. The model is conceived for adhesives with brittle-elastic behaviour and highly resistant elastic adherends (joint failure occurs before significant permanent deformation). The goal is to use such model to obtain as much information as possible about the influence of the geometry of the glued area on the rupture force for a given adhesive/adherend system with a minimum number of experiments. The influence of the thickness of the adhesive layer and other important parameters such as adherends laminate lay-up) is not considered on this preliminary analysis. The material constants considered in the constitutive equations are simple to be identified experimentally for a given adhesive thickness and the resulting mathematical problem is also very simple (the model equations can be solved analytically). One interesting result is that, under certain conditions, the rupture forces (in the case of monotonic loading) and lifetimes (in the case of cyclic loading) of two joints with different overlap lengths can be correlated through a shape factor.

The adhesive/adherend system considered in this paper consists of carbon/epoxy laminates bonded with epoxy adhesive. Results from experimental testing of single lap joints with different sizes and bonded areas presented in (Quaresimin and Ricotta, 2006c) are compared with model prediction and showing a good correlation. It is important to note that the present study is mainly focused on the influence that the size and the geometry of the glued area have on the joint strength. The preliminary results for static strength and high cycle fatigue are quite promising.

2. MATERIALS AND METHODS

The observations made by Landrock (1985) that the rupture force of a glued joint varies almost linearly with the overlap width, only a variable overlap length was considered in the analysis.

In order to quantify influence of the overlap length L on the joint strength, three different kinds of specimen with overlap length L respectively of 20, 30 and 40 mm were considered. The width W was 24 mm, the adhesive layer thickness was L_{ads} = 1,65 mm, the distance between grips was H = 260 mm and the laminate lay-up was for all specimens. Joints were manufactured from autoclave-moulded laminates (carbon fibres fabric/ epoxy matrix) bonded with a two-part epoxy adhesive. The specimens are characterized by the edge of the glue (SE = Squared Edge, or F = Spew Fillet Edge), and by the surface preparation on the static strength (PP = peel-ply, GB-D = grit blasted and degreased). Three kinds of specimens with different dimensions (see Table 1) were tested on a servo hydraulic machine at room temperature. In order to adopt the model proposed in the next section, the joints with W = W_{ref} = 24 mm and L= L_{ref} = 30 mm were taken as reference.

Group	Width [mm]	Length [mm]		Area [mm^2]
F PP	24	20	L_{ref}	480
		30	$3/2\ L_{ref}$	720
		40	$4/2\ L_{ref}$	960
F GB-D		20	L_{ref}	480
		30	$3/2\ L_{ref}$	720
		40	$4/2\ L_{ref}$	960
SE PP		20	L_{ref}	480
		30	$3/2\ L_{ref}$	720
		40	$4/2\ L_{ref}$	960

Table 1 – Specimens dimensions

Fatigue tests were performed at room temperature on a servo-hydraulic MTS 809 machine with 10/100 kN load cells and carried out under load control, with a sinusoidal wave, nominal load ratio R = 0.05 and a test frequency variable in the range of 10–15 Hz depending on the applied stress level. Tab ends were used to improve alignment and also to assure that the extremities are clamped (i.e. no rotation or lateral displacement occurs).

3. MODELLING

3.1. Static failure analysis

The model is developed within the framework of Continuum Damage Mechanics and the constitutive equations can be derived from thermodynamic arguments and follow a procedure successfully used to model tensile tests in the presence of other nonlinear phenomena as in work Costa-Mattos (2008).

The goal is to propose model equations simple enough to allow their usage by designers while retaining the capability of describing a complex non-linear mechanical behaviour. The focus is to use this model to analyse the influence of the overlap length on the strength of a given joint in a tensile test. The basic idea of the model is to approximate the stiffness K of the real joint subjected to tensile loading through a nonlinear “equivalent” spring system.

It is possible to prove, as in recent work Costa-Mattos (2011), that, if the free energy and the dissipated power of both systems (joint and equivalent spring) coincide at any time instant, then the external force F(t) necessary to impose a given elongation history δ(t) will be the same in both systems. Or, in other words, they will have the same stiffness at any time instant. Hence, it is necessary to postulate adequate expressions for the free energy and the dissipated power of the “equivalent” spring system in order to obtain a curve F x δ that is similar to the one obtained in a tensile test of the joint.

In order to account for the dissipative mechanism of rupture, a macroscopic auxiliary damage variable D ∈ [0,1], related to the loss of the global stiffness of the joint due to the damage is introduced.

Initially the joint has a linear behaviour with stiffness Ko. If D = 0 , the joint is undamaged (K=Ko) and if D = 1, it can no longer resist to mechanical loading (K = 0). The evolution of the auxiliary variable D can be evaluated by using the relation F=K δ=(1−D)Ko δ and measuring the variable joint stiffness K in a tensile test as shown in fig. 1a.

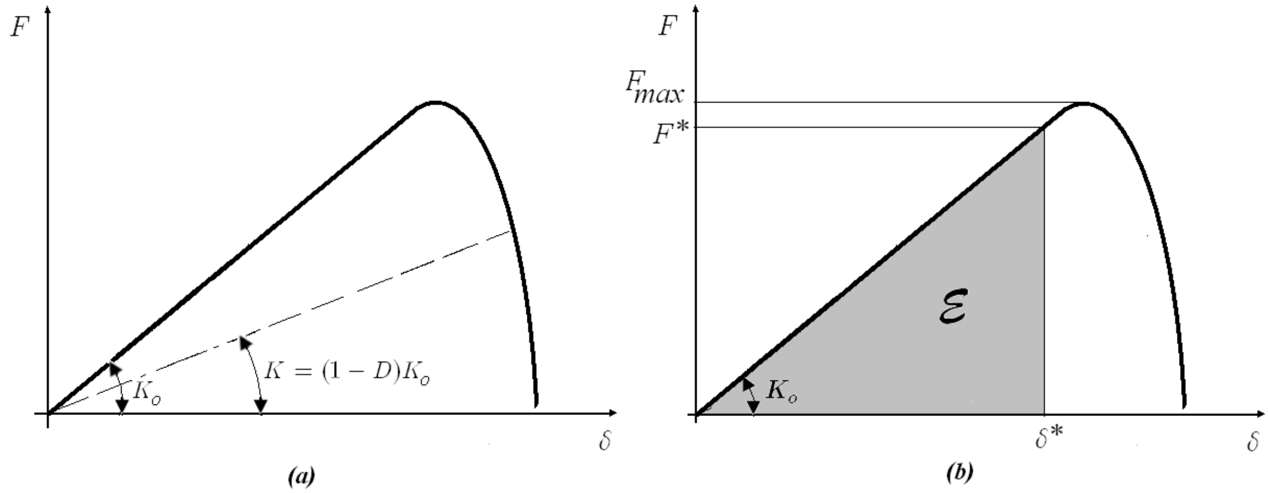

Figure 1: (a) Definition of the variable D during a tensile test. (b) Definition of the energy ε in a tensile test.

The force F that corresponds to an elongation δ(t) on the "equivalent" spring is also supposed to obey the law F = (1 - D)Ko δ. Hence, the modelling of the "equivalent" spring is reduced to the choice of an adequate expression for the evolution of the damage variable D. This choice is made within a thermodynamic context which will not be discussed in this paper.

The present analysis considers a single lap joint under tension submitted to a prescribed monotone elongation δ(t) . The "equivalent" spring system is supposed to be governed by the following model equations:

$$F = (1 - D)K_0\delta \tag{1}$$

$$\dot{D} = \frac{1}{c}\left\langle G - \varepsilon \right\rangle; \;\; D(t=0) = 0; \quad 0 \le D \le 1 \tag{2}$$

$$\left\langle G - \varepsilon \right\rangle = \max\left\{ (G - \varepsilon), 0 \right\} \tag{3}$$

$$G = \frac{1}{2} K_0 \delta^2 \tag{4}$$

$$\varepsilon = \varepsilon_{ref} \frac{K_0^{ref}}{K_0} \eta^2 ; \;\; \eta = \frac{W\sqrt{L}}{W_{ref}\sqrt{L_{ref}}} \tag{5}$$

Equations (1 - 5) form a set of governing equations for any family of single lap joints with the same adhesive layer thickness L_{ads}, length H and adherend laminate lay-up. c is a positive material parameter that does not depend on the joint geometry (it depends only on the adherend and adhesive materials, on the adhesive thickness and, eventually, on the surface treatment of the glued surfaces). ε is the energy corresponding to the area under the linear portion of the curve $F \times \delta$ (which will be called the critical energy , see fig 1b). The critical energy ε must depend on the overlap length and overlap width to account for size effects.

K_0 is the stiffness of the undamaged joint measured in a tensile test which is dependent of the joint geometry (including the overlap length L) and of the adherend and adhesive elastic properties (see Fig. 1). In most cases, it is not possible to provide an exact analytic expression for the stiffness K_o. Nevertheless, as it is shown in the following analysis, the constitutive relation (5) that relates K_o and ε allows circumventing this problem and provides all the additional necessary information about the joint dissipative behaviour.

The first expression in (5) relates the critical energy ε of an arbitrary joint with the critical energy ε_{ref} of a reference joint ($W = W_{ref}, L = L_{ref}$) which can be measured experimentally in a tensile test. η is called the size (or shape) factor.

From Silva et al (2009a), (2009b), it comes that $\dot{D} = 0$ if $G \le \varepsilon$. As a consequence, considering the initial condition $D(t = 0) = 0$, it is possible to conclude that $D = 0$ if $G \le \varepsilon$ and hence the behaviour is linear according to eq (1). The force F^* beyond which the curve $F \times \delta$ is no longer linear (here called the proportional limit) is given by the following expression:

$$F^* = K_0\delta^* = K_0\sqrt{\frac{2\varepsilon}{K_0}} \Rightarrow \varepsilon = \frac{\left(F^*\right)^2}{2K_0} \tag{6}$$

Introducing (6) into constitutive equation (5), it is possible to obtain an expression relating the proportional limit F^* of an arbitrary joint with the limit F^*_{ref} of a reference joint :

$$\varepsilon = \varepsilon_{ref}\frac{K_0^{ref}}{K_0}\eta^2 \Rightarrow \frac{\left(F^*\right)^2}{2K_0} = \frac{\left(F^*_{ref}\right)^2}{2K_0}\eta^2 \Rightarrow F^* = \eta F^*_{ref} \tag{7}$$

Considering a tensile test with prescribed elongation $\delta(t) = \alpha t$ ($\alpha > 0$), it is possible to obtain the analytic solution of eq. (2) until rupture ($D = 1$) at instant t^r :

$$D(t) = 0\text{, if } t \le t^* = \frac{1}{\alpha}\sqrt{\frac{2\varepsilon}{K_0}} \tag{8}$$

$$D(t) = \frac{1}{c}\left[\frac{1}{6}K_0\alpha^2\left(t^3 - \left(t^*\right)^3\right) - \varepsilon\left(t - t^*\right)\right]\text{, if } t^* < t < t^r \tag{9}$$

Constant c is related to the viscosity of the adhesive layer and can be identified from one rupture test performed in a joint with arbitrary geometry (generally the reference joint, for which $K_o = K_o^{ref}$ and $\varepsilon = \varepsilon_{ref}$ are known) using eq. (9) and condition $D(t = t^r) = 1$:

$$c = \left[\frac{1}{6}K_0^{ref}\alpha^2\left[\left(t^r\right)^3 - \left(t^*\right)^3\right] - \varepsilon_{ref}\left[t^r - t^*\right]\right] \tag{10}$$

Equations (1 - 5) allow describing some rate dependency. Nevertheless, for most epoxy resins, c is very small (less than 0.001 N mm s). If $c \to 0$, the joint does not present significant rate dependency and the behaviour is brittle ($t^r \to t^*$, $\delta^r \to \delta^*$ and $F^* \to F_{\max}$). Since $F^* \to F_{\max}$ in this case, using eq. (7), it is possible to conclude that the rupture force $F_{\max}$ for an arbitrary joint can be obtained from the rupture force $F^{ref}_{\max}$ of a reference through a shape factor η :

$$F_{\max} \approx \eta F^{ref}_{\max}\,;\eta = \left(\frac{\sqrt{L}W}{\sqrt{L_{ref}}W_{ref}}\right) \tag{11}$$

From equation (11) it is possible to conclude that the rupture forces (F_1, F_2) obtained in tensile tests performed in two lap joints with different geometry of the glued area ((W_1, L_1), (W_2, L_2), respectively), are related through the shape factor η :

$$\frac{F_1}{F_2} = \eta \text{ with } \eta = \frac{W_1\sqrt{L_1}}{W_2\sqrt{L_2}} \tag{12}$$

3.2. Fatigue analysis

Considering a joint with a quasi-brittle behaviour in tension ($c \to 0$), it is possible to propose a preliminary damage model for fatigue analysis. In this case, the fatigue is supposed to be almost independent of the frequency. The proposed damage "evolution law" in terms of cycle N is:

$$\frac{\delta D}{\delta N} = \left[\left(\frac{\sqrt{L_{ref}}W_{ref}\Delta F}{\sqrt{LW}} \right)^{-1/a} 10^{(b/a)}(1-R) \right]^{-1} \tag{13}$$

Where, $\Delta F = \max(F) - \min(F)$ and $R = (\min(F)/\max(F))$ and a , b are positive parameters for a fixed set of adhesive layer thickness L_{ads}, length H and adherend laminate lay-up. Once defined the adherend and adhesive materials, the model is supposed to hold for $10^4 \le N_F \le 10^6$. Since D = 1 at failure (N=N_F), it follows the equation (13) that

$$\int_{D=0}^{D=1} \delta D = \int_{N=0}^{N=N_F} \left[\left(\frac{\sqrt{L_{ref}}W_{ref}\Delta F}{\sqrt{LW}} \right)^{-1/a} 10^{(b/a)}(1-R) \right]^{-1} \delta N \tag{14}$$

Hence,

$$N_F = \left[\left(\frac{\sqrt{L_{ref}}W_{ref}\Delta F}{\sqrt{LW}} \right)^{-1/a} 10^{(b/a)}(1-R) \right] \tag{15}$$

Or

$$\left(\frac{\sqrt{L_{ref}}W_{ref}}{\sqrt{LW}} \right) \Delta F = 10^b \left[\frac{N_F}{1-R} \right]^{-a} \tag{16}$$

From the equation (16), it is possible to verify that the parameters a and b can be identified on a log(ΔF) x log(N_F) curve for a reference joint with glued area A_{ref}=$L_{ref}W_{ref}$ for which the shape factor is equal to 1 as can be seen in figure 2.

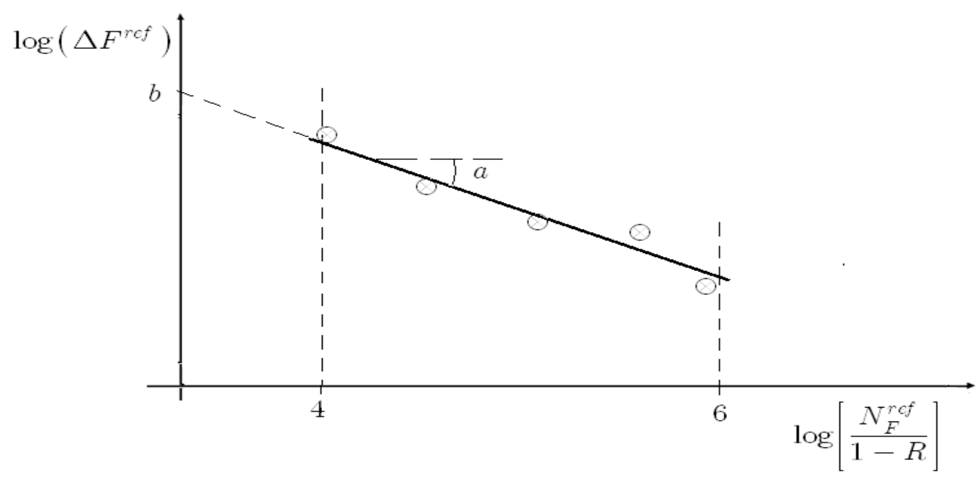

Figure 2: Experimental identification of parameters a, b using a reference joint with glued area A_{ref}=$L_{ref}W_{ref}$

$$\log(\Delta F^{ref}) = -a \log\left[\frac{N_F^{ref}}{1-R}\right] + b \tag{17}$$

From equations (16) and (17) it is possible to relate the force amplitude ΔF of an arbitrary joint (arbitrary L and W, but fixed adhesive layer thickness L_{ads} , length H and adherend laminate lay-up to the force amplitude ΔF^{ref} of a reference joint. In particular, the force amplitude ΔF related to a given life N_F can be obtained from the amplitude in a fatigue test with the same load ratio *R* with the introduction of a shape factor:

$$N_F = N_F^{ref} = \bar{N} \Rightarrow \Delta F = \left(\frac{\sqrt{LW}}{\sqrt{L_{ref}W_{ref}}}\right)\Delta F^{ref} = \left(\frac{\sqrt{LW}}{\sqrt{L_{ref}W_{ref}}}\right)10^b\left[\frac{\bar{N}}{1-R}\right]^{-a} \tag{18}$$

4. RESULTS

4.1. Static failure analysis. Comparison with experimental results

Table 2 presents the experimental and predicted rupture forces for the three kinds of joints defined in Table 1. The rupture force $F_{\max}$ for each joint was obtained using the procedure proposed in section 3.1.

Specimen	Shape factor	Predicted rupture force (N)	Experimental rupture force (N)	Error %
F PP				
L = 20 mm Reference	$F_{\max}^{ref} = 408.9$			
L = 30 mm	1.225	501	502	0.2%
L = 40 mm	1.414	578	552	-4.9%
F GB-P				
L = 20 mm Reference	$F_{\max}^{ref} = 375.2$			
L = 30 mm	1.225	460	426	-7.8%
L = 40 mm	1.414	531	491	-8.0%
SE PP				
L = 20 mm Reference	$F_{\max}^{ref} = 347.9$			
L = 30 mm	1.225	426	408	-4.5%
L = 40 mm	1.414	492	440	-11.8%

Table 2 – Rupture force. Comparison between model predictions and experimental results

The comparison between the predicted rupture forces and the measurements shows that the theoretical model does indeed offer an accurate estimate.

4.2. Fatigue failure analysis. Comparison with experimental results

As can be seen in figure 2, the fatigue curves were obtained for two Squared Edge Joints. As a result of that, a preliminary analysis of the proposed modelling methodology can be done.

Considering a joint with an overlap length of 20 mm as reference, it is possible to compute the shape factor η for the other joint . The predicted curve for the joint with overlap length of 40 mm obtained using the eq. (16) shows a good correlation.

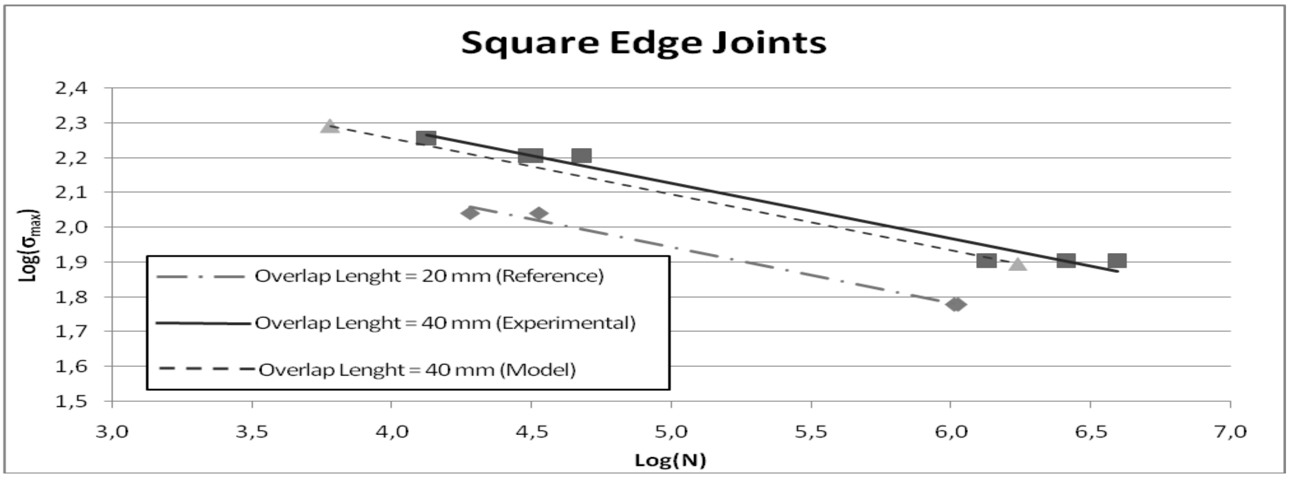

Figure 2: Fatigue tests Comparison between experiments and model prediction.

5. CONCLUSIONS

The purpose of this work is to provide a method to easily perform failure analysis (static and fatigue) of a family of adhesive single lap joints in composite materials. The main goal of the preliminary study is to account the effect of the overlap length on the strength of the joint.

From the model it is possible to obtain a shape factor that accounts for size (geometric) effects – mainly comprised by the overlap width and the overlap length. In the case of joints with highly resistant adherends and brittle adhesives, the analysis shows that the rupture forces (F_1, F_2) obtained in tensile tests with two lap joints with different dimensions of the overlap width and overlap length ((W_1, L_1) and (W_2, L_2),respectively), but built in the same condition with the same adhesive layer thickness L_{ads}, length H and adherend laminate lay-up, can be related thought a shape (or geometric) factor

$$\frac{F_1}{F_2} = \frac{W_1\sqrt{L_1}}{W_2\sqrt{L_2}} \tag{19}$$

In the case of fatigue tests of joints with the same load ratio ($R_1 = R_2 = \bar{R}$), the analysis shows that the force amplitudes (ΔF_1, ΔF_2) can also be related thought the shape factor if $N_F^1 = N_F^2 = \bar{N}$.

$$\frac{\Delta F_1}{\Delta F_2} = \frac{W_1\sqrt{H_1}}{W_2\sqrt{H_2}} \tag{20}$$

The preliminary results indicate that the proposed model allows a quick estimate of both static and fatigue strength of an adhesive joint with arbitrary area from a reference joint. It is quite difficult to compare other models predictions with the ones performed using the shape factor, since, as far as the authors know, the material parameters that arise in more sophisticate (and eventually more accurate) theories are generally difficult to be identified experimentally. The only reasonable source of comparison are the works of Quaresimin and Ricotta (2006a and 2006b) and Meneghetti et al (2010), where the joint lifetime is described as the sequence of a crack nucleation phase followed by a propagation phase.

Nevertheless, it is important to remark that such a simplified model is global and that it does not take into account the micro-mechanics of the fatigue damage evolution. Although the expression proposed for the shape factor in the present paper works only for a particular family of joints, it may be used to obtain a preliminary estimate of the adhesive area (always brittle-elastic adhesives) before a more adequate (but eventually more complex) analysis including the effects of layer orientation, lap joint length and water immersion on the fatigue behaviour (such as in Goyal 2008, Ascione, 2009a and 2009b, for instance).

The purpose of this work is to provide a method to easily perform failure analysis (static and fatigue) of a family of adhesive single lap joints in composite materials. Such a theory is first step of a research program that aims at proposing practical formulas for bonded joint design calculation as simple, safe and reliable as the ones usually adopted for designing solder lap joints. The main motivation is the replacement of welded joints by glued joints in situations where the production of heat and/or sparkling is forbidden such as in offshore oil platforms.

The goal is to propose model equations simple enough to allow their usage by designers while retaining the capability of describing a complex non-linear mechanical behaviour. In this preliminary study it is accounted only the effect of the overlap length on the strength of the joint. Results from experimental testing of carbon/epoxy laminates bonded with epoxy adhesive with different sizes and bonded areas have been compared with model prediction, showing a good correlation. The preliminary results in both cases (static strength and fatigue strength) are quite promising. It is important to remark that the authors do not claim general validity for any of these preliminary results. Nevertheless, these results justify a careful prosecution of the study and the development of an intensive research program for the experimental validation of the proposed models, mainly in the case of high cycle fatigue. A more detailed study concerning how adhesive thickness L_{ads} (or variable length H) affects the strength of an arbitrary joint will be presented in a forthcoming paper.

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