A MATLAB® BASED TOOL FOR THE QUICK DESIGN AND ANALYSIS OF CONJUGATE CAM MECHANISMS

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Abstract*. The main objective of the Multiweave project was the design of a weaving machine to produce an innovative multidirectional woven fabric. This is a textile structure intended to compensate the usual anisotropic properties of conventional biaxial fabrics which is envisaged of great importance for some technical applications such as in the reinforcement of high technical components made from composite materials requiring improved mechanical properties. While biaxial fabrics interlace two sets of yarns, warps and wefts, the Multiweave fabric is obtained by interlacing 4 sets of yarns: the conventional warps and wefts at 90º and other two sets of bias yarns at +45º and -45º. This process results in a structure that doesn't collapse when pulled at 45º and is much less anisotropic. Fabrics of different fibres were prepared using the Multiweave development prototype using high tenacity polyester 440 tex on warp and bias and either 440 tex HT polyester, 220 tex aramid (kevlar®) or 800 tex carbon on the weft. The concept of the Multiweave development prototype contains several mechanisms such as the shed formation mechanism, including the heddle rising and lowering, the beating mechanism and the weft insertion, all in perfect synchronization, which are driven by means of conjugate or desmodromic cam mechanisms. The proper design of cam mechanisms is one of the most important and difficult steps in machine design, namely in weaving. In the case of Multiweave, a reliable actuation, both on the advancement and returning movements is required. Therefore, the use of conjugate cams was recommended. This work is concerned with the process of designing and manufacturing such mechanisms. A model of conjugate cams with oscillating roller follower was designed, including the basic design parameters, namely, cam base diameter, contact roller diameter, distance from cam axis to follower pivot, oscillating follower arms length, follower arms angle and the coordinates of follower pivot in relation to cam axis. A program was created in MATLAB® environment with many advantage, namely: a comparison of several common methods used for cam design could be performed in a very small amount of time, therefore allowing an optimization process before the final design of the mechanism. The program was later extended to be capable of computing single or conjugate cams with oscillating or translating roller followers and different contact elements. It contains equations for 10 different methods. Furthermore it allows inputting an unlimited number of sections (rise, dwell, return) and all calculations are performed with a very high accuracy. From the cam profile obtained by Matlab as a matrix of points it is possible to design the cam transferring the data to a CAD software, SolidWorks® for example. The final part can be obtained directly from a CNC machine using the CAD file. The work resulted in a very useful tool that can be applied in other projects involving the design and manufacturing of simple or complex cam mechanisms.*

Keywords*: multiweave; multidirectional fabric; cam mechanism, conjugate cams*

1. INTRODUCTION

The Multiweave project has been developed at the University of Minho, Centre for Mechanical and Materials Technologies, in Guimarães, Portugal, funded by the European Commission under the CRAFT programme of the Framework VI. The final goal of the project was to design a weaving machine able to produce multiaxial fabrics (Lima *et al*, 2009), (Lima *et al*, 2007a), (Lima *et al*, 2007b). This is an innovative textile structure intended to compensate the usual anisotropic properties of conventional biaxial fabrics which is of great importance for technical applications such as in the reinforcement of composite materials.

While biaxial fabrics interlace two sets of yarns, warps and wefts, the Multiweave fabric, as represented in the model of figure 1, is obtained by interlacing 4 sets of yarns, warps, wefts and other two sets of bias yarns at +45 and -45 degrees. Fabrics of different fibres were manufactured using the Multiweave development prototype. Figure 2 shows a sample with polyester 220 tex on the warp and bias and 800 tex carbon on the weft.

The Multiweave development prototype is shown in figure 3. The concept of the machine contains several essential mechanisms for functions such as the bias yarns feeding with crisscross insertion, heddle rising and lowering with the warps to form the shed, weft insertion across the open shed, reed beating including a false beating and fabric taking-up. All these functions are provided in the first place by the rotation of the main shaft, which is driven by an electric motor.

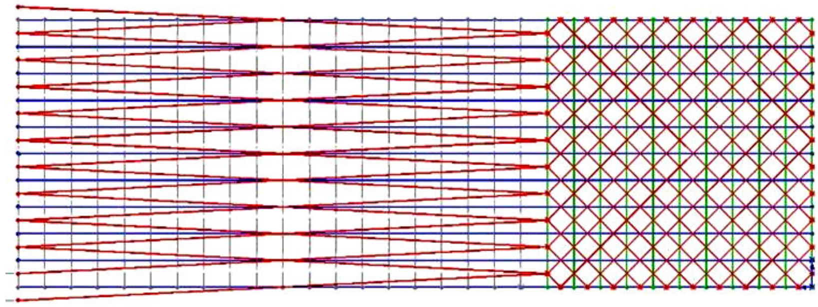

Figure 1. Geometric model of the Multiweave fabric

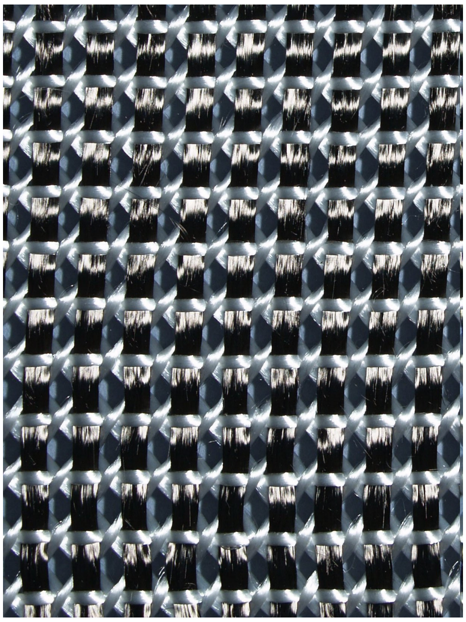

Figure 2. Multiweave fabric sample

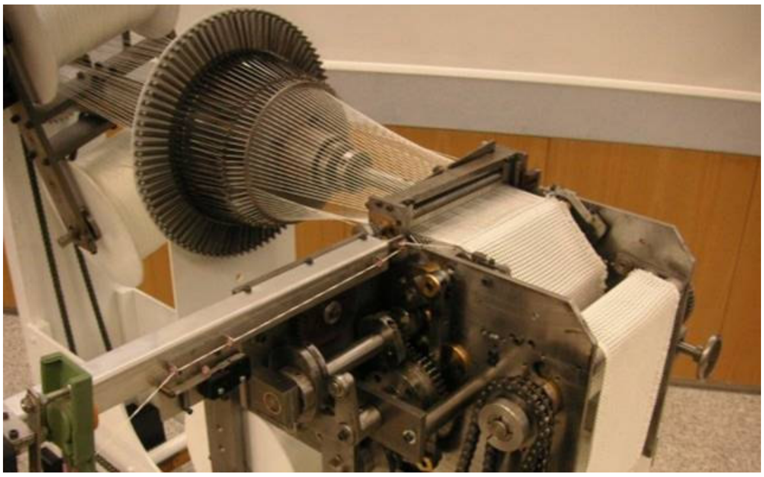

Figure 3. Partial view of the Multiweave development prototype

The movements of the main functions were achieved by means of conjugate cam mechanisms. The main reason is that this solution allows periods of rest which is easily obtained by appropriate dwells in the cams profiles. Also the designer is able to adjust the cinematic characteristics of the end effectors by a criterions selection of the type of movement given by existing mathematical models. The more conventional single cam with spring return follower is not acceptable. One of the reasons is the excess force needed to overcome the spring, considerably magnified by the high ratio of the mechanism; the other reason, possibly the most important, is that conjugate cams assure that the return movement is always achieved and is not dependent on the spring actuation. A possible jam on the return movement could cause a devastating collision, particularly serious between the reed and the weft insertion element if any of them goes out of synchronism. Therefore the design of these mechanisms is an important task, even if this not yet at an industrial scale. The requirements for synchronism and precision are however the same as in a full scale machine.

The early design of the weft insertion mechanism is shown in some detail in figure 4.

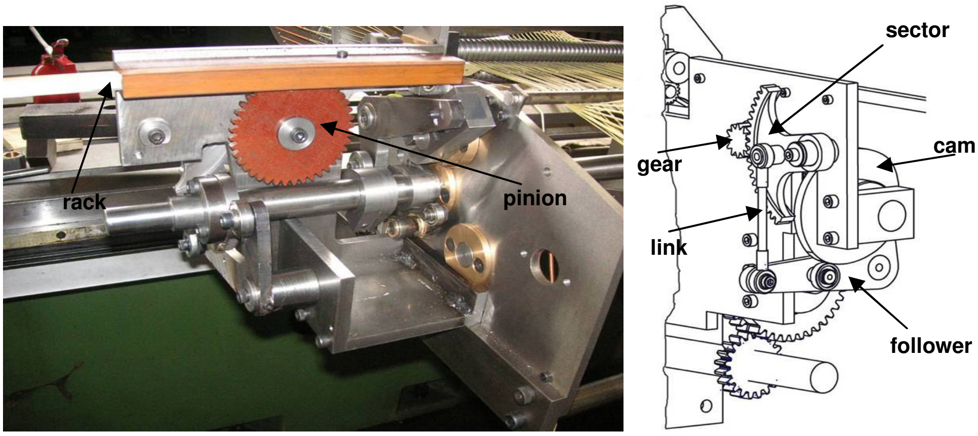

Figure 4. The weft insertion mechanism

In the case of a prototype such as this, the design would have to be changed several times during the development stages. Therefore the main demand was for a quick and low-cost design process. Matlab® came as an ideal tool to handle mathematical equations that translate the cinematic characteristics of cam mechanisms, perform quick comparisons, optimize the design and obtain the cam profiles. Also it was important to consider requests for single-parts manufacturing. On the other hand, durability was not to be an important requirement due to the limited usage of the machine. In the case of the Multiweave development prototype the expected operating speed was relatively slow which also turned the problem easier.

2. DESIGN STEPS

The design of a cam mechanism is a complex task, usually containing the following steps:

- Definition of the cam type;
- Establishment of basic dimensions;
- Creation of s-v-a-j diagrams;
- Determination of pressure angle diagram;
- Construction of cam surface profile;
- Dynamic analysis;
- Stress analysis.

The first three steps involve inserting input data. Even though some parameters are given by the knowledge of the working requirements of the different systems (e.g. a timing diagram of the machine) each new design requires working out several parameters. In the end, it is up to the designer's skill how the design will look like. In a conventional design process, a large number of calculations has to be performed in order to obtain the desired shape and cinematic properties of the cam. Many times the obtained results are unacceptable and the whole process must be repeated again with a new approach for the input data.

In the case of the Multiweave weft insertion mechanism, besides the timing diagram, several requests where part of the specifications. One of the basic demands is to ensure a permanent contact between the cam and the follower. The use of a simple cam with spring return is not acceptable in this situation. The reason is that, if for any malfunction the weft needle gets stuck before completing the return movement, a collision with other moving parts is unavoidable causing a possible destruction of the machine, especially at high speeds. A grooved faced cam could be used, but in terms of space saving and precision a conjugate cam offers a better choice. A model of a conjugate cam with oscillating roller follower is shown in figure 5. In the same figure the basic design parameters are highlighted, namely,

D_b – Cam base diameter;
D_r – Roller diameter;
c – Distance from cam axis to follower pivot;
l – Follower arms length;
β – Follower arms angle;
x_b, y_b – Coordinates of follower pivot from cam axis;

Anticipating low operating speeds for the development prototype and from the experience of previous designs it was possible to simplify the whole process by not considering the dynamic and stress analysis at this stage.

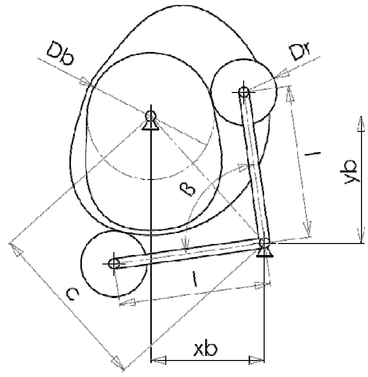

Figure 5. Conjugate cam geometric model

2. THEORETICAL CONSIDERATIONS

The basic step of the cam design is the creation of *s-v-a-j* diagrams. These diagrams are built on the basis of the timing diagram and describe the kinematics properties of the follower. They are stroke or displacement, *s*, velocity, *v*, acceleration, *a*, and jerk, *j*. These equations are obtained by successive derivatives of the stroke or displacement equation. They are all expressed in dimensionless relative values independent of the angular velocity of the cam. The equations, for a constant velocity of the cam shaft are,

$$\delta = \delta(t) = \delta(\theta(t)) = \delta_0 + s(\theta(t)) \qquad \delta_0 = const \tag{1}$$

$$v = \frac{ds}{d\theta} \qquad \dot{\delta} = v \cdot \dot{\theta} \tag{2}$$

$$a = \frac{dv}{d\theta} \qquad \ddot{\delta} = a \cdot \dot{\theta}^2 \tag{3}$$

$$j = \frac{da}{d\theta} \qquad \dddot{\delta} = j \cdot \dot{\theta}^3 \tag{4}$$

where θ is the angular position of the cam, δ is the angular position of the oscillating follower and t is time. The real kinematics properties can be obtained by multiplying the relative values by the corresponding power of the cam angular velocity.

For the mathematical description of the rising and falling sections various different equations can be used (Norton, 2002), (Harold, 1956). The most common types of cam profiles are cycloidal, trapezoid, modified trapezoid, modified sine, 345-polynomial and 4567-polynomial. As an example, the acceleration curves for each of them, which are usual used to evaluate the dynamic behaviour of the mechanism, are comparatively represented in figure 6.

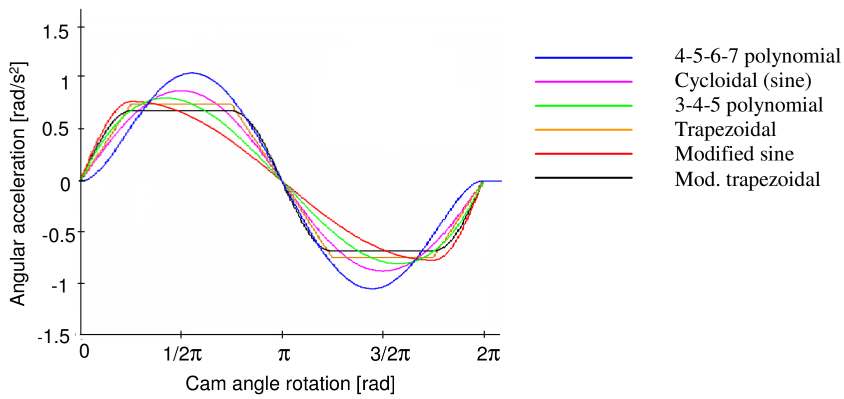

Figure 6. Comparison of the acceleration curves for different methods

Another comparison of all mentioned methods is shown in figure 7. The first three parameters are the earlier mentioned kinematics properties. The last parameter marked as ϕ represents the pressure angle. The scale is expressed relatively to the worst method for each of the parameters, which are assumed to be 1 and dimensionless.

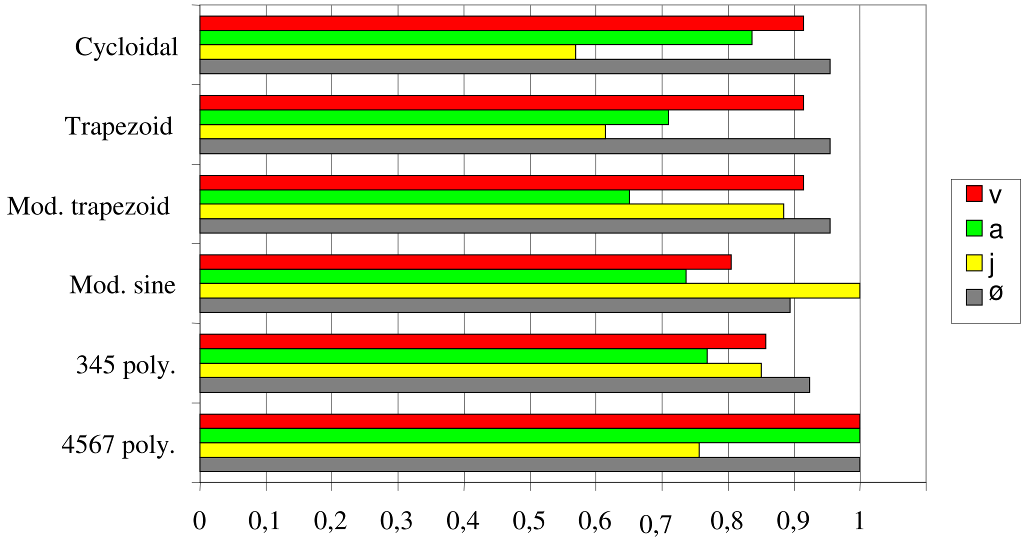

Figure 7. Relative comparison of the different methods for *v, a, j* and ϕ

From the comparison chart the following conclusions can be drawn: Cycloidal equations are the most simple to calculate and also provide the lowest jerk. Therefore this method is very often used for simple cams design. Trapezoid equations provide a combination of low jerk and acceleration. Modified trapezoid equations are one of the most complicated to calculate, but they offer the lowest acceleration. Modified sine equations give the lowest velocity and pressure angle. Therefore this method is recommended for slow cams with big loads and was selected for the design. 345 polynomial equations have good dynamic properties with low pressure angle. Finally, 4567 polynomial method is the only one of the presented methods which shows a continuous jerk profile, which could be important to minimise vibrations in high speed cams.

Modified sine equations (Norton, 2002) were used in our application. They are applied in three different intervals, as follows:

$$0 \le \theta \le \frac{1}{8}\beta$$

$$s = h\left[0.43990085\frac{\theta}{\beta} - 0.0350062\sin\left(4\pi\frac{\theta}{\beta}\right)\right] \tag{5}$$

$$v = 0.43990085\frac{h}{\beta}\left[1-\cos\left(4\pi\frac{\theta}{\beta}\right)\right] \tag{6}$$

$$a = 5.5279571\frac{h}{\beta^2}\sin\left(4\pi\frac{\theta}{\beta}\right) \tag{7}$$

$$j = 69.4663577\frac{h}{\beta^2}\cos\left(4\pi\frac{\theta}{\beta}\right) \tag{8}$$

$$\frac{1}{8}\beta < \theta \le \frac{7}{8}\beta$$

$$s = h\left[0.28004957 + 0.43990085\frac{\theta}{\beta} - 0.31505577\cos\left(\frac{4\pi}{3}\frac{\theta}{\beta} - \frac{\pi}{6}\right)\right] \tag{9}$$

$$v = 0.43990085\frac{h}{\beta}\left[1 + 3\sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} - \frac{\pi}{6}\right)\right] \tag{10}$$

$$a = 5.5279571 \frac{h}{\beta^2} \cos\left(\frac{4\pi}{3} \frac{\theta}{\beta} - \frac{\pi}{6} \right) \tag{11}$$

$$j = -23.1553 \frac{h}{\beta^2} \cos\left(\frac{4\pi}{3} \frac{\theta}{\beta} - \frac{\pi}{6} \right) \tag{12}$$

$$\frac{7}{8}\beta < \theta \le \beta$$

$$s = h \left[0.560099150 + 0.43990085 \frac{\theta}{\beta} - 0.0350062 \sin\left[2\pi \left(2\frac{\theta}{\beta} - 1 \right) \right] \right] \tag{13}$$

$$v = 0.43990085 \frac{h}{\beta} \left[1 - \cos\left[2\pi \left(2\frac{\theta}{\beta} - 1 \right) \right] \right] \tag{14}$$

$$a = 5.5279571 \frac{h}{\beta^2} \sin\left[2\pi \left(2\frac{\theta}{\beta} - 1 \right) \right] \tag{15}$$

$$j = 69.4663577 \frac{h}{\beta^2} \cos\left[2\pi \left(2\frac{\theta}{\beta} - 1 \right) \right] \tag{16}$$

2. IMPLEMENTATION

2.1. Earlier design

On the basis of the mathematical support for the design of cams, a program has been developed in MATLAB®. An aspect of the program interface is displayed in figure 8.

At present the program is capable of computing single or conjugate cams with oscillating roller followers. It contains equations for 10 different methods. Furthermore it allows inputting an unlimited number of sections (rise, dwell, return) and calculations are carried out with a degree of accuracy that can be selected in the program, namely the angular interval of each calculation. As an input the program requires the basic dimensions and the timing diagram. These are the base diameter, the roller diameter, angle of follower arms, length of follower arms and distance from centre of cam to pivot of follower. Then the method to be used is selected. Finally, the required output is decided.

The program is currently able to compute s-v-a-j diagrams, pressure angle, pitch curves (red in figure 8) and cam profile (blue in figure 8). In addition it is possible to carry out an automatic comparison of all methods. After inserting the necessary input data the program can be run and the required output obtained almost immediately.

The data obtained for the cams surfaces is then exported from MATLAB® into an Excel file and then, after some manipulation, into a text file. This file is subsequently modified to the required format and loaded into a CAD software as a xyz curve. In the case of Multiweave project SolidWorks® has been used. These curves are then transformed into sketches which are finally used to create the complete 3D design of the cams. This principle easily allows using more than one curve in one part which is needed for the design of conjugate cams. The final CAD file can be then directly used as a source data for the CNC manufacturing.

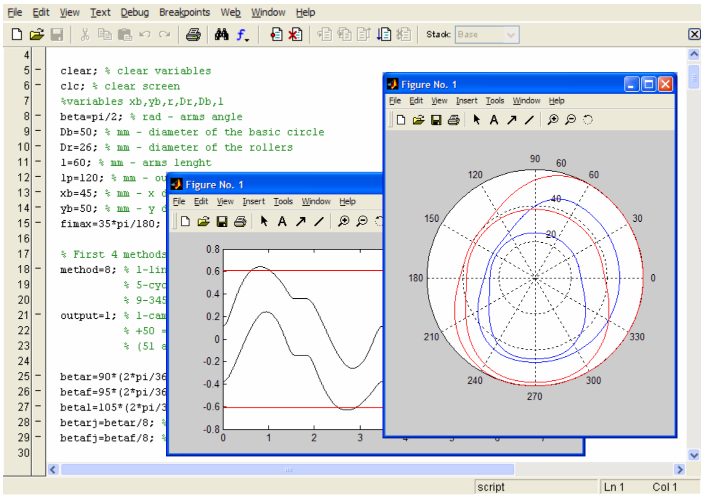

Figure 8. Program interface in MATLAB® with outputs

Figure 9 represents the 3D virtual design of a pair of conjugate cams whose profile surfaces are represented in figure 8, having been added a spacer between the two cams and a hole for the shaft where the assembly is mounted. The final part, as obtained from a CNC machine, is represented in figure 10.

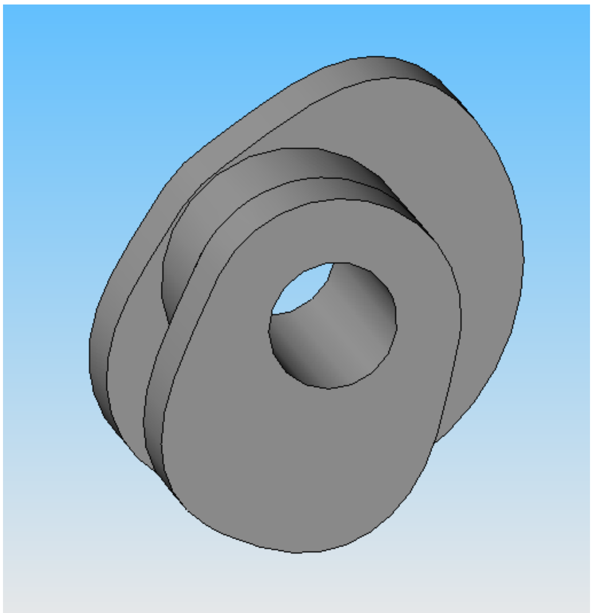

Figure 9. Final 3D design of the conjugate cams

Figure 10. Conjugate cams obtained by CNC

2.2. New design

A second generation prototype for the Multiweave project is now under development. For this machine a new set of cam mechanisms was designed using the program. As an example, the data for the conjugate cams of the new weft insertion mechanism are as follows:

Cam base diameter, D_b = 150,56 mm;
Roller diameter, D_r = 40 mm;
Rising angle, $\beta_r = 50°$;
Return angle, $\beta_f = 35°$,
Begin of return angle, $\beta_1 = 57.5°$, (corresponding to a dwell = $\beta_1 - \beta_r = 7.5°$);
Follower arms length, l = 70 mm;
Distance from cam axis to follower pivot, c = 130 mm;
Follower radial rising, z = 25 mm
Follower angular rising, s_{max} = 0,359312859 rad (corresponding to z = 25 mm).
This value of s_{max} was worked out using an appropriate geometric transformation.
Follower arms angle, $\beta = 112°$ (1.954768762 rad).

The value worked out for β was in fact 111,43°. For easy manufacturing this angle was rounded up to 112°, which caused the value of the cam base diameter, D_b to be raised from 150 to 150,56 mm;

The data was inserted in the program as a matrix named *tgdm*, which, in this case, took the following shape:

tgdm = [0*deg, 0; (initial position)
50*deg, smax; (rising in 50º)
57.5*deg, smax; (dwell in 7.5º)
92.5*deg, 0]; (return at 92.5º, after 35º rotation of the cam)

The output of the program, when selecting “cam profile” could be immediately obtained, as represented in figure 11.

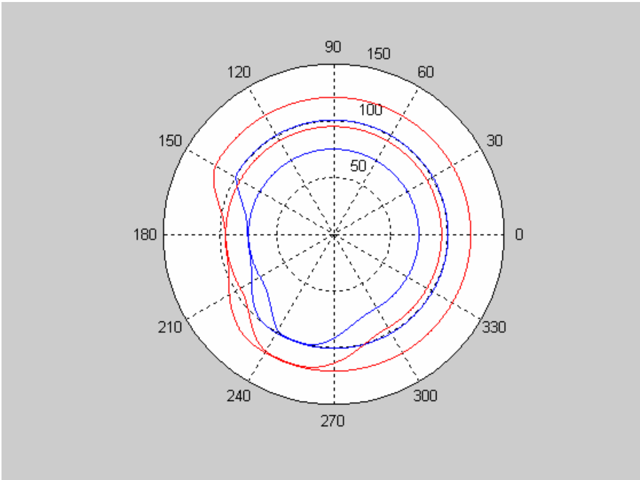

Figure 11. Cam profiles and pitch curves as obtained from MATLAB

The complete 3D SolidWorks® design of one of the two conjugate cams of figure 11 is represented in figure 12, after adding several elements such as a large centre hole for reduced mass and 4 fixing holes. There is another hole aligned with the end of the rising angle that is used to precisely match the two conjugate cams relative position when assembling.

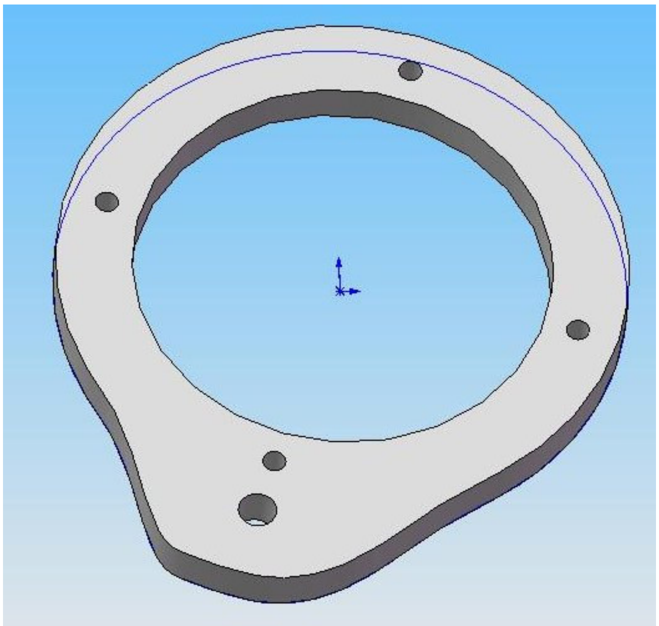

Figure 12. Cam of the new design

Finally, one of the cam boxes of the new prototype, with two sets of conjugate cams can be seen in figure 13, partially immersed in the lubrication oil. Also clearly visible are the swinging followers with two arms and the respective contact rollers.

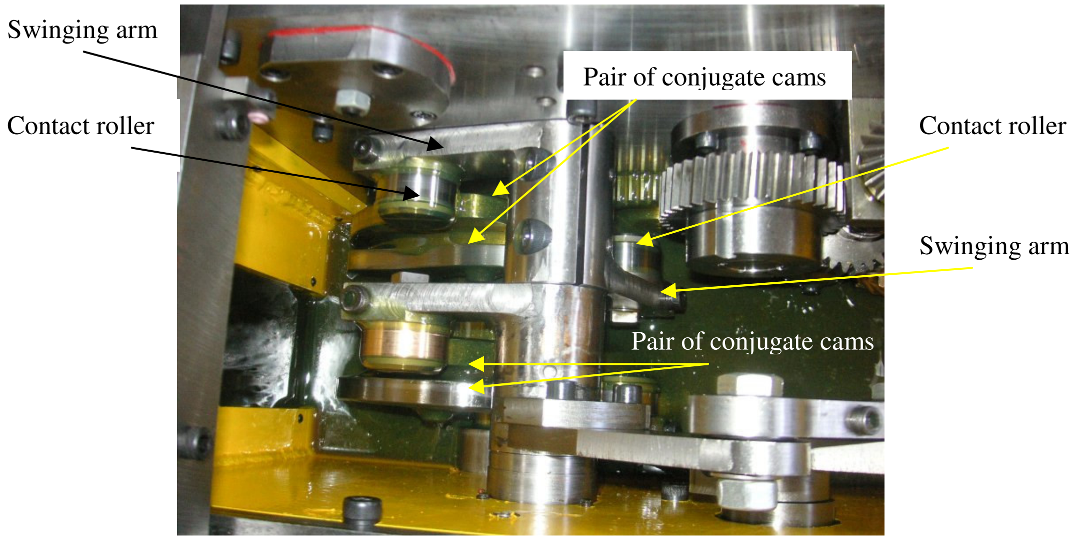

Figure 13. New conjugate cams in cam box

4. CONCLUSIONS

The decision to create a program in MATLAB® produced several benefits. The most important aspect is a very fast design process. The designer can quickly obtain and compare several different designs, and, on the basis of the analysis of the kinematics characteristics, improve the mechanism. Once the input parameters are decided, the whole design process can be done within a few minutes. The other advantage is the possibility of an easy modification and extension of the program. As an example it is possible to involve calculations of connected mechanisms, including optimization or export the obtained cam shape to any other CAD-CAM software.

The inclusion of the comparison of several different methods is also very helpful. The comparison represented in figure 7 clearly shows strengths and weaknesses of each method. This provides a good base for deciding which method is more appropriate for a particular application.

The program has already been successfully used for the design and manufacture of conjugate cam mechanisms with oscillating roller follower, always with very good results.

5. ACKNOWLEDGEMENTS

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