COMPARYING THE SOLUTIONS FOR THE SINGULARITY IN THE INTEGRAL OF THE BOUNDARY ELEMENT METHOD APPLIED TO THE SOUND RADIATION PROBLEM

Marcio Eustáquio Maria professormarcinho@yahoo.com.br
Ester Naves Machado Borges ester@des.cefetmg.br
Márcio M. Afonso marciomatias@des.cefetmg.br

Centro Federal de Educação Tecnológica de Minas Gerais, Av. Amazonas, 7675, 30.510-000 Belo Horizonte, MG, Brazil.

Abstract: *This work compares two ways for treating the singularity which appears in the integral of the fundamental solution of the problem of the sound radiation by a rigid cylinder. The singularity is solved using an expression obtained by a coordinates system change and by varying the number of Gauss' points used in the integration process. The numerical technique employed is the boundary elements method. A comparison of the obtained results of the computational simulations show that the results obtained when the coordinates system is changed are more precise and closer to those of the analytical solution when the number of Gauss' points is altered.*

Keywords: Acoustic radiation, vibrating cylinder, number of Gauss' points

1.INTRODUCTION

Let's consider an infinite cylinder of radius $r = a$, whose surface vibrates in the monopole way, i.e. vibrates uniformly with an amplitude V_o, radiating sound waves at all the directions. The inner region of the cylinder is represented by Ω, the exterior one by Ω_0, and the boundary by S. In this work, the medium is considered homogeneous, without losses or viscosity effects, hence the velocity of the particles of the fluid can be expressed as the gradient of a scalar function, the velocity potential, designed by φ. The velocity potential $\varphi(p)$satisfies the Helmholtz equation, at a point $p(x,y)$ of the boundary S or of the outer region Ω_0, Ziomek (1995).

$$\nabla^2\varphi(p)+\kappa^2\varphi(p)=0 \tag{1}$$

As the surface of the cylinder is considered rigid, Neumann's condition is satisfied:

$$\frac{\partial\varphi(p)}{\partial n}=V_0 \tag{2}$$

In Equation (2), V_0 is the fluid velocity amplitude at the cylinder's surface and its value was fixed as 1. The analytical solutions for the velocity potential are given by the following expression, Morse (1986):

$$\phi(r,\theta)=-\frac{V_0}{\kappa}\frac{H_0^2(\kappa R)}{H_1^2(\kappa a)}, \tag{3}$$

In Equation (3), $H_0^2(\kappa R)$, $H_1^2(\kappa a)$, κa and κR are respectively the Hankel's functions and the normalized frequencies.

The integral formulation for the velocity potential $\varphi(p)$ radiated by the infinite cylinder is given by the following expression, Maria *et al.* (2010):

$$\varphi(p)+\int_S\varphi(p)\frac{\partial G(p,q)}{\partial n}dS=\int_S G(p,q)\frac{\partial\varphi(p)}{\partial n}dS \tag{4}$$

In Equation (4), $G(p,q)$ is the Green's function for bi-dimensional problems, and has the expression, Ziomek (1995):

$$G(p,q)=H_n(\kappa R) \tag{5}$$

In the above equation R is the distance between the observation point $p(x,y)$ of the radiated velocity potential and the source point $q(x,y)$. In this work, the observation points are placed in the cylinder's surface. As $\frac{\partial\varphi(p)}{\partial n}=V_0=1$, it is allowed to write:

$$\frac{1}{2}\varphi(p)+\int_S \varphi(p)\frac{\partial G(p,q)}{\partial n}dS=\int_S G(p,q)dS \tag{6}$$

2. THE SINGULARITY IN THE FUNDAMENTAL SOLUTIONS

When the observation point $p(x,y)$ and the source point $q(x,y)$ are located at the same element and coincide, the distance R between them is null, a singularity in the Green's function $G(p,q)$ occurs. In this work, the surface of the cylinder is discretized using a mesh of constant elements. In order to obtain an expression for the singular integral of the fundamental solution, a change in the coordinate system is made, as shown in Fig. 1.

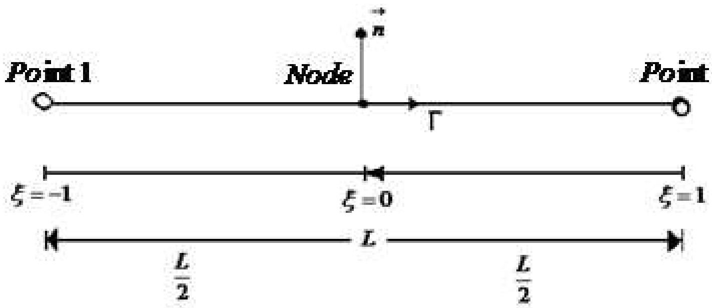

Figure 1- Coordinate system at the elements

The integral of the fundamental solution $G(p,q)$ in a given element of the mesh used in the discretization procedure of the cylinder's surface has the following expression:

$$G_{ii}=\int_{point1}^{point2} G(p,q)dS_i=2\int_{node}^{point2} H_0^1(\kappa R)dR \tag{7}$$

If the substitution $R=\frac{\kappa L}{2}\xi$ is made, Eq. (7) can be rewritten:

$$G_{ii}=\frac{iL}{4}\int_0^1 H_0^1\left(\kappa\frac{L}{2}\xi\right)d\xi \tag{8}$$

The Hankel's function $H_0^1(\kappa R)$ is defined as $H_0^1(\kappa R)=J_0(\kappa R)+iY_0(\kappa R)$. Integrating the functions $J_0(\kappa R)$ and $Y_0(\kappa R)$ an expression for the singular integral G_{ii} is obtained as following:

$$G_{ii}=\frac{L}{2\pi}\left(\left(\frac{i}{2}-\frac{1}{\pi}\left(\gamma+\ln\left(\frac{\kappa L}{4}\right)-1\right)+\sum_{m=1}^{\infty}\frac{(-1)^m(\kappa L)^{2m}}{2^{4m}(m!)(2m+1)}\left(\frac{i}{2}-\frac{1}{\pi}\left(\gamma+\ln\left(\frac{\kappa L}{4}\right)-H_m-\frac{1}{2m+1}\right)\right)\right)\right) \tag{9}$$

In Equation (9), L represents the size of the constant element used to discretize the cylinder's surface. The singular integral G_{ii} can be evaluated by using Eq. (9) or by using the gaussian quadrature method and changing the number of gauss' points used in order to get a more accurate solution for the problem of the acoustic radiation. In this work, two ways for evaluating the singular integral are compared and the variation of the number of Gauss's points at the surface and for points located in the exterior region of the cylinder was used.

2.1. THE INFLUENCE OF THE FREQUENCY IN THE VALUE OF THE VELOCITY POTENTIAL AT THE CYLINDER'S SURFACE

The value of the velocity potential was numerically obtained for points at the cylinder's surface, of radius $R = 1$ by using a mesh of 32 constant elements. Simulations were made for frequencies of 75, 100, 115, 160, 225, 250, 425, 590 and 670 hertz using 2, 4, 6 and 8 gauss' points. The solution of the acoustic radiation problem using the singular integral is compared with the solution obtained by varying the number of gauss' points to treat the singularity problem. The obtained results are presented in Fig. 2.

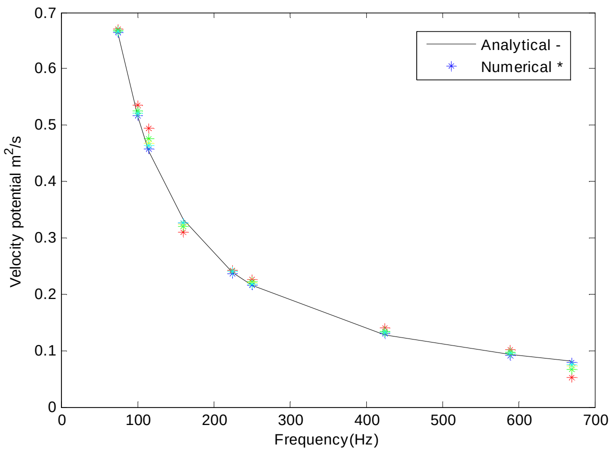

Figure 2- Velocity potential evaluated for several Gauss points

In Figure 2, the analytical expression of the velocity potential is represented by the continuous black line, the velocity potential obtained with the use of the expression for the singularity is represented by the blue line and the values of the velocity potential obtained by 2, 4, 6 and 8 gauss' points are represented respectively by the red, yellow, green and gray lines.

Analyzing the results presented by Fig. 2, it can be noticed that the results obtained for the singular integral are more precise than those obtained with different gauss' points. This conclusion is confirmed by the analysis of the relative error, defined by:

$$e_r = \frac{|\varphi_A - \varphi_N|}{|\varphi_A|} \tag{10}$$

In Equation (10), φ_A and φ_N are, respectively, the analytical and numerical values of the velocity potential in each point. The relative errors for the results obtained by the values of the velocity potential using both the singular integral and varying the number of gauss' points are compared. In this test, the minor and the major value for the frequency range considered in this work were used, which are 75 Hz and 670 Hz. The results are shown in the next figures.

Analyzing the results plotted in Fig. 3 and Fig. 4, it can be observed that the relative error diminishes as the number of gauss' points used becomes greater. However, these errors are always greater than the relative errors obtained with the use of the integral singular expression. An increase of the number of gauss' points establishes a greater distance of the origin in which the singularity occurs, diminishing the relative error.

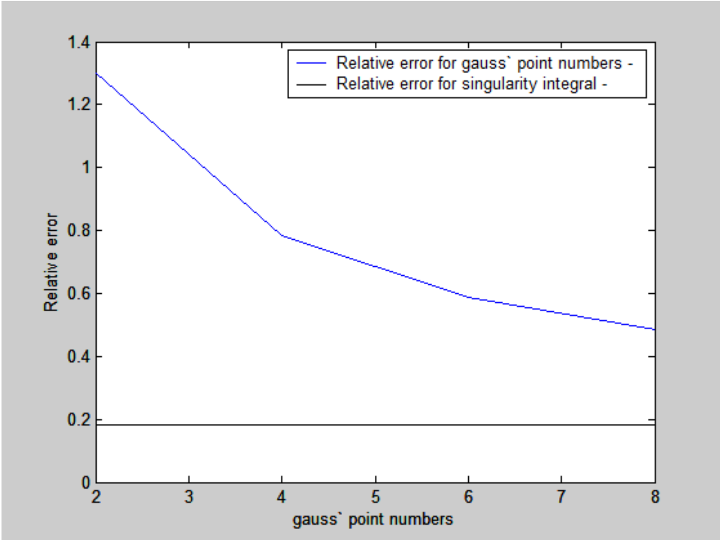

Figure 3- Comparing the relative errors for the frequency of 75 Hz

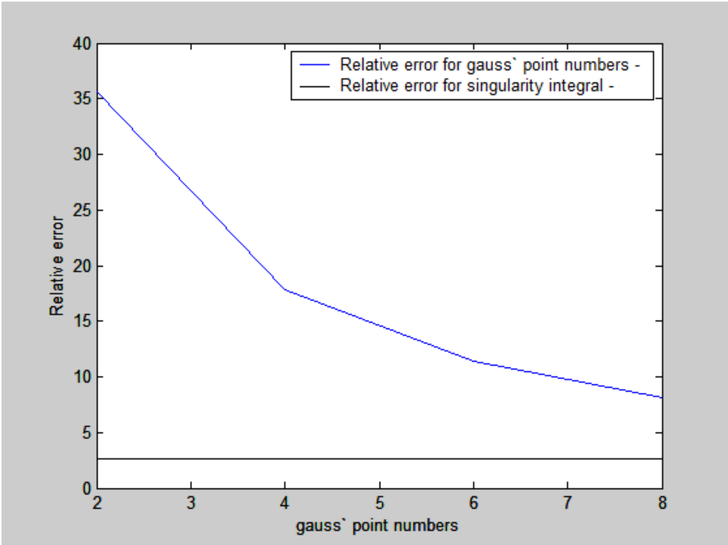

Figure 4 - Comparing the relative errors for the frequency of 670 Hz.

2.2. THE INFLUENCE OF THE CILYNDER'S RADIUS SIZE IN THE VALUE OF THE RADIATED VELOCITY

A further analysis showed how the radiated velocity potential is affected by the variation of the cylinder's radius size. The radius' values chosen for the tests varied in steps of 1 meter, from 1 to 10 m, independently of the frequency. The boundary surfaces were also discretized using a 32 constant elements mesh. The analytical solution for the acoustic radiation problem was compared with the numerical solution obtained with the variation of the number of gauss' points. The results of those computational simulations are presented in the graphics shown in Fig. 5.

In Figure 5 the black line represents the analytical velocity potential; the dark blue line represents the radiated velocity potential got by the use of the singularity expression and the numerical results of the radiated velocity potential, using 2, 4, 6 and 8 gauss' points are respectively represented by the blue, red, green and yellow lines.

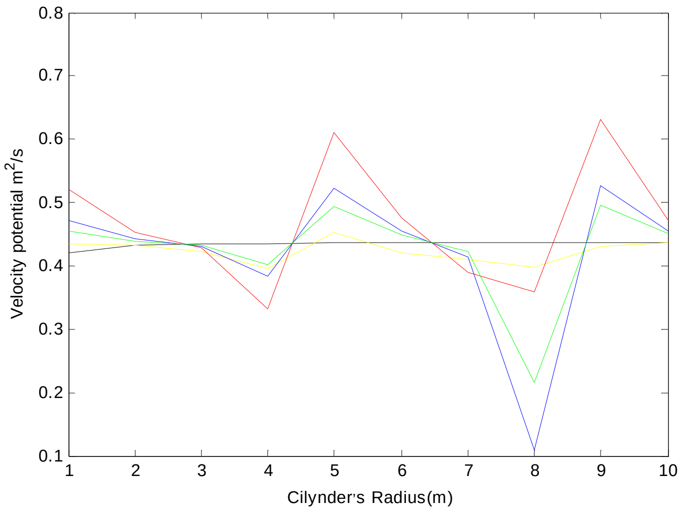

Figure 5. Comparing numerical velocity potential at different values of the cylinder's radius

The analysis of the graphic plotted in Fig 5 shows that increasing the cylinder's radius produces an increasing of the oscillation of the value of the radiated velocity potential. This oscillation occurs due to the increasing of the cylinder's radius size without the corresponding increasing of the number of the elements of the mesh used for discretize the cylinder's surface. However, increasing the number of gauss' points used in the solution of the singular integral reduces the oscillation, making the results more precise. To verify the efficiency of the way of treating the integral singularity, the solution which uses the expression for singular integral was compared with the solution containing 8 gauss' points and the results are presented in Fig. 6.

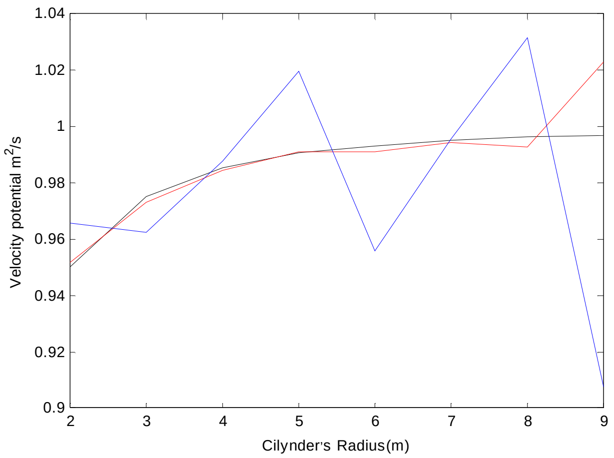

Figure 6: The velocity potential for different values of the cylinder's radius.

In Figure 6, the black line represents the analytical velocity potential, the red line represents the potential got by using the expression for the singularity integral and the blue line represents the value of the velocity potential obtained by using 8 gauss' points.

The analysis of Fig. 6 shows the results obtained by using the singular integral expression are more precise than those got by varying the number of gauss' points.

3. CONCLUSIONS

The computational simulations show that the results obtained by using the singular integral expression are more precise than those got by varying the number of gauss' points. For a fixed value of the radius and varying the frequency, the relative error decreased with the increasing of the number of gauss' points used, but it was greater than the relative error obtained by using the expression of the singular integral. Fixing the frequency and varying the radius, the numerical values of the velocity potential oscillates around the analytical values of the velocity potential as the cylinder's radius increases. However, the velocity potential values obtained by using the singularity expression are closer to those of the analytical values got by varying the number of gauss' points. These conclusions confirm that the expression for the singular integral is more efficient than the variation of the number of the gauss' points to solve the sound radiation problem by using the Boundary Elements Method.

4. ACKNOWLEDGEMENTS

The authors would like to thank CEFET-MG for the opportunity to make this work.

5. REFERENCES

Greco, M., 2000, Análise do problema harmônico de radiação e difusão acústica, usando o Método dos Elementos de Contorno. Dissertação de mestrado, USP, São Carlos.

Maria, M.E, Borges, E.N.M, Afonso, M.M., 2009, "Analytical and numerical study of the acoustical scattering using boundary element method", X Congresso Ibero Latino Americano de Métodos Numéricos Computacionais em Engenharia, Rio de Janeiro.

Morse, P.M, Ingard, K. Uno. 1986. "Theoretical acoustics". Princeton, N.J.: Princeton University Press.

Pappini, G. S. (1999). "Estudo Numérico de Barreiras Acústicas". Dissertação de Mestrado, UFMG, Belo Horizonte

Costa, E, S.(2008). "Modelagem e simulação numérica da radiação acústica de um cilindro pulsante". Dissertação de Mestrado, CEFET-MG, Belo Horizonte, 2008.

Yoon, W.S., Park, J.M., Eversman,W., 1990. Two- Dimensional radiation and scattering at short wave lenght, Journal of Vibration and Acoustics, V. 112,

Ziomek, L.J.,1995., "Fundamentals of acoustic field theory and space-time signal processing", Boca Raton CRC,.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the content of the printed material included in their paper.